Primordial plasma: influence of non-ideality on equilibrium radiation

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The asymptotic behavior of the equilibrium radiation in Maxwellian plasma is investigated for the region of low frequencies. It is shown, that already for a weakly non-ideal plasma the equilibrium radiation can essentially deviate from the Planck law. The deviation from the Planck distribution is described by the transverse dielectric permittivity which takes into account both frequency and spatial dispersion. The influence of plasma non-ideality increases with increase of the non-ideality parameter in the dielectric permittivity. The spectral energy distribution of the equilibrium radiation essentially changes in the region of possibly observable frequencies. At asymptotically low frequencies there is transfer from logarithmic to power behavior of the equilibrium radiation. The results indicate that in the primordial plasma of Early Universe the spectral energy distribution of radiation could be different than the Planck one.

PACS number(s) : 42.50.Ct, 52.25.Os, 52.40.Db, 51.30.+i

Introduction

The spectral energy distribution of the equilibrium radiation established by M. Planck [1] corresponds to an idealized model of an absolutely black body, which exists in a cavity filled with radiation and bounded by an absolutely absorbing substance. It is assumed that the radiation is in thermodynamic equilibrium with the substance, although the effects of the interaction of photons with the substance bounding the cavity are not considered [2].

The practical implementation of the Planck distribution, as a rule, is associated with the consideration of a macroscopic body in thermal equilibrium with the "black" radiation surrounding it [2]. Great success has been achieved in solving this problem, which is directly related to Kirchhoffs law (see in more detail [3-5] and the literature cited there). At the same time, lack of attention has been paid to the question of the spectral energy distribution of equilibrium radiation (SEDER) in a substance (see [6] and the literature cited there). The solution of this problem was mainly restricted to the analysis of transparency regions at low photon momentum. This approach seems to be limited, since it is clear from physical considerations that in order to establish the thermodynamic equilibrium of radiation in matter, it is necessary to take into account the effects of radiation absorption.

Recent investigations have been devoted to the sequential consideration of the effect of an absorbing plasma medium on the spectral energy density of equilibrium radiation (SEDER) in a substance [7-9]. In these works, the consideration was carried out both for a completely equilibrium system of non-relativistic charged particles and photons [7,8] in the quantum electrodynamic (QED) approximation, and based on a generalization of a more traditional approach using the fluctuation-dissipation theorem [9]. In [10, 11], based on [7, 8], the frequency-asymptotic behavior of SEDER was studied. It was found that at low frequencies the SEDER in a plasma medium has a logarithmic (integrable) singularity. In this case, the radiation energy remains finite. However, the works [10, 11] were based on the part of the SEDER that is related to the influence of plasma on the photon distribution function through the transverse permittivity of charged particles. At the same time, as was shown in [12], there is one more contribution to the SEDER, which must be taken into account. This contribution is related to the interaction between intrinsic fluctuations of currents and fields and leads to a contribution of the same order as the one considered earlier. Below, when calculating the low-frequency behavior of SEDER, we use the general approach developed in [10]. The analytical asymptotic based on this approach changes the logarithmic low-frequency asymptotic singularity on more singular, but still integrable $1/\omega^{2/3}$. For simplicity, we restrict ourselves to the consideration of radiation in an electron gas.

I. GENERAL FORMULAE FOR THE SEDER IN ELECTRON GAS

According to [12], the full SEDER in plasma medium $E(\omega)$, which includes the zero fluctuations in plasma is the sum of two terms

$$E(\omega) = E^{(1)}(\omega) + E^{(2)}(\omega) \tag{1}$$

$$E^{(1)}(\omega) = \frac{V\hbar\omega^3}{\pi^2 c^3}\coth\left(\frac{\hbar\omega}{2T}\right)\frac{c^5}{\pi\omega}\int_0^\infty dkk^4 \frac{\mathrm{Im}\varepsilon^{tr}(k,\omega)}{(\omega^2\mathrm{Re}\varepsilon^{tr}(k,\omega) - c^2k^2)^2 + \omega^4(\mathrm{Im}\varepsilon^{tr}(k,\omega))^2}, \tag{2}$$

$$E^{(2)}(\omega) = V\sum_a \frac{\hbar\omega^2\omega_{p,a}^2\coth\left(\frac{\hbar\omega}{2T}\right)}{\pi^3}\int_0^\infty dkk^2 \frac{\mathrm{Im}\varepsilon^{tr}(k,\omega)}{\mid \omega^2\varepsilon^{tr}(k,\omega) - k^2c^2 \mid^2} \tag{3}$$

Here $\varepsilon^{tr}(k,\omega)$ is the transverse dielectric permittivity (TDP) of non-relativistic plasma, which takes into account spatial and frequency dependence of electromagnetic field as well as arbitrary strong interaction between charged particles in the system. However, the TDP for strongly interacted plasma is unknown and we use for further calculations the most accurate form of TDP for the case of a weak Coulomb interaction between charged particles in quasiclassial and quasineutral plasma medium [13]. The value $\omega_{p,a} = \sqrt{4\pi n_a^2 z_a^2 e^2/m_a}$ is the plasma frequency for the particles of species a.

As easy to see Eq. (1) contains also the Planck distribution, which corresponds to the limit of negligible particle density. We have stress that the zero fluctuations in plasma may in principle differ [14] from the vacuum zero fluctuations and the form of these fluctuations strictly speaking, is not known. Both functions $E^{(1)}(\omega)$ and $E^{(2)}(\omega)$ are definitely positive for arbitrary form of TDP and for arbitrary frequencies due to inclusion of zero fluctuations. For the low-frequency region, considering in this paper, zero oscillations are negligible as we show below.

In this work, using the results of analytical and numerical calculations [13] for TDP of almost ideal plasma, we consider the influence of non-ideality on low frequency behavior of the full SEDER (1) when the parameter, characterized the plasma non-ideality (or Coulomb interaction parameter) $\Gamma = e^2n^{1/3}/T$ increases. The concrete form of the TDP $\varepsilon^{tr}(k,\omega)$ which we use for weakly non-ideal and non-relativistic electron gas is determined by the relations

$$\mathrm{Re}\varepsilon^{tr}(X,W)=1-\frac{2\Gamma_e\eta_e^{2/3}}{W^2}-\frac{\Gamma_e\eta_e^{2/3}}{W^2}\left\{\left[\frac{W}{X^2}+\frac{X^2}{2}\right]\times\right.$$
$$\left.\left[{}_1F_1\left(1,\frac{3}{2};-\left(\frac{W}{2X}-\frac{X}{2}\right)^2\right)-{}_1F_1\left(1,\frac{3}{2};-\left(\frac{W}{2X}+\frac{X}{2}\right)^2\right)\right]\right\}+\frac{\Gamma_e\eta_e^{2/3}}{W^2}\left\{\left[1+\frac{X^2}{2}\right]\times\right.$$
$$\left.\left[{}_1F_1\left(1,\frac{3}{2};-\left(\frac{W}{2X}-\frac{X}{2}\right)^2\right)+{}_1F_1\left(1,\frac{3}{2};-\left(\frac{W}{2X}+\frac{X}{2}\right)^2\right)\right]\right\},\quad(4)$$

$$\mathrm{Im}\varepsilon^{tr}(X,W)=\frac{\sqrt{\pi}\,\Gamma_e\eta_e^{2/3}}{W^2}\left\{\frac{1}{X}+\frac{X}{2\pi}\right\}\left\{\exp\left[-\left(\frac{W}{2X}-\frac{X}{2}\right)^2\right]-\exp\left[-\left(\frac{W}{2X}+\frac{X}{2}\right)^2\right]\right\}\quad(5)$$

where $\eta_e = n_e\Lambda_e^3$, $W = \hbar\omega/T$, $X = k\Lambda_e/\sqrt{4\pi}$, $\Lambda_a = (2\pi\hbar^2/m_eT)^{1/2}$ is the thermal de Broglie wavelength for electrons and ${}_1F_1(\alpha,\beta,z)$ is the degenerate hypergeometric function. As is clear the description of electromagnetic fluctuations in presence of plasma medium characterizes not only temperature, as in the case of SEDER for free photon gas, but also the values of plasma density, electron mass and charge through the TDP. Equations (2) and (3) describes TDP in thermodynamic limit when volume V and the full number of particles N_e tends to infinity such a way that the ratio $N/V \to Const$ equals a homogeneous average electron density $n_e = \lim_{N\to\infty,V\to\infty}(N/V)$. It is necessary to stress that non-relativistic form for $\varepsilon^{tr}(X,W)$ can be used only if $\omega < ck$. As known for non-collisional plasma system, corresponding to generalized random phase approximation of relativistic plasma medium, for the region of frequencies $\omega \geq ck$ the condition $\mathrm{Im}\varepsilon^{tr}(X,W) \equiv 0$ is fulfil [15]. Therefore, for $\omega \geq ck$ photons are non-damping, since the phase velocity of wave exceeds the light velocity c. This means integration on k in Eqs. (2), (3) should be splitted into two integrals: from zero to ω/c with $\mathrm{Im}\varepsilon^{tr}(X,W) \equiv 0$ and from ω/c to ∞. As easy to see for asymptotically small ω, considering in this paper only the integral from ω/c to ∞ is essential. Wherein, the lower limit can be taken equal to zero and for non-relativistic electrons Eq. (5) is applicable.

II. ASYMPTOTICAL BEHAVIOR OF THE SEDER AND THE COULOMB INTERACTION INFLUENCE

Let us introduce the dimensionless SEDER $e(\omega)$ determined by the equalities $E(\omega) = (VT^3/\pi^2c^3\hbar^2)e(\omega)$. As is known [10,11] the low-frequency asymptotic of the part $e^{(1)}(\omega)$ of

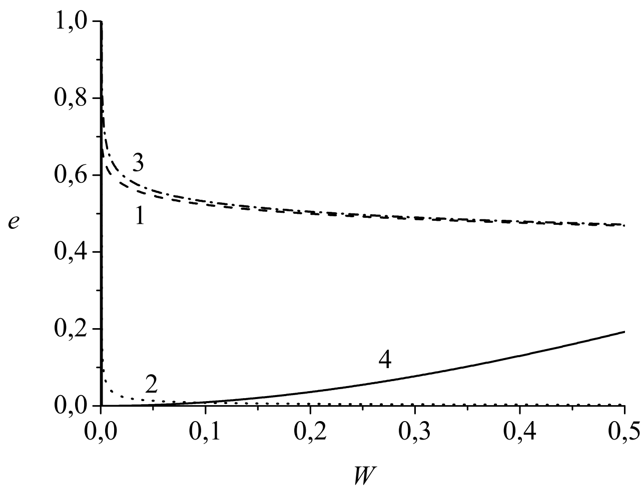

Figure 1: The different terms of the SEDER for $\eta = 0.1$ and $\Gamma = 0.02$. $e^{(1)}(W)$ - the dashed line —1; $e^{(2)}(W)$ - the dotted line —2; the full SEDER in a plasma $e^{(1)}(W)+e^{(2)}(W)$ - the dashed-dotted line —3. Calculations are valid for $W \ll 1$, however formally are continued for the region $W \simeq 0.5$. The Planck SEDER -solid line —4.

the SEDER for the dimensionless variable $W \ll 1$ has the logarithmic singularity

$$e^{(1)}(\omega)\mid_{\omega\to 0}\to \frac{\sqrt{2}\,\Gamma\eta^{2/3}}{3\sqrt{\pi\alpha}}\left[4\ln 4-3\gamma+\frac{6}{\pi}-2\ln(\alpha\sqrt{\pi}\Gamma\eta^{2/3}W)\right]\simeq 91,1\,\eta^{1/3}\Gamma^{2}[28-2\ln(\frac{\eta^{4/3}W}{\Gamma})] \quad (6)$$

where $\gamma = 0,577$ is the Euler's constant. The term $W^3/2$ in $e^{(1)}(\omega)$ corresponding to zero vacuum oscillations is omitted as negligible in $e^{(1)}(\omega)$ in the low-frequency limit. The low-frequency asymptotic $W \ll 1$ of the part $e^{(2)}(\omega)$ of the full SEDER [12] for electron gas can be found analytically by the method similar to that proposed in [10] for the part $e^{(1)}(\omega)$ and reads

$$e^{(2)}(\omega)\mid_{\omega\to 0}\to \frac{4\Gamma^2\eta^{4/3}\sqrt{2\pi\alpha}}{3\sqrt{3}\,(\alpha\sqrt{2}\pi\Gamma\eta^{2/3}W)^{2/3}}\simeq 7.32\frac{\Gamma^{5/3}\eta^{7/9}}{W^{2/3}}, \quad (7)$$

where $\alpha \equiv \mathrm{Ry}\,\eta^{2/3}/\pi m_e c^2\Gamma$ (Ry is the Rydberg constant equals $m_e e^4/2\hbar^2$).

As easy to see from Figures 1 and 2 the Coulomb interaction increase leads to the increase of the SEDER for $W < 0.5$. This value of the SEDER is essentially higher than the Planck for small $W \ll 0.5$. In turn, the term of zero vacuum oscillations $W^3/2$ much smaller than

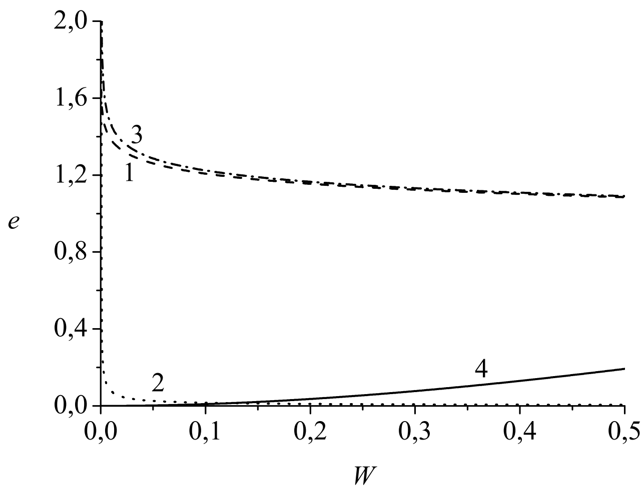

Figure 2: The different terms of the SEDER for $\eta = 0.1$ and $\Gamma = 0.03$. Notations are the same as in Figure 1.

the Planck low frequency representation $e_P = W^3/(\exp W - 1) \simeq W^2$. The last one is small in comparison with the calculated on the basis of Eqs. (6), (7) asymptotical curves of the SEDER as it seen on Figures 1 and 2. In the region of frequencies $W \ll 0.1$ the singular behavior $\sim 1/W^{2/3}$ obliged to the term $e^{(2)}(W)$ is dominant. For larger W the crucial role plays the logarithmic singularity of $e^{(1)}(W)$. The Planck distribution is essential for the frequencies of order $W \simeq 0.3 \div 0.5$ and even more, where the asymptotic relations (6) and (7) are strictly speaking invalid.

III. CONCLUSIONS

In conclusion, we emphasize that the above results take place for homogeneous and isotropic Coulomb system that is in thermodynamic equilibrium with a thermostat at a given temperature T. This system consists charged particles and a quantized electromagnetic field that interact with each other. Note, that the effective use of the QED approach which we use is applied to many problems of statistical physics, including fluctuations in [16]. However, calculation of quantum fluctuations of the electromagnetic field is not enough

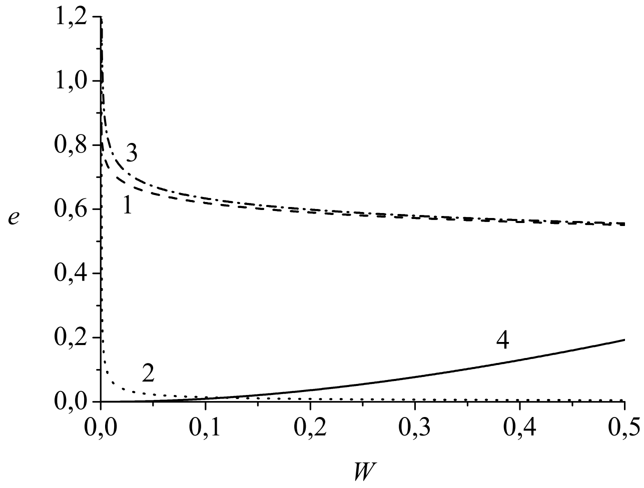

Figure 3: The different terms of the SEDER for $\eta = 0.2$ and $\Gamma = 0.02$. Notations are the same as in Figure 1.

to find the SEDER in plasma medium. We consider the case of non-relativistic and non-degenerate charges under conditions when the accounting of quantum effects in TDP are fundamental for the convergence of the respective integrals at large values of wave vector. In the above general expressions for the spectral energy density of the equilibrium radiation these quantum effects are associated with the spatial and temporal dispersion of the transverse TDP. The explicit analytical expressions for the low-frequency asymptotical regimes for the SEDER are valid for plasma with a small interaction parameter $\Gamma < 1$. Since the results depend on the interaction parameter Γ we can estimate the role of Coulomb interaction on the SEDER. It is shown that increase of the parameter Γ leads to increase of the SEDER at low frequencies, where the role of plasma particles is dominant and the SEDER crucially differs in comparison with the Planck distribution.

In application to cosmic microwave background radiation (CMBR), the considered effects can be essential in observable region of radiation. This problem has been formulated in the simplest approximation for the SEDER neglecting the spatial dispersion of the TDP in [6], [17]. In this approximaion damping is absent, the photon spectrum is equal to

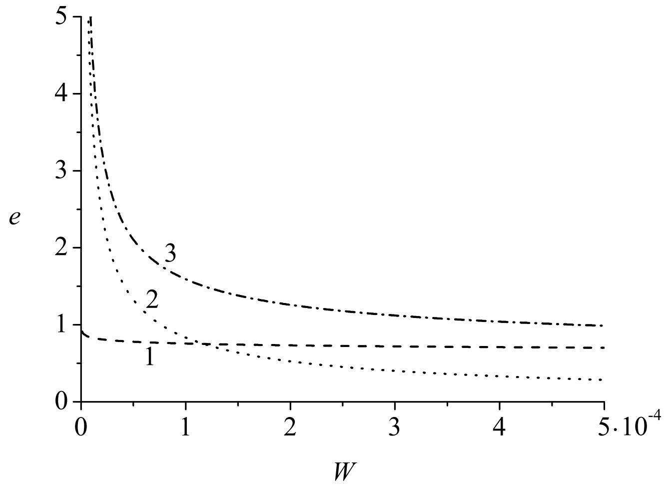

Figure 4: The different terms of the SEDER for $\eta = 0.1$ and $\Gamma = 0.02$ for $W < 5 \cdot 10^{-4}$. $e^{(1)}(W)$ - the dashed line — 1; $e^{(2)}(W)$ - the dotted line —2; the full SEDER in a plasma $e^{(1)}(W) + e^{(2)}(W)$ - the dashed-dotted line —3. For the region of very small $W < 10^{-4}$ the term $e^{(2)}(W)$ exceeds $e^{(1)}(W)$. Planck's curve for the SEDER is negligible for the considering ω and does not shown.

$\omega = \sqrt{\omega_p^2 + c^2k^2}$ and the SEDER turns to zero at $\omega < \omega_p$ [6, 17-19]. Considering the root of equation $\omega^2 \mathrm{Re}\varepsilon^{tr}(k, \omega) - c^2k^2$ or in the dimensionless form $\alpha W^2 \mathrm{Re}\varepsilon^{tr}(X, W) - 2X^2$ we can formally determine the "photon spectrum". For small X the dependence of the root $W(X)$ is close to the case without spatial dispersion. At the same time the SEDER is crucially different from the Planck one due to non-zero $\mathrm{Im}\varepsilon^{tr}(X, W)$.

The conclusion about influence of plasma medium on the different frequency ranges of the equilibrium radiation and opportunity to observe this influence needs to take into account the spatial dispersion of the TDP and interaction between primordial plasma particles into epoch before recombination. Note also, that the calculation of TDP in this work is based on the traditional normalization of the photon distribution function on zero chemical potential. The alternative variants (see, e.g., [19]) can be considered separately in the framework of the developed approach, accounting the spatial dispersion of the TDP. The maximal value of the dimensionless frequency $W = \hbar\omega/T$ is, according to the Planck distribution at $T \simeq 2,72$

K, equals $W_{max} \simeq 2,58$ and lies outside the asymptotical region of frequencies. However, for a fixed frequency, in the hot primordial plasma before the recombination epoch, the equilibrium radiation could be different from the Planck distribution since for any fixed frequency ω the respective dimensionless frequency at the peak W_{max} is shifted to the low-frequency region. This difference can be essential for the description of the Universe evolution.

Acknowledgment

The authors are thankful to A.M. Ignatov and A.G. Zagorodny for the useful discussions. S.Trigger is grateful for support of the general analytical part of this work to the State program for Joint Institute for High Temperatures no. 075-00892-20-00. S.Maslov is thankful to the Ministry of Science and Higher Education of the Russian Federation grant in the framework of which the numerical calculations and application to the primordial plasma has been done.

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