Stability of shock wave structures and uniqueness of solution to the Riemann problem for the generalized Hopf equation

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The Korteweg–de-Vries–Burgers (KdVB) equation is proposed as a prototype of partial differential equations with dynamic discontinuities (shock waves). Depending on the relation between the parameters of dispersion and dissipation, the KdVB equation with a special non-convex potential can have a finite number of types of stationary solutions representing structures of special discontinuities (i.e. discontinuities with extra relations following from the demand for structure existence). The number of such special discontinuities of different types grows as the relative influence of dispersion is enhanced as compared with dissipation. This causes multiple non-uniqueness of solutions when these discontinuities are used to construct solutions of self-similar problems [1].

We give detailed analysis of the steady travelling wave solutions from a viewpoint of discontinuities. An Evans function formulation to study the linear stability is used. Integrations of the initial value problem illustrating the instabilities are presented. Relying on the results of [2, 3] we construct a self-similar solution of the problem of arbitrary discontinuity disintegration. This solution consists of only discontinuities with a stable stationary or non-stationary structure. A final criterion is formulated for choosing an admissible discontinuity (discontinuities) to be the result of the Cauchy data evolution in the problem of arbitrary discontinuity disintegration.