High-energy processes in super-strong laser fields

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EMMI, 14-15 May 09, JIHT, Moscow
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Radiation dominated dynamics in Thomson scattering

Classical effect of radiation reaction / radiation damping.

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Can radiation reaction effects be observable in strong laser fields?
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Is the radiation reaction always perturbation?
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Is the radiation reaction always perturbation?

Does exist an interaction regime when the electron dynamics is determined by the radiation reaction?
Radiation dominated dynamics in Thomson scattering

In non-relativistic classical electrodynamics the radiation reaction is perturbation:

\[ m \ddot{\mathbf{v}} = e \mathbf{E} + \frac{e}{c}[v \mathbf{H}] + \frac{2e^2}{3c^3} \mathbf{v} \]

\[ f = \frac{2e^3}{3mc^3} \dot{\mathbf{E}} + \frac{2e^4}{3m^2c^4}[\mathbf{EH}] \]

\[ r_0 \ll \lambda \quad E \ll E_c / \alpha \]

perturbation conditions

\[ r_0 \ll \lambda_c \ll \lambda \quad E \ll E_c \ll E_c / \alpha \]

the realm of the classical physics

\[ \lambda_c = h / mc \]

- Compton wavelength
Radiation dominated dynamics in Thomson scattering

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Lorentz-Abraham-Dirac equation:

\[ mc \frac{du^i}{ds} = \frac{e}{c} F_{ik} u_k + g^i. \]

\[ g^i = \frac{2e^2}{3c} \left( \frac{d^2 u^i}{ds^2} - (u^i u^k) \frac{d^2 u_k}{ds^2} \right). \]

In relativistic electrodynamics the radiation reaction is perturbation in the rest frame.

\[ \gamma E \ll E_c / \alpha \]
Radiation dominated dynamics in Thomson scattering

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\[ f_x = - \frac{2e^4}{3m^2c^4} \frac{(E_y - H_z)^2 + (E_x + H_y)^2}{1 - v^2/c^2} \]


\[ f_{\text{rad}} / f_L \sim \alpha \gamma^2 E / E_c \sim 1 \]

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Radiation dominated regime
Radiation dominated regime in strong laser fields:

\[ R = \frac{2}{3} r_0 \gamma_0 (1 + \beta_0) \xi^2 \omega / c \sim 1 \]

\[ \delta \varepsilon_{rad} \sim \varepsilon_0 \]

\[ \xi = eE / mc \omega \sim 1 \]

\[ \gamma = 300, \xi = 100 (\varepsilon \sim 150 \text{ MeV}, I \sim 10^{22} \text{ W/cm}^2) \]

J. Koga et al. PP 12, 093106 (2005)
Radiation dominated dynamics in Thomson scattering

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\[ \gamma_{\text{drift}} \sim \xi \sim \gamma_0 \gg 1 \]

\[ \delta p_z \sim p_z \]

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Radiation dominated dynamics in Thomson scattering

Exact solution of Landau-Lifshitz equation in a laser field:

\[ R = \left( \frac{2}{3} \right) r_0 \gamma (1 + \beta) \xi^2 \omega / c \]

\[ \xi = eE / mc \omega \]

\[ u^{\mu}(\phi) = \frac{1}{h(\phi)} \left( \begin{array}{c} \gamma_0 + \frac{\omega_0}{2m\eta_0} [h^2(\phi) - 1 + \xi^2 I^2(\phi)] \\ 0 \\ -\beta_0 \gamma_0 + \frac{\omega_0}{2m\eta_0} [h^2(\phi) - 1 + \xi^2 I^2(\phi)] \\ -\xi I(\phi) \end{array} \right) \]

In this expression \( \eta_0 = \omega_0 \gamma_0 (1 + \beta_0) / m \) and

\[ h(\phi) = 1 + R \int_{\phi_0}^{\phi} d\zeta \psi^2(\zeta), \quad E = E_0 \psi(\phi) \]

\[ I(\phi) = \int_{\phi_0}^{\phi} d\zeta \left[ h(\zeta) \psi(\zeta) + \frac{R}{\xi^2} \frac{d\psi(\zeta)}{d\zeta} \right] \]
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\[ R \ll 1 \]

\[ u_y(\phi) > 0 \]

\[ 2R \xi^2 g(\phi) > 4 \gamma^2 - \xi^2 I_0^2(\phi) > 0 \]

\[ I \approx I_0(\phi) + R g(\phi) \]

Radiation dominated dynamics in Thomson scattering

Radiation dominated regime in strong laser fields:

\[ \frac{\delta \varepsilon_{\text{rad}}}{\varepsilon_0} \sim 1, \quad \omega \tau \sim 1 \]

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J. Koga et al. PP 12, 093106 (2005)

Radiation dominated dynamics:

\[ \gamma_{\text{drift}} \sim \xi \sim \gamma_0 \]

\[ \delta p_z \sim p_z \]

at \( R \ll 1 \)

\[ R \sim \left( 4 \gamma_0^2 - \xi^2 \right) / 2 \xi^2 \]

\[ \gamma = 80, \quad \xi = 150 \quad (\varepsilon \sim 40 \text{ MeV}, \quad I \sim 5 \times 10^{22} \text{ W/cm}^2) \]

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Radiation dominated dynamics in Thomson scattering

Electron trajectory

Maximum of intensity at $\theta=80^\circ$

$\gamma=80$ $\xi=150$

($ I \approx 5 \times 10^{22}$ W/cm$^2$)

$\pi \gamma^2 v/\omega_c \approx \rho/\gamma$

Angle resolved radiation spectra

Dependence on the electron position in laser focus

$w_0=2.5 \mu$m

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Radiation dominated dynamics in Thomson scattering

**Electron trajectory**

Maximum of intensity at $\theta=80^\circ$

- $10^9$ electrons in beam
- $10^4$ photons per pulse
- 1% of electrons contribute

**Angle resolved radiation spectra**

Dependence on the electron position in laser focus

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Quantum vacuum is a region of space-time which contains no real particles (electrons, positrons, photons etc). Virtual particles are present:

$$\delta x \sim \lambda_c = h/mc \approx 3.86 \times 10^{-11} \text{ cm}$$
$$\delta t \sim h/mc^2 \approx 10^{-21} \text{ s}$$
Vacuum polarization in laser fields

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$$eE\lambda_c = mc^2$$

$$E = E_c = m^2c^3/\epsilon h = 1.3 \times 10^{16} \text{ V/cm}$$

$$I_c = cE_c^2/8\pi = 2.3 \times 10^{29} \text{ W/cm}^2$$

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\[ \xi = e \sqrt{A_{\mu} A^{\mu} / mc^2} = eE \lambda_c / \omega \]

\[ \xi = 1 / \omega \tau ; \tau = m / eE \]

\[ \chi = \frac{e \sqrt{(F_{\mu \nu} p^{\nu})^2}}{(mc^2)(mc)} \lambda_c = \frac{eE \lambda_c}{mc^2} \bigg|_{r.f.} = \frac{E}{E_{cr}} \bigg|_{r.f.} \quad \text{or} \quad \frac{\Omega}{m} \frac{E}{E_{cr}} \bigg|_{r.f.} \]
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\[ \xi = e \sqrt{A_\mu A^\mu} / m = e E \lambda_c / \omega \]

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\( \chi = 1, I \approx 10^{29} \text{ W/cm}^2 \)

Spontaneous electron-positron pair production

\( \chi < 1 \) vacuum is stable, however electron-positron pair exist virtually during the Heisenberg uncertainty time.

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$$\eta = \chi / \xi = (kk_0) / m^2 = 2 \omega \omega_0 / m^2$$
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Vacuum is polarizable

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\( \chi = 1, \left| I \right| \approx 10^{29} \text{ W/cm}^2 \)

Spontaneous electron-positron pair production \( E/E_c \) and \( \omega/m \)

\( \chi < 1 \) vacuum is stable, however electron-positron pair exist virtually during the Heisenberg uncertainty time.

Vacuum is polarizable
**Light-by-light diffraction**

Euler-Heisenberg Lagrangian density:

\[
L = \frac{1}{2} (E^2 - B^2) + \frac{2\alpha^2}{45m^4} \left[ (E^2 - B^2)^2 + 7(\vec{E} \cdot \vec{B})^2 \right]
\]

Polarization current:

\[
\nabla^2 \vec{E} - \partial_t^2 \vec{E} = \vec{J}; \quad \vec{J} \propto F^3
\]
Interaction of an x-ray beam with a strong standing wave

In far zone the probe diffraction is important

Di Piazza et al. PRL 97, 083603 (2006)
Interaction of an x-ray beam with a strong standing wave

In far zone the probe diffraction is important

In near zone: \(2\varepsilon = \omega l (n_\perp - n_\parallel) \sin 2\theta\)

In far zone: \((D_0 D_p)^{1/2}\) times smaller

\[D_0 = \frac{w_0^2}{z\lambda_p} \ll 1\]

\[D_p = \frac{w_p^2}{z\lambda_p} \ll 1\]

Probes polarization before the interaction

Probes polarization after the interaction

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Enhancement of vacuum polarization effects in plasma

When a strong laser pulse propagates through plasma near the threshold of the plasma transparency the vacuum polarization effects are enhanced.

In the proximity of this singular point $\omega \rightarrow \omega_p$, the plasma refractive index tends to zero, the field increases and the vacuum refractive index becomes more visible.

\[ n^2 = \varepsilon \mu \approx \varepsilon_p + \frac{2\alpha}{45\pi} \frac{E^2}{E_{cr}^2} \left( 1 - \varepsilon_p^2 \right) \]

\[ \varepsilon_p = 1 - \frac{\omega_p}{\omega^2} \]

In plasma:
\[ \varepsilon_p \rightarrow 0 \Rightarrow n_{pl} \approx \sqrt{\frac{2\alpha}{45\pi} \frac{E^2}{E_{cr}^2}} \]

In vacuum:
\[ \varepsilon_p \rightarrow 1 \Rightarrow n_{vac} \approx 1 + \frac{\alpha}{45\pi} \frac{E^2}{E_{cr}^2} \]

\[ n_{pl} \gg n_{vac} \]

Di Piazza et al. PP 14, 032102 (2007)
VPEs in a plasma (approach)

Equations of a two-fluids, cold, collisional and relativistic plasma including VPEs

\[ \partial \cdot \mathbf{E} = -e(N_e - ZN_i) + \rho_{vac}, \]
\[ \partial \cdot \mathbf{B} = 0, \]
\[ \partial \times \mathbf{E} + \partial_t \mathbf{B} = 0, \]
\[ \partial \times \mathbf{B} - \partial_t \mathbf{E} = -e(N_e \mathbf{v}_e - ZN_i \mathbf{v}_i) + \mathbf{J}_{vac}, \]
\[ \partial_t N_e + \partial \cdot (N_e \mathbf{v}_e) = 0, \]
\[ \partial_t N_i + \partial \cdot (N_i \mathbf{v}_i) = 0, \]

VPEs can be described mathematically as a ‘current’ but it contains no particles quantities like velocity. It contains only the electromagnetic field to the third power.

\[ \partial_t \mathbf{p}_e + (\mathbf{v}_e \cdot \partial)\mathbf{p}_e = -e(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \nu_e m_r (\gamma_e \mathbf{v}_e - \gamma_i \mathbf{v}_i) \]
\[ \partial_t \mathbf{p}_i + (\mathbf{v}_i \cdot \partial)\mathbf{p}_i = Ze(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \nu_i m_r (\gamma_i \mathbf{v}_i - \gamma_e \mathbf{v}_e) \]

Collisional effects are important in the regime we are interested in but for simplicity they are neglected here (they can be treated perturbatively).

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A possible (ideal) experimental setup

Laser data: \( \omega = 1 \text{ eV} \)
- \( I_{0,1} = 7 \times 10^{21} \text{ W/cm}^2 \)
- \( I_{0,2} = 3 \times 10^{22} \text{ W/cm}^2 \)

Plasma data:
- \( Z = 46 \) (palladium)
- \( N_{0,1} = 10^{23} \text{ cm}^{-3}, N_{0,2} = 2I_{0,1} \)
- \( L = 100 \mu\text{m}, n_{p0} = 5 \times 10^{-2} \)

Numerical results and comments:

Rotation of laser polarization: \( 6.8 \times 10^{-8} \text{ rad} \) (more than one order of magnitude with respect to the case of diffraction)

Measurable nowadays

Large densities required because we require close to plasma frequency and high laser intensities

\[
\Delta n_{pl} = \frac{\alpha}{45\pi} \frac{E_2^2 - E_1^2}{E_{cr}^2} \left(1 - \frac{n_{pl,0}^2}{2n_{pl,0}}\right)^2
\]

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Photon fusion during laser and proton beam collision

\[ \Omega \approx 2\gamma \omega_L; \quad E \approx 2\gamma E_L \]
\[ \Omega \sim m; \quad E \sim E_{cr} \]

Nonliner QED

\[ \chi = \frac{2\Omega}{m} \frac{E}{E_{cr}} \gg 1 \]

Di Piazza et al. PRL 97, 083603 (2006); PRA 78, 062109 (2008)

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Photon fusion during laser and proton beam collision

\[ \Omega \approx 2 \gamma \omega_L; \quad E \approx 2 \gamma E_L \]
\[ \Omega \sim m; \quad E \sim E_{cr} \]

Nonlinear QED
\[ \chi = \frac{2 \Omega}{m} \frac{E}{E_{cr}} \gg 1 \]

Perturbative:
\[ c_n \sim \chi^{2n} \]

Nonperturbative:
\[ c_n \sim \chi^{2/3} \]

Opening of multiphoton channels:
\[ R_n \sim 1/n^5 \]
Proton beam \hspace{1cm} Laser beam

**Tevatron:** Proton energy 980 GeV; \( N_p = 10^{11} \)

XUV Laser: \( I = 4 \times 10^{22} \) W/cm\(^2\), \( \omega = 70 \) eV

Second harmonic: 500 events/h
4th: 7 events/h

**LHC:** Proton energy 7 TeV; \( N_p = 10^{11} \)

Laser: \( I = 3 \times 10^{22} \) W/cm\(^2\), IR

Second harmonic: 400 events/h
4th: 6 events/h

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Pair creation in counterpropagating laser waves

\[ A = A_0 \left[ \sin(wt - kz) + \sin(wt + kz) \right] = 2A_0 \sin(wt) \cos(kz) \]

Dipole approximation \( \cos(kz) \approx 1 \) and \( B=0 \) is applicable only if

\[ l_c << \lambda \Rightarrow \xi = \frac{eE}{m\omega} >> 1 \]

\[ l_c \sim \frac{m}{eE} \text{ is the pair formation length} \]

For XFEL/Compton radiation sources \( \omega \leq m \) and \( \xi \leq 1 \), the DA is not valid.

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Overview: Pair creation in an oscillating electric field

- Pure two level system due to momentum conservation
  \[ q_0(p) = \frac{1}{T} \int_0^T dt \sqrt{(p - eA(t))^2 + m^2} \]
  \[ m^* \equiv q_0(p = 0) \approx 1.21m \quad \text{for} \quad \xi = 1 \]

- Rabi-oscillations

- Resonances enforced by energy conservation
  \[ 2q_0(0) = n\omega \]

\[ w_n \sim \hat{J}_n(U_p) \]
\[ n = m + U_p \]
\[ U_p = a m\xi^2 \]
\[ n < U \]
\[ \xi \gg 1, \quad w_n \sim e^{-Ec/E} \]

see, e.g. V. S. Popov,
JETP Lett. 18, 255 (1973) etc
EMMI, 14-15 May 09, JIHT, Moscow
The influence of the magnetic-field component

The resonance peaks are shifted and split, due to non-zero photon momentum: For example, \( n = 5 = 3 \) (from left) + 2 (from right) = 4 + 1

\[
\omega = m^* (n_+ + n_-) / 2n_+ n_-
\]

M. Ruf et al. PRL 102, 080402 (2009)

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The influence of the magnetic-field component

\[ \omega = m_+ (n_+ + n_-) / 2n_+ n_- \]
Laser-driven collider

\[ L = \left[ \frac{N_e(N_e - 1)}{a_b^2} + \frac{N_e}{a_w^2} \right] f \]

Luminosity is enhanced due to the coherent component

\[ r = 1 \text{ fm} = 10^{-13} \text{ cm} \]

\[ \varepsilon \sim \frac{\text{ch}}{r} \sim 1 \text{ GeV} \]

\[ L \sim 10^{26} - 10^{27} \text{ cm}^{-2} \text{ s}^{-1} \]

Laser wakefield accelerators? \[ L \sim 10^{21} \text{ cm}^{-2} \text{ s}^{-1} \]

B. Henrich et al. PRL 93, 013601 (2004)
K. Hatsagortsyan et al. EPL 76, 29 (2006)

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Laser-driven collider

Short recollision time ~ T/2
Wave packet spreading is not large: \( a_0 < 4a_B \)
Scattering energy: \( mc^2 \xi \)

Coherent collisions with Ps: \( N_{Ps} < (a_b/a_w)^2 \sim 10^{11} \)
Reaction events per pulse: \( 10^{-7} \) at \( N_{Ps} = 10^7 \); \( n=10^{15} \) cm\(^{-3}\)
\( 10^{-4} \) at \( n=10^{18} \) cm\(^{-3}\)  
One reaction event per sec at \( f=1 \) kHz
Eff. Luminosity: \( L_{eff} = 10^{24}-10^{27} \) cm\(^{-2}\)s\(^{-1}\)

Incoherent collisions with e+e- plasma:
Reaction events per pulse: \( 10^{-9} \) at \( n = 10^{15} \) cm\(^{-3}\); \( \tau=30 \) fs  

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Conclusion

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Radiation dominated dynamics below $R << 1$
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Diffraction decreases the ellipticity due to vacuum polarization

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Radiation dominated dynamics below $R<<1$

Diffraction decreases the ellipticity due to vacuum polarization

Enhancement of the visibility of vacuum polarization effect in plasma

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Radiation dominated dynamics below $R \ll 1$

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Photon merging in laser-proton beam collision

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Laser-driven collider; muon production

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Thank you for your attention