



## **EXPERIMENTS ON THE STABILITY OF TAYLOR-COUETTE FLOW WITH RADIAL TEMPERATURE GRADIENT**

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### **ABSTRACT**

The flow between two concentric cylinders with the inner one rotating and with an imposed radial temperature gradient is studied using Digital Particle Image Velocimetry (DPIV) method. Four models of the outer cylinder without and with different numbers of slits(6, 9 and 18) are considered, and the radius ratio and aspect ratio of each models were 0.825 and 48, respectively. The flow regime in the Taylor-Couette flow was studied by increasing the Reynolds number. The results showed that smaller number of slits have no obviously effect on the transition process, which only change the shape of the vortex, and that the transition to turbulent Taylor vortex is accelerated as the number of slit increases in both isothermal and non-isothermal conditions. It is also shown that the presence of temperature gradient increased the flow instability obviously as the Froude number larger than 0.003.

**Keywords: Taylor-Couette flow, Slit wall, DPIV, Constant temperature gradient, Froude number**

### **INTRODUCTION**

The flow between two concentric cylinders with the inner one rotating or both rotating, called Taylor–Couette flow, which has been first studied by Taylor(1923). Since then, many researchers have studied the instability causing Taylor vortices. Different methods were given for solving this eigenvalue problem by Jones(1985), Wereley and Lueptow (1998), Rigopoulos and Sheridan(2003), Marques and Lopez(2006). These studies have been performed under plain wall and isothermal conditions. A few works have been addressed on the stability of the cylinder's wall shape and non-isothermal conditions. Especially little experimental work was done. Lee and Kim (2007) studied Taylor-Couette system with different numbers of slits in the outer cylinder by using DPIV method. However, their study was limited to the isothermal conditions. The temperature of the two rotating cylinders need not remain the same in many engineering problems.

Lepiller et al.(2008) studied the influence of radial temperature heating on the stability of the circular Couette flow, in their experiment, the Grashof number varied from -1000 to 1000, and they found that a radial temperature gradient destabilizes the Couette flow leading to a pattern of traveling helicoidal vortices occurring only near the bottom of the system. They also found that the size of the pattern increases as the rotation frequency of the cylinder is increased. Takhar et al. (1988) studied the

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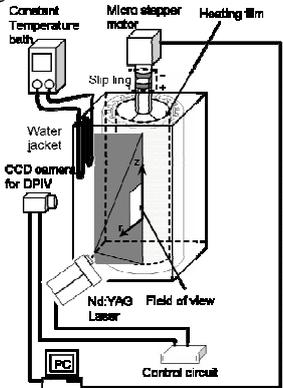
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effects of radial temperature gradient on the stability of flow in a narrow-gap annulus with constant heat flux at the inner rotating cylinder. Lee and Minkowycz(1989) studied heat transfer characteristics by using the naphthalene sublimation technique in the annular gap between two short concentric cylinders which had either two smooth walls or one smooth and one axially slit wall. This study yielded qualitative information regarding heat transfer but did not address the flow phenomena inside the annular gap.

The present work explores the effect of a constant radial temperature gradient quantified by the Grashof number and the presence of slits on the transition process of Taylor-Couette flow. We used digital particle image velocimetry (DPIV) to measure the flow field inside the gap. We believe this study can help not only to improve the performance of machinery but also to understand flow instability phenomena in Taylor-Couette flow.

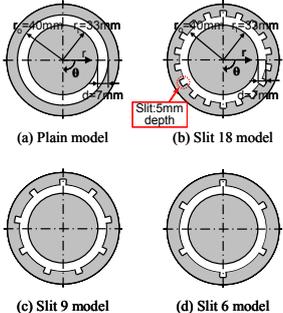
**APPARATUS AND EXPERIMENTAL METHOD**

The test section of the apparatus is similar to that used in the previous study (Lee et. al., 2009) except that the current one was modified chiefly by adding two closed circuits, which were used to generate temperature gradient between the inner and outer cylinders. The general arrangement of the apparatus is shown schematically in Figure. 1.



**Figure 1 Schematic diagram of experimental apparatus and setup**

The radius of the inner cylinder ( $r_i$ ) and the inner radius of the outer cylinder ( $r_o$ ) were 33mm and 40mm, respectively. The annular gap ( $d$ ) between the cylinders was 7mm and the length of the cylinders ( $L$ ) was 336mm. The radius ratio and aspect ratio of the models were 0.825 and 48, respectively. The inner cylinder was rotated with micro stepping motor having a resolution of 400,000 steps per revolution. This micro stepper motor was controlled through a computer, which allowed for the precise control of angular velocity ( $\Omega$ ) and the acceleration to the preset velocity. The outer cylinder was kept stationary. A total of four concentric cylinder models were used for investigating the slit wall effect. One was a plain wall model for comparison purposes and the other three models had different numbers of slits. Figure 2 shows the geometries of experimental models used in this study. For all models, the inner cylinder was made of polypropylene and outer cylinder was made of acrylic pipe.



**Figure 2 Geometries of experimental models**

The complex geometry and curved surfaces of the outer cylinders hindered obtaining clear images via the particle image velocimetry (PIV) method because of the difference in refractive indexes. Therefore, careful refractive index matching between the annulus and the working fluid allows PIV systems to obtain the clear images is needed before the DPIV experiment. Refractive index matching is achieved when the fluid mixture has the same refractive index with the outer cylinder material. We used aqueous solutions of sodium iodide for working fluid and matching the refractive index. The kinematic viscosity of the sodium iodide solution was  $\nu=1.52\text{cSt}$  at  $24^\circ\text{C}$ .

During the experiment, the temperature of the inner and outer cylinders must be controlled carefully to keep the imposed temperature gradients constant. The heating film wrapped around the inner cylinder was used for generating the constant heat flux, which has the thickness of  $300\ \mu\text{m}$  and the resistance of  $59\ \text{ohm}$ . A water jacket was installed in the space between the enclosure box and the outer cylinder, which is connect with the constant temperature bath, and formed another closed circuit. This circuit mainly has two jobs, one of the jobs is to maintain the constant temperature condition between the enclosure box and outer cylinder, another job is to remove the unnecessary heat flux generated by the heating film when the inner cylinder is fixed at a constant temperature. In our experiment, the temperature between the enclosure box and outer cylinder was maintained at  $T_{\text{out}}=24^\circ\text{C}$ , and after the inner cylinder was heated about 10 hours, the temperature near the inner cylinder was tending towards stability. Finally, the temperature of inner cylinder was maintained at  $T_{\text{in}}=25.2^\circ\text{C}$ . Because of these two closed circuits, the temperature gradient between two concentric cylinders was kept constantly. The effect of temperature gradient is parameterized by the Grashof number, defined as  $Gr = d^3 \beta g \Delta T / \nu$ , where  $g$  is the acceleration due to gravity,  $\beta$  is the coefficient of thermal expansion of working fluid, and  $\Delta T$  is the temperature change across the gap.  $\Delta T$  is defined as  $(T_{\text{in}} - T_{\text{out}})$  and thus is considered to be positive if the inner cylinder is hotter than the outer. In this paper, the  $\Delta T$  is  $1.2^\circ\text{C}$ , and the  $\beta$  is equal to  $0.00057$ , which obtained indirectly measuring the density variation. Finally, the Grashof number in our research is  $995$ .

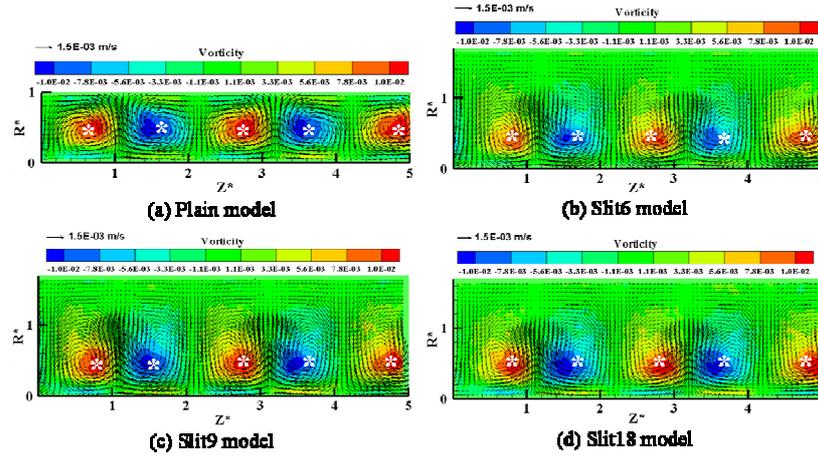
In our experiment, the critical condition may be expressed as a critical Reynolds number  $Re$ , defined as  $Re=r_i d \Omega / \nu$ , where  $\Omega$  is the angular velocity of the inner cylinder.

## EXPERIMENTAL RESULTS AND DISCUSSION

The radial and axial velocity fields between the inner and outer cylinders with and without temperature gradient were measured as the rotating Reynolds number was increased from  $Re=57$  to  $Re=2863$ . Various flow regimes were classified quantitatively by measuring the velocity fields.

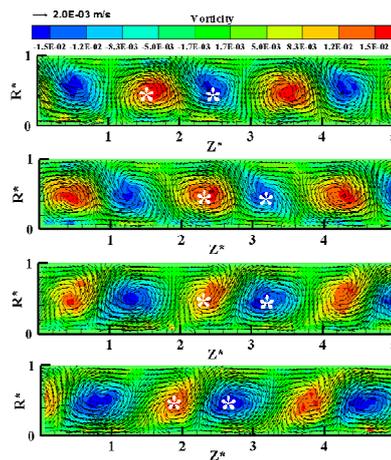
### WITHOUT TEMPERATURE GRADIENT

Figure 3 shows the ensemble averaged velocity fields of Taylor vortex flow on the radial-axial plane at  $Re=115$ . The axial position,  $Z^*=z/d$ , and the radial position,  $R^*=(r-r_{in})/d$ , are normalized by the annular gap width so that it varies from  $R^*=0$  at the inner cylinder to  $R^*=1$  at the outer cylinder of plain model. The region,  $R^*>1$ , refers to the axial slit space in the outer cylinder. Vectors indicate velocity in the axial radial plane. The background represents the vortex distribution. The axial size of the vortices was similar to the annular gap width as  $1.1d$  in all cases. In slit wall configurations, vortices expanded into the axial slit space and vorticity became weaker as the number of slits decreased.

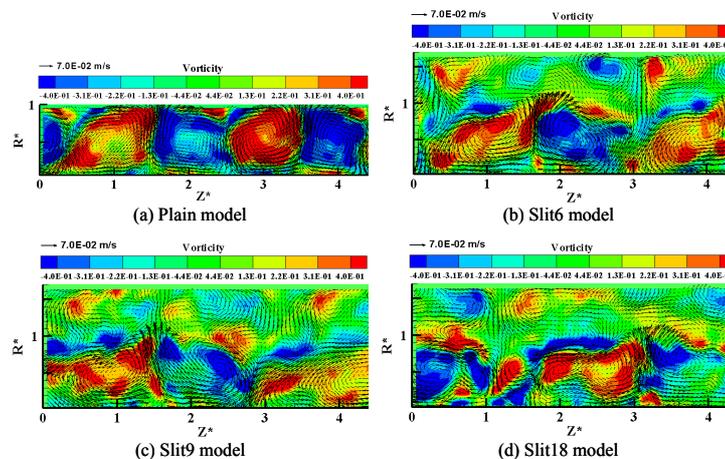


**Figure 3 Average velocity field of Taylor vortex flow at Re=115**

Upon further increasing the Reynolds number, another transition is made to wavy vortex flow (WVF) at approximately  $Re = 124$  of plain model, which is shown in figure 4. In this figure, the vorticity of adjacent vortices was similar and the vortices almost maintained a circular shape in any axial position. The WVF is a time-periodic supercritical bifurcation of the TVF characterized by unsteady traveling waves. This is a first time-dependent solution with breaking of the azimuthal symmetry.



**Figure 4 Instantaneous velocity fields of wavy vortex flow at Re=132**



**Figure 5 Instantaneous velocity fields of turbulent Taylor vortex flow at Re=2863**

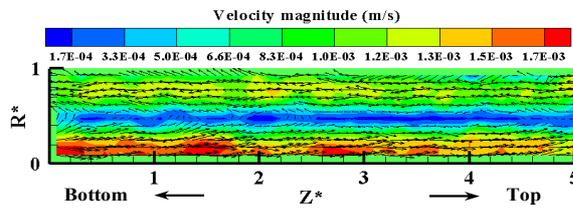
For the plain and slit6 models, as the Reynolds number is increased further, modulated wavy

vortex flow (MWVF) is formed at  $Re = 1145$ . This is doubly periodic; and there are two incommensurable frequencies. At much higher Reynolds number, the flow becomes chaotic or "weakly turbulent." Thereafter, with a continuous increase of Reynolds number, the flow becomes "fully turbulent",  $Re = 1909$ .

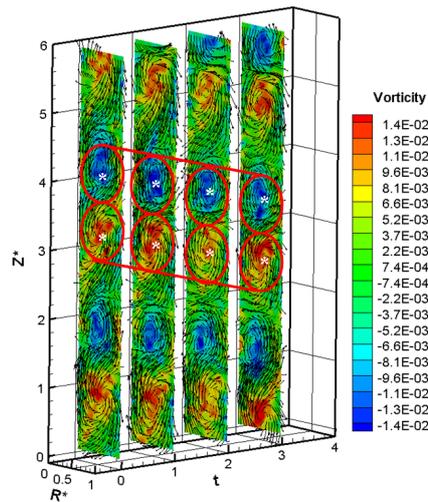
For the slit9 and slit18 models, the transition to turbulent Taylor vortex flow happened at smaller Reynolds number. The wavy vortex flow transitioned to turbulent Taylor vortex flow at  $Re=897$  in the slit9 model, and at  $Re=143$  in the slit 18 model. The slit models had more irregularly-shaped vortices than those of the plain model. Instantaneous velocity fields of turbulent Taylor vortex flow for all models at  $Re=2863$  are shown in figure 5.

### WITH THE TEMPERATURE GRADIENT

Because of the existence of the constant temperature gradient, natural convection current is set up. Before the measuring velocity field as the rotating Reynolds number increases, natural convective flow was measured when the concentric cylinders are stationary, which is shown in figure 6.



**Figure 6 Instantaneous velocity field of natural convective flow at  $Re=0$**

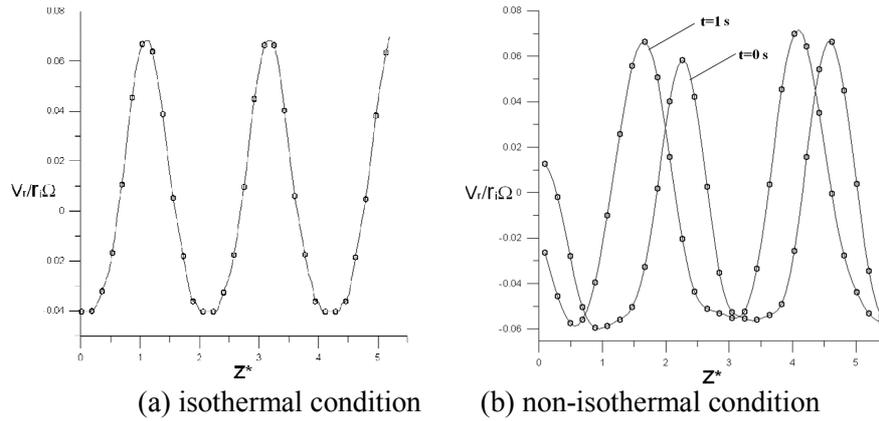


**Figure 7 Time-dependent flow fields of plain model at  $Re=115$**

Figure 7 Shows the time-dependent flow fields of plain model at  $Re=115$ , at this Reynolds number, the helical vortex flow appeared.

Distribution of the radial velocity components along the axial direction of plain model is compared between the isothermal and non-isothermal condition at  $Re= 115$  in figure 8. Radial velocity is denoted by  $v_r$ , and axial location by  $Z^*$ . From these two figures, we can see the difference between Taylor vortex flow and helical vortex flow. In Taylor vortex flow, the vortex circulates around a fixed center, but the vortex in helical vortex flow has a tendency of moving along the axis. In both figure, the peak corresponding to the outflow is narrower and stronger than the one corresponding to the inflow. This is the common characteristics not only in the isothermal condition but also in the non-isothermal condition. This is because of the distance between the two vortex centers of a vortex pair is shorter than that of two adjacent vortex pairs, and the outflow between the vortex centers of a vortex pair is stronger than the inflow between two adjacent vortex pairs as explained in figure 3 and

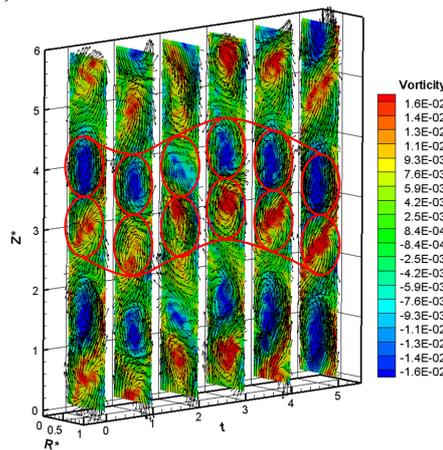
figure 7.



**Figure 8 Radial velocity components along the axial of isothermal condition**

Figure 9 shows the time-dependent flow fields of plain model at  $Re=132$ . At the same Reynolds number, wavy vortex flow appeared under isothermal condition. However, helical wavy vortex flow appeared by the influence of convection flow. In our experiment, near the inner region has higher temperature than the outer region, therefore, the direction of the convection current is upwards near the inner cylinder and downwards near the outer cylinder (Deters and Egbers, 2005). As a result of this, the Taylor cell that has the same direction of circulation as the natural convection current increases in size. The counter rotating cell, on the other hand, decreases in size. Because of this reason, the helical wavy vortex flow was formed.

As increasing Reynolds number, helical wavy vortex flow and random helical vortex flow appeared in plain and slit 6 models. And we could see helical wavy vortex flow again at  $Re=344$  of both models. In addition, the wavy vortex flow regime appeared in both the plain model and the slit6 model before the transition to turbulent Taylor-Couette flow at  $Re=574$ . And above  $Re=574$ , the flow regimes under the positive temperature gradient were similar to that of isothermal condition. This result shows that the effect of the temperature gradient can be ignored when the Reynolds number is larger than  $Re=574$ . The flow finally transitioned to turbulent Taylor vortex flow as the increasing rotating Reynolds number to 1909, this is the same as that of isothermal condition.

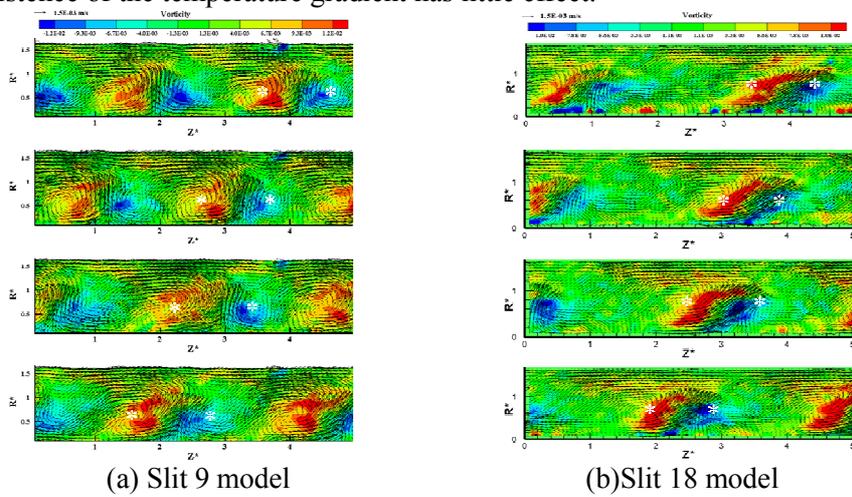


**Figure 9 Time-dependent flow fields of Helical Wavy Vortex Flow at  $Re=132$**

In case of Taylor-Couette flow system with temperature gradient of plain model, two aspects affect the flow instability of Taylor-Couette flow system. One is rotational force, which is caused by the inner cylinder rotation. The other is radial gravity (Tagg and Weidman, 2007) which is caused by the presence of a radial temperature gradient. Here Froude number  $\sigma$  is introduced to distinguish which force is dominant (Ball et. Al., 1989), defined as  $\sigma = Gr/Re^2$ . In our research, the radial temperature gradient remains constant, so the Grashof number is invariable during the experiment. From

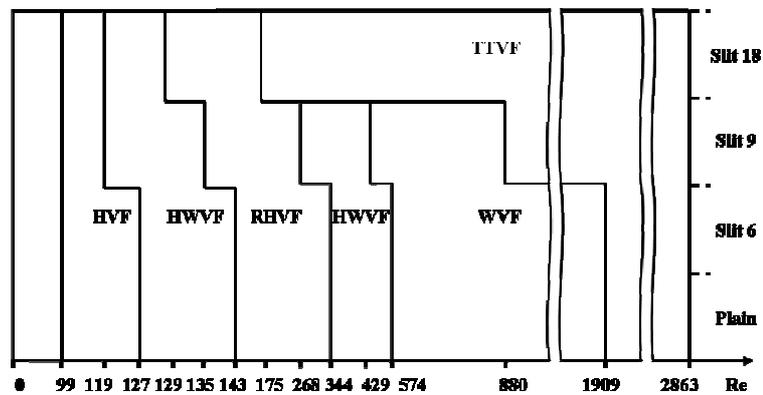
experimental results, when Reynolds number is smaller than 574, that is  $\sigma$  larger than 0.003, both the temperature gradient and the rotational forces affect the flow instability in our experiment. As the rotation speed of the inner cylinder increases, therefore, the  $\sigma$  becomes smaller than 0.003, the rotational forces are predominant, and this can be easily observed from the wavy vortex flow appeared at Reynolds number larger than 574.

Figure 10 shows the instantaneous velocity fields of slit9 model and slit 18 model at  $Re=115$ . In slit wall configurations, vortices expanded into the axial slit space and vorticity became stronger as the number of slits increase d. However, the transition from helical vortex flow to helical wavy vortex flow is a little earlier than that of plain model, at the slit9 and slit18. This transition was occurred at  $Re=119$ . As the Reynolds number increases, the flow regime changes to random helical vortex flow, which occurred at  $Re=135$  and  $Re=129$  of slit9 and slit18, respectively. And then in slit9 model, the wavy vortex flow appeared again at  $Re=574$ . We also found that the transition to turbulent Taylor vortex flow was accelerated like that under the isothermal condition, at  $Re=880$  and 175 respectively, which means that the transition to turbulent Taylor vortex flow is the more sensitive with the presence of slits, the existence of the temperature gradient has little effect.



**Figure 10 Instantaneous velocity fields of Helical Vortex Flow at  $Re=115$**

We summarized the transition processes of Taylor-Couette flow with constant temperature gradient in the slit wall configuration according to the rotating Reynolds number in figure 11. Compared with the results of isothermal condition, we found that the radial gravity caused by temperature gradient has more influence on the plain and slit6 models.



**Figure 11 Flow type in plain model and slit models for various  $Re$**

(HVF: helical vortex flow. HWVF: helical wavy vortex flow. RHVF: random helical vortex flow. WVF: wavy vortex flow. TTVF: turbulent Taylor vortex flow )

## CONCLUSIONS

Experimental study has been performed for two-dimensional Taylor-Couette flow with various slits under the isothermal and non-isothermal condition. Through adjusting the rotation speed of the inner cylinder gradually, we investigated different flow motions between two cylinders from laminar flow to turbulent flow.

From the results, we found that when Reynolds number is smaller than 574, the radial temperature gradient imposed on the concentric cylinders has the strong effect on the flow field. Transition process from laminar to turbulent flow includes helical vortex flow, helical wavy vortex flow, random helical vortex flow, wavy vortex flow and turbulent vortex flow. The reason of the wavy vortex flow appeared again in the non-isothermal conditions is that the temperature gradient has little effect when the Reynolds number is larger than 574. Compared between the flow field in plain model and models of the different slit numbers, we can conclude that the smaller number of slits have no apparent effect on the transition process, which just slightly change the shape of the vortex. However, the effect of more numerous slits is obvious through accelerating early transition to turbulent flow in isothermal and non-isothermal conditions.

## ACKNOWLEDGEMENT

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