INFLUENCE OF ENERGY RELEASE IN NONEQUILIBRIUM MEDIUM UPON THE STRUCTURE OF SWIRLING FLOWS

Nikolay Vinnichenko¹, Alexey Osipov², and Alexander Uvarov²

ABSTRACT

This paper is devoted to modification of parameters of a single columnar vortex or a swirling flow in a pipe due to energy release from the internal degrees of freedom in nonequilibrium medium. The problem is addressed using direct numerical simulations by first- and second-order Godunov methods. For axisymmetric evolution of a free columnar vortex analytical solutions are also found for limiting cases of quick and slow relaxation. It is shown that in the result of initial excitation of internal degrees of freedom a columnar vortex is transformed into a new stationary state, it does not disappear or break down. The process of vortex evolution includes relaxation of the initial excitation, propagation of the wave, which takes away extra mass, and (in non-axisymmetric case) formation of a spiral and falling of the hot gas region towards the vortex centre. Also the development of Rayleigh-Taylor instability is possible. In the case of a swirling flow in a pipe heating results in slowing down of the axial flow and dramatic vorticity decrease in the vortex core, whilst vorticity increases at the vortex periphery. This result can be interpreted as a vortex breakdown in a pipe.

Keywords: Swirling flows, energy release, direct numerical simulation, compressible fluid, plasma aerodynamics

INTRODUCTION

The swirling flows are ubiquitous in nature and technical applications. Frequently they interact with energy releases of various kinds: chemical energy release in combustion, or energy release from internal degrees of freedom in gas discharge, or Joule heat in electrohydrodynamics, or the heat, associated with a phase transition, or simply energy release from some heating device. Swirl is often used to stabilize combustion in burner devices and inside the combustion chambers of jet engines (see the book by Gupta et al. (1984)). Using nonequilibrium state of the medium created with the discharge is often proposed as a flow control tool in aviation. It is desirable to control the flow inside the engines, the tip vortices at the wings and the length of the vortical wake behind the aircraft using non-mechanical techniques, first of all, thermal and electrical ones. There is even an impressive example of vortex—energy release interaction in nature: formation of a tropical storm, which is presumably driven by the influence of the heat from condensation upon the structure of a cyclone.

¹ Corresponding author: Moscow State University, Faculty of Physics, e-mail: nickvinn@yandex.ru
² Moscow State University, Faculty of Physics
Despite the practical importance, little is known about the influence of nonequilibrium energy release (and even simple heating) upon the parameters of vortices and swirling flows. Since vortices are essentially subsonic flows (the axial velocity may exceed the speed of sound, but the azimuthal component is always subsonic), most studies use incompressible fluid approximation. It can yield the influence of the flow structure upon the temperature distribution in the flow, but not vice versa. As heating results in mass transfer which is neglected in incompressible fluid approximation, it is necessary to use the complete system of equations for compressible fluid for a correct description of heating effects on parameters of the flow.

Vortex breakdown is often mentioned in the context of vortical flow parameters modification due to heating. However, it should be stated that vortex breakdown takes place only in swirling flows (vortices with axial velocity), where a zone of slow axial flow can be created by the energy release. Dramatic change of azimuthal velocity is associated with these changes of axial flow structure, and thus is impossible in columnar vortices without axial velocity. In the latter case only changes of azimuthal velocity due to mass redistribution (taking into account total angular momentum conservation) are possible.

First, the most simple flow configuration — a columnar vortex — will be considered. Since there is no axial velocity for this configuration, which can provide heat removal, constant or periodic heating (or energy pumping into internal degrees of freedom) would result in gradual heating of the whole fluid volume, which is physically irrelevant. Therefore, we will consider the evolution of initial excitation of the internal degrees of freedom in some spot, the center of which can either coincide with the vortex center (axisymmetric case) or not (nonaxisymmetric case). Also, the case of finite length of excitation region along the vortex axis and possible development of Rayleigh-Taylor instability for periodic energy pumping in annular region will be considered. Then we will address the problem of the influence of local heating upon the structure of swirling flow in a pipe. The presence of axial velocity and a wall results in quite different behavior of the flow. This investigation is still under way, but some of the first results will be given here.

EVOLUTION OF A COLUMNAR VORTEX DUE TO INITIAL EXCITATION OF INTERNAL DEGREES OF FREEDOM

Problem Formulation

Evolution of the vortex is governed by hydrodynamics system of equations (1) complemented by one equation of relaxation of internal degrees of freedom. We do not consider detailed kinetics model because it is usually specific for some particular conditions and suffers from the lack of reliable data about reaction rates (relaxation times). Our goal is to understand the basics of vortex evolution from the initial nonequilibrium state, so we pay more attention to gas dynamics and model kinetics with just one effective relaxation process. One of the advantages of this approach is that the results become generally applicable to problems, involving vortices interacting with other types of energy release (combustion problems, tropical storm formation, etc.). Since the parameters of vortex are changed by the sound waves (or shock waves) due to energy release, the process of vortex evolution is quick and the influence of viscous dissipation is negligible for typical vortex parameters. Moreover, analytical solutions, which will be presented below, are obtained for inviscid fluid. Numerical simulations for viscous fluid showed that viscous dissipation and influence of energy release are independent: very small vortices die out because of viscous dissipation before the influence of energy release becomes remarkable, so in this case there is no vortex left which can be modified by the energy release. For these reasons the results will be presented for inviscid fluid only. Equations for axisymmetric case and infinitely long excitation region (in polar coordinates) are given here for brevity, in the case of finite excitation region length there is also
momentum equation for $z$-axis (parallel to vortex axis), in nonaxisymmetric case and in simulations of Rayleigh-Taylor instability the problem is treated in cartesian coordinates.

$$\frac{\partial \rho}{\partial t} + \frac{\rho v_r}{r} + v_r \frac{\partial \rho}{\partial r} + \rho v_r \frac{\partial v_r}{\partial r} = 0$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_r^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

$$\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_r v_\phi}{r} = 0$$

$$c_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} \right) - \frac{1}{\rho} \left( \frac{\partial p}{\partial t} + v_r \frac{\partial p}{\partial r} \right) = \frac{\varepsilon - \varepsilon_{eq}(T)}{\tau}$$

$$\frac{\partial \varepsilon}{\partial t} + v_r \frac{\partial \varepsilon}{\partial r} = -\frac{\varepsilon - \varepsilon_{eq}(T)}{\tau}$$

$$p = \frac{R}{\mu} \rho T.$$
where \( \hbar \sigma \) is energy quantum, \( m \) is mass of a molecule.

The following boundary conditions were used:

- at the axis \( v_r = v_\phi = \frac{\partial T}{\partial r} = \frac{\partial \rho}{\partial r} = \frac{\partial \epsilon}{\partial r} = 0 \),
- at the outer boundary \( v_r = \frac{\partial^2 v_\phi}{\partial r^2} = \frac{\partial^2 T}{\partial r^2} = \frac{\partial^2 \rho}{\partial r^2} = \frac{\partial^2 \epsilon}{\partial r^2} = 0 \).

The evolution of single columnar vortices in nonequilibrium medium was investigated by Soukhomlinov et al. (2005) and by Zavershinskii et al. (2009). Both studies used conception of stationary (for motionless fluid) nonequilibrium medium, where energy release (density-dependent or temperature-dependent) from the internal degrees of freedom is compensated by constant heat loss. Unfortunately, some simplifications were made in both cases concerning compressibility of the fluid. The obtained results — fast disappearance of the vortex and its transformation into diverging or converging flow with axial velocity — seem to be caused by these oversimplifications. Detailed discussion of these results and a correct description of vortex evolution in a stationary nonequilibrium medium can be found in our recent paper (Vinnichenko et al., 2009). The present model of an initial locally nonequilibrium state of the medium is more realistic one, the level of excitation is not constant any more, but allowed to change according to relaxation equation.

**Results for Axisymmetric Case**

The problem was solved using first order Godunov method (see the book by Kulikovskii et al., 2000) with exact Riemann solver. Nonuniform 500-points grid was used in most calculations, the grid step varied from 0.02 near the axis to 0.14 near the outer boundary \( r = 40 \) (hereafter the distances are given in typical radii of the vortex). The initial azimuthal velocity profile corresponds to Gauss vortex:

\[
v_{\phi 0}(r) = v_\phi r \exp \left( \frac{1 - r^2}{2} \right)
\]  

(4)

First of all, relaxation of the initial excitation takes place. It may be quick or slow, depending on the value of relaxation time. If relaxation time is small (quasi-thermal case), relaxation takes place so quickly that only temperature and pressure change, the other variables are not modified. Then, according to imbalance between the pressure gradient and centrifugal force, a wave is formed which propagates away from the vortex axis, taking away extra mass. Since the total angular momentum is conserved, mass transfer from the vortex axis results in modification of azimuthal velocity profile. After the wave leaves, the vortex remains in a new stationary state with modified parameters. Possibility to divide in time relaxation and propagation of the wave for the case of small relaxation time allows to construct analytical solution describing the process of vortex evolution. It is constructed in the form of Fourier-Bessel expansion analogous to the case of stationary nonequilibrium medium (see Vinnichenko et al., 2009). The expression is too lengthy, so it is not given here. It follows from the analytical solution that the final change of azimuthal velocity is proportional to initial vorticity distribution and the proportion factor is negative. Therefore, Gauss vortex, which has a core of positive vorticity, surrounded by an annulus of negative vorticity, slows down in the core and is accelerated in the peripheral region. Comparison of analytical and numerical solutions for an intermediate time moment, when the wave is already formed and propagates away, is shown in Fig. 1. Mach number is 0.28, temperature of undisturbed medium \( T_0 \) is 300 K, \( \Delta T_{\text{int}} \) is 400 K, \( D = 3 \), relaxation time is \( 10^{5} \) s. Values of \( v_r \) and \( v_\phi' \) are normalized using \( v_0 \), values of density — using \( \rho_\infty \), values of pressure — using \( \rho_\infty v_0^2 \), temperature is given in kelvins, time — in seconds, \( r \) — in radii of initial vortex. Solid line corresponds to analytical solution, circles — to numerical one. (To obtain a quantitative criterion of “quick or slow” relaxation, one can find the ratio
of typical time scales: relaxation time $\tau$ and the time of wave propagation $R_{\text{vor}} / c_s$, where $R_{\text{vor}}$ is the vortex radius and $c_s$ is the speed of sound.)

$$\frac{v_R}{v_R} = \frac{R_{\text{vor}}}{v_R}$$

**Fig. 1.** Radial profiles of a) change of azimuthal velocity, b) radial velocity, c) density, d) pressure, e) temperature, f) temperature of internal degree of freedom.

If relaxation is slow, the forming wave is much more weak, because the energy has enough time to be redistributed. Relaxation takes place simultaneously with propagation of the wave, so it is impossible to divide two processes in time. Nevertheless, it is possible to construct another analytical solution, taking into account that the wave is weak and the pressure remains almost the same during the whole process. It appears that the final state does not depend on the value of relaxation time: weak waves in the case of slow relaxation eventually take away the same mass and result in the same change of azimuthal velocity as the intense wave formed in the case of quick relaxation. This result becomes even more impressive if we simulate a real energy distribution in gas discharge: part of the energy ($\sim 25\%$) is quickly thermalized, the remaining part goes to internal degrees of freedom and relaxates relatively slowly. Despite the small amplitude of the waves driven by slow relaxation of $75\%$ of energy (it is so small that these waves are not observed in the experiments and often are simply neglected), these waves cause larger change of azimuthal velocity than the initial wave due to direct thermalization of $25\%$ of energy.

**Finite Length of Excitation Region**

One of the simplifications that we have used above is supposition that the initial excitation depends only on $r$, i.e. it is uniform along the vortex axis. Though it is usual for gas discharge chambers that the discharge is bounded in axial direction only by the walls of the chamber, local excitation is much more interesting for aviation applications. In this subsection we will see what additional effects appear if we consider the same problem taking into account $z$-axis and using the initial excitation of the form

$$T_{\text{int}}(r,z) = T_\infty + \Delta T_{\text{int}} \exp\left(-\frac{r^2}{D_z^2}\right) \exp\left(-\frac{z^2}{D_z^2}\right)$$

(5)

instead of (2). The problem was solved using second order Godunov method (Kulikovskii et al., 2000) with van Leer TVD-limiter. The first order method appeared to possess too large numerical viscosity for
solving problems on a 2D-grid. 200x100-points grid was used for calculations, a supplementary layer of 15 points in each direction with rapidly increasing step provided the absence of reflection at the outer boundaries.

Temporal evolution of distributions of azimuthal velocity, pressure and density in meridional plane \( r-z \) is shown in Fig. 2. \( v_0 = 20 \) m/s, \( R_{vor} = 0.1 \) m, \( \Delta T_{int} = 2700 \) K, \( D = D_z = 1 \), \( \tau = 3 \cdot 10^{-5} \) s. Values of velocity are given in m/s, values of pressure — in Pa, values of density — in kg/m\(^3\), time is given in seconds.

\[
\begin{align*}
\text{Fig. 2. Snapshots of distributions of a) azimuthal velocity (m/s), b) pressure (Pa), c) density (kg/m}^3)\text{ for different time moments. t=0.0045 s corresponds to the moment, when the primary wave leaves the domain, t=0.09 s — to the final state.}
\end{align*}
\]

One can see that due to finite length of excitation region when the primary wave leaves the domain (second column in Fig. 2) there is a non-zero pressure gradient along the vortex axis. Up to this stage the solution corresponds to 1D-problem solution for each \( z \) (so, the azimuthal velocity is decreased in excitation region), but non-zero axial pressure gradient results in a secondary flow: gas moves upwards and downwards from the excitation region along the vortex axis and, to keep the radial pressure gradient, from the periphery towards the axis. The mass transfer is reflected by evolution of the shape of low-density region — initially circular, it elongates along the vortex axis. The radial movement of gas towards the vortex axis results in increase of azimuthal velocity. Since the pressure in the final state does not depend on \( z \) and the stationary state satisfies \( \partial p / \partial r = \rho v_{\phi}^2 / r \), the value of \( \rho v_{\phi}^2 \) does not depend on \( z \) and thus is equal to the value for undisturbed vortex. Hence, if density decreases, azimuthal velocity is eventually increased. So, one can state that for a finite excitation region the effect has the opposite sign due to secondary flow.

**Nonaxisymmetric Case: Spiral Formation and Falling to the Center**

If the center of excitation spot does not coincide with the center of the vortex, the first stage — relaxation of the excitation and propagation of the wave — take place just as in axisymmetric case. The difference is that the hot gas region, which finally remains at the place of initial excitation, is not in the center of the vortex in this case. Two new effects arise: first, the hot gas region is transformed into spiral,
and secondly, it falls towards the vortex center. The reason of spiral formation is purely kinematic: in the vortex the angular speed of rotation is less for points situated further from the axis. Any spot contains points which are situated more close to the axis and rotate faster and the others which fall behind. Thus, any passive spot in the vortex velocity field is transformed into a piece of a spiral, bounded by minimal and maximal distances from the axis, which must remain the same if the spot is really passive. But for a considerable initial heating, the hot gas region is not passive: it has low density, which affects the velocity field. Since the radial pressure gradient does not change much and the centrifugal force is decreased because of decrease of density, the balance between the pressure gradient and the centrifugal force is violated and the hot light gas falls to the center of the vortex. The process is analogous to rising of a light bubble in the gravity field, even the shape of the bubble with two side tails looks the same. Depending on the value of initial excitation, falling to the center may be quick, then the spiral does not have enough time to be formed. Density evolution in two different cases is shown in Figs. 3 ($\Delta T_{\text{int}} = 2000\, \text{K}$) and 4 ($\Delta T_{\text{int}} = 2700\, \text{K}$). In the first case (Fig. 3) falling is slow and a pronounced spiral shape can be seen, whereas in the second case (Fig. 4) falling to the center is much more quick.

Fig. 3. Density field ($\text{kg/m}^3$) evolution for $\Delta T_{\text{int}} = 2000\, \text{K}$: spiral formation

Fig. 4. Density field ($\text{kg/m}^3$) evolution for $\Delta T_{\text{int}} = 2700\, \text{K}$: falling to the center

Finally, the hot gas mixes in the central part of the vortex, and the final state of the vortex appears to be axisymmetric. Presumably, formation of the spiral, which has large perimeter, and final mixing must lead to increase of the role of dissipative processes. This hypothesis is supported by the results of numerical simulations, but unfortunately, the numerical viscosity is too large to show this quantitatively.

**Development of Rayleigh-Taylor Instability**

Another effect, which should be considered for a heated vortex, is possible development of
Rayleigh-Taylor instability. It is well-known (see e.g. Sipp et al., 2005) that the vortices with heavy core (i.e. with negative radial density gradient) are unstable. The instability is analogous to classical Rayleigh-Taylor instability of two-layer configuration, when the heavier fluid is situated above the lighter one. In a vortex the role of gravity is played by centrifugal force and “above” means “closer to the axis”. The negative radial density gradient may be created if one heats an annular region of the fluid, concentric with the vortex. The forming waves take the mass away from the heated annulus and it becomes lighter than the central part of the vortex. Thus, the conditions necessary for instability development may be created. However, development of instability without any special eigenmode disturbance of a finite amplitude is a long process, it takes several dozens of periods of vortex rotation. The negative density gradient has to be maintained during this time. So, instead of initial excitation in this case we simulated a periodic energy pumping into internal degrees of freedom. It results in gradual heating of the whole vortex, but the maximal achieved temperature was only 620 K, much lower than typical temperatures in gas discharge. So, the energy necessary for development of Rayleigh-Taylor instability is rather moderate. Temporal evolution of density field is shown in Fig. 5. One can see that the mode with azimuthal wave number 2 appears to be dominant, though at early stages also mode with azimuthal wave number 3 can be observed. The vortex transforms into elliptical one with two spiral arms. If energy pumping is stopped, falling to the center and mixing take place, and the vortex becomes circular once again.

Fig. 5. Development of Rayleigh-Taylor instability: temporal evolution of density field (kg/m³)

INFLUENCE OF LOCAL HEATING UPON THE STRUCTURE OF SWIRLING FLOW IN PIPE

It was shown above that one should spend a lot of energy (in other words, to provide considerable heating) in order to obtain remarkable changes of azimuthal velocity for a columnar vortex. Here we will see that the heating can be more effective for a swirling flow. There are two main differences from a columnar vortex:

- presence of axial velocity,
- presence of side wall.

Presence of axial flow allows to use constant or periodic heating, still obtaining stationary or periodic flow regimes. Besides, complemented with the presence of the wall, it leads to possible vortex breakdown even without heating. The problem becomes much more complex (especially, stability issues), but new opportunities arise associated with vortex breakdown control using moderate energy release. Of course, it is tempting to benefit from relatively unstable flow regime, which can be controlled by moderate resources. The third difference from a free columnar vortex is that the swirling flow in a pipe can be realized and measured experimentally. So, this is also a possibility to check the results of simulations. The
influence of energy release upon parameters of a swirling flow was investigated by Kazakov (1998). It was shown that the energy release can lead to decrease of the axial velocity and a considerable decrease of azimuthal velocity. Since the flow was bounded not by a pipe wall, but by a cocurrent flow, no true vortex breakdown phenomena could be observed without energy release. Moreover, only the parabolic system of equations for stationary flows was considered, hence no recirculation zones (with negative axial velocity) could be obtained. So, these results should be considered as a benchmark for further investigations.

The flow is supposed to be axisymmetric, governed by Navier-Stokes equations for compressible fluid in cylindrical coordinates. Correct formulation of boundary conditions appears to be very important. First of all, there is a problem of boundary layer at the wall. For high Reynolds numbers, which are typical for the experiments, the boundary layer is very thin. A complete description of boundary layer requires grid refinement, which results in dramatic increase of calculation time. Since the internal structure of boundary layer is not important for the considered problem, it is preferable to use simplified description of the boundary layer. At the moment, we use following boundary conditions at the wall surface: 

\[ v_r = \partial v_z / \partial r = \partial v_\varphi / \partial r = \partial T / \partial r = 0, \quad \partial p / \partial r = \rho v^2 / r. \]

It is not an ideal choice, because it actually eliminates friction at the wall surface, allowing fluid to slip freely. The result is the absence of vortex breakdown without energy release. So, further improvement of the boundary condition is necessary. Next, the inlet boundary conditions are prescribed as follows: 

\[ v_z = v_{z0}(r), \quad v_\varphi = v_{\varphi0}(r), \quad \partial v_z / \partial z = 0, \]

\[ T = T_0, \quad \partial p / \partial z = 0. \]

Here \( v_{z0}(r) \) and \( v_{\varphi0}(r) \) are fixed distributions of velocity components at the inlet which can be obtained experimentally. At the outlet convective boundary conditions of the form 

\[ \partial A / \partial t + v_z \partial A / \partial z = 0 \]

are used for density, pressure and velocity components. At the axis we use 

\[ v_r = v_\varphi = \partial v_z / \partial r = \partial \rho / \partial r = \partial T / \partial r = 0. \]

The use of boundary condition \( \partial p / \partial z = 0 \) instead of a fixed pressure profile at the inlet is due to the fact that heating may change the pressure even in points situated upstream of the heating region. In a real experiment this may depend on apparatus, which creates the swirling flow. However, it is quite complex and can hardly be modeled. Hence, we allow pressure to change freely, though an experimental verification of this boundary condition is necessary.

The problem is solved on a 100x100-points grid by second order Godunov method. The radius of the pipe is 17.6 mm, its length is equal to 5 radii, radial profiles of axial and azimuthal velocity components at the inlet are taken from experiment by Klimov and Moralev in High Temperature Institute of RAS, the maximal values are reached near the wall and are equal to 15 m/s for axial velocity and 19 m/s for the azimuthal component. The center of the heating region is situated at the axis, 3 radii downstream from the inlet, the length of the heating region is 1 radius of pipe and its typical radius is 0.2. The heating power is \( 3 \times 10^8 \) Wt/kg. With boundary conditions specified above, the stationary state without energy release is almost columnar. When the heating is on, the axial velocity is decreased in the heating region, which agrees well with results by Kazakov, azimuthal velocity is also decreased and the pressure is decreased in the whole computational domain (Fig. 6). The axial component of vorticity is redistributed: it is damped near the axis and is increased at the vortex periphery. The intensity of the vortex decreases, whereas its effective radius increases. Such phenomena are usually referred to as vortex breakdown though there is no strict definition for this term. It should be noted that the situation is surprisingly different from 1D-flow without swirl, which is accelerated by heating according to conservation law \( \rho v = \text{const.} \) The pressure is also increased in 1D-flow. Obviously, this difference is associated with the effect of swirl and must vanish when azimuthal velocity is decreased. It should be noted also that deceleration of the axial flow by heating in a swirling flow leads to a positive feedback between the heating and decrease of axial velocity: more heating results in stronger deceleration, which causes extra heating because the time it takes for a fluid particle to pass the heating region increases and the fluid gets more heat from the same power source. For large heating power this positive feedback may cause
formation of recirculation zones and instability of the flow due to overheating. Anyway, it enhances the efficiency of changing flow parameters by energy release.

Fig. 6. Stationary distributions of a) axial velocity (m/s), b) azimuthal velocity (m/s), c) radial velocity (m/s), d) axial vorticity (s⁻¹), e) pressure (Pa), f) temperature (K) in meridional plane $r-z$ without and with energy release. The flow is from bottom to top.
CONCLUSIONS

The problem of vortex modification by a local energy release was considered for a columnar vortex without axial velocity and for a swirling flow in a pipe. It was shown that a columnar vortex is transformed into a new stationary state in the result of relaxation of nonequilibrium initial state, it does not disappear or break down. The central part of the vortex has high temperature and low density in the final state. There is certain modification of azimuthal velocity (increase in the case of finite length of excitation region along the vortex axis), but the efficiency of vortex modification by energy release appears to be low. It was shown that the result of vortex evolution does not depend on the value of relaxation time, though for slow relaxation the process takes much more time and only very weak waves can be observed. Different effects were analyzed for axisymmetric and nonaxisymmetric initial excitation: relaxation of the initial excitation, propagation of the wave, transformation of hot gas region into spiral and its falling towards the vortex center. Also, the possible development of Rayleigh-Taylor instability was demonstrated. It was shown that efficiency of vortex modification may be higher in the case of swirling flow in a pipe due to the presence of axial velocity and possibility of vortex breakdown. First results, though incomplete, clearly show that local heating can cause slowing down of axial flow (in contrast to the case of 1D-flow) and consequent modification of vorticity structure. This can provide opportunity to control the vortex breakdown using local moderate heating. Some problems concerning the correct formulation of boundary conditions for a compressible swirling flow in a pipe were also discussed.

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