On the admissibility of the discontinuities in the generalized Hopf equation with non-convex flux functions

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We investigate the admissibility of the discontinuous solutions of the generalized Hopf equation

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0, \quad u = u(x,t), \tag{1}$$

in the case of a non-convex flux function $f(u)$. The structure of the discontinuities is described by the generalized Korteweg–de Vries–Burgers equation

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = \frac{\partial}{\partial x}\mu\frac{\partial u}{\partial x} - m\frac{\partial^3 u}{\partial x^3}, \tag{2}$$

where μ is the dissipation coefficient which can be constant or depend on x, t, u and m is the constant dispersion coefficient. Some discontinuities in (1) do not satisfy the Lax condition and are called special. The stability analysis of the structures with monotonic profile is used to confirm their admissibility. If their velocity is known, it can be determined whether other special discontinuities monotonic or not.