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## **DIGITAL HOLOGRAPHY BASED PARTICLE TRACKING VELOCIMETRY**

Kazuo Ohmi<sup>1</sup>, Sanjeeb Prasad Panday<sup>2</sup> and Basanta Joshi<sup>2</sup>

### **ABSTRACT**

Digital holography appears to be the next-generation technology for holographic diagnostics of particle fields and has been applied to holographic particle tracking flow measurements. In the digital holography, the Fresnel or Fourier holograms are recorded directly by the CCD or CMOS cameras and stored digitally. No film material involving wet-chemical or other processing is necessary as in optical holography. The reconstruction of the wave-field, which is done optically by illumination of a hologram, is performed by numerical methods. Digital holography also enables the use of complex amplitude information inaccessible by optical reconstruction, thereby allowing flexible reconstruction algorithms to achieve optimization of specific information. However, due to the poor pixel resolution of CCD or CMOS cameras as compared to that of holographic films, the digital holography gives poor depth resolution for images. This problem severely impairs the usefulness of digital holography especially in densely populated particle fields. In this paper the authors present a simple digital in-line holography recording setup and numerical reconstruction process based on the Fourier transform and convolution schemes. The use of the complex amplitude, available from these reconstructed waves, in particle extraction significantly improves particle axial-location accuracy compared with other numerical reconstruction schemes that are merely traditional optical reconstruction. This method also alleviates the speckle noise problem intrinsic to in-line particle holography since speckles and particles can be clearly differentiated in complex wave-field.

**Keywords: Digital holography, in line holography, complex amplitude, particle tracking velocimetry, speckles noise**

### **INTRODUCTION**

Holography is a 3D imaging process that instantaneously captures the volumetric information of a test object. With traditional holography, particle fields are recorded on holographic plates or films with pulsed laser beams, while the developed holograms are brought back to the reference beams to optically reconstruct the 3D images of the particle fields. A CCD or CMOS camera then scans the 3D image volume mechanically, plane by plane, and the particle field information is computed from the scanned images. A good example of this is the holographic particle image velocimetry system developed by Pu

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<sup>1</sup> Corresponding author: Dept. of Information Systems Engineering, Osaka Sangyo University, Daito-shi, Osaka 574-8530, Japan, e-mail: ohmi@ise.osaka-sandai.ac.jp

<sup>2</sup> Dept. of Information Systems Engineering, Graduate Student of Faculty of Engineering, Osaka Sangyo University, Daito-shi, Osaka 574-8530, Japan

and Meng (2000) and Pu *et al.* (2000). While holography is potentially the most powerful tools for 3D imaging, the conventional procedure of film-based holography, which includes film-based recording, wet chemical processing, and optical reconstruction, has severely restricted the user-friendliness of the holographic imaging technique.

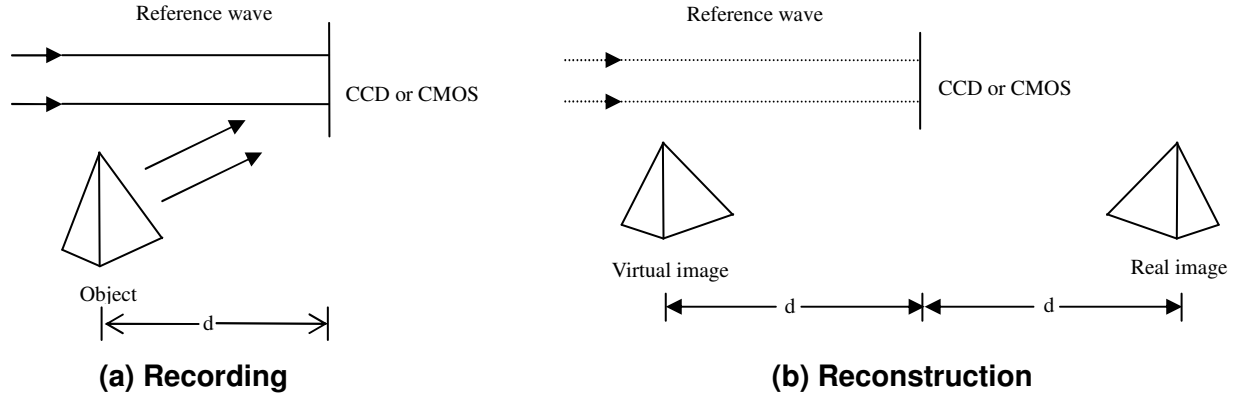
To circumvent the drawbacks associated with optical holographic recording and reconstruction, over the past few years, digital holography has been explored as an alternative. Digital holography (DH) looks very promising in making holography an easy-to-use technique and is a rapidly growing opto-digital method for imaging, microscopy and metrology. In DH, the hologram which is the interference of object and reference waves is recorded in digital form using CCD or CMOS sensors and then numerically reconstructed on the computer. With recent developments in mega-pixel CCD or CMOS cameras and the fast, efficient reconstruction algorithm, it is now possible to record digital holograms in real time. Use of fast computers in numerical reconstruction makes DH more flexible in terms of hologram processing. The ability of numerical evaluation of both amplitude and phase information is its main advantage over other optical imaging methods. The limited pixel size of the commercially available digital detectors restricts the angle between the object and reference beams to a few degrees for hologram recording. Thus in-line geometry proposed by Gabor (1951) becomes the preferred choice for digital holography. Also, in-line DH makes efficient use of full sensor area for hologram recording in comparison to off-axis geometry. The main problem with in-line geometry is the overlapping of zero order and twin image waves with the real-image wave during reconstruction process. Various digital methods have been proposed to suppress these unwanted waves (Kreis and Juptner, 1997; Takaki *et al.*, 1999).

Various optical techniques are developed for instantaneous 3D measurement of particle distribution (Fournier *et al.*, 2004) including stereoscopic particle tracking (Virant and Dracos, 1997), defocusing digital particle image velocimetry (Pereira and Gharib, 2000) forward scattering particle image velocimetry (Ovryn, 2000), and holographic techniques such as holographic particle image velocimetry (Pu and Meng, 2000; Bishop *et al.*, 2001; McIntyre *et al.*, 2003). These techniques have limitations either with volume size or particle density or the need for multiple exposures. However, in recent years DH has shown its capability in providing high-resolution, instantaneous measurements of a large number of particles in a 3D volume (Fournier *et al.*, 2004). In-line DH for particle imaging has the potential for the measurement of size, position and velocity of particles distributed in 3D. The advantage with in-line DH is that full CCD or CMOS sensor area is utilized for real-image reconstruction of the objects.

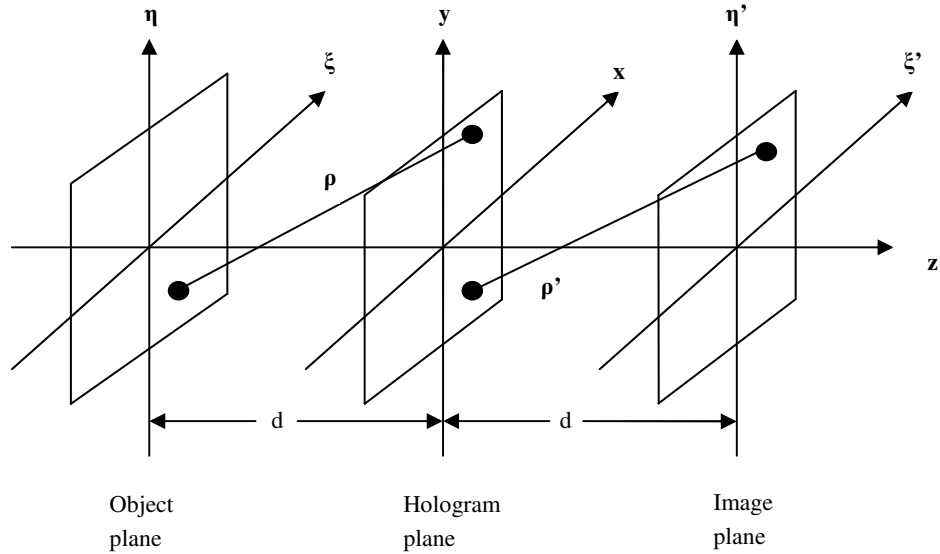
The main aim of this paper is to demonstrate the numerical reconstruction process of particles holograms in different planes based on the Fourier transform (FT) and convolution (CV) schemes as these are helpful in visualizing micro-particles along the flow field. Numerically reconstructed images using both methods along with in focus and out of focus particles are shown. Then, a comparative study of numerical reconstruction using FT and CV methods is presented. This approach to digital hologram recording and numerical reconstruction not only eliminates wet chemical processing and mechanical scanning but also enables the use of complex amplitude information that is inaccessible in optical reconstruction. The use of this complex amplitude in particle extraction significantly improves particle axial-location accuracy compared with other numerical reconstruction schemes that are merely traditional optical reconstruction (Pan and Meng, 2003). Moreover, speckles and particles can be clearly differentiated in complex wave-field and as a result these reconstruction methods also alleviate the speckle noise problem intrinsic to in-line particle holography. Furthermore, only a single hologram is necessary to display the particles in different planes along the depth of focus and no other curve-fitting or processing is required.

## GENERAL PRINCIPLES OF DIGITAL HOLOGRAPHY

The concept of digital hologram recording is illustrated in Fig. 1(a). A plane reference wave and the wave reflected from a three dimensional object placed at a distance  $d$  from a CCD or CMOS are interfering at the same surface. The resulting hologram is electronically recorded and stored. In optical reconstruction as shown in Fig. 1(b), a virtual image appears at the position of the original object and the real image is also formed at a distance  $d$ , but in the opposite side from the CCD or CMOS. Fig. 2 shows the coordinate system for numerical hologram reconstruction in DH.



**Fig. 1. Digital holography**



**Fig. 2. Coordinate system for numerical hologram reconstruction**

The diffraction of a light wave at CCD or CMOS is described by the Fresnel-Kirchoff integral (Goodman, 1968).

$$\Gamma(\xi', \eta') = \frac{i}{\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) R(x, y) \frac{\exp\left(-i \frac{2\pi}{\lambda} \rho'\right)}{\rho'} dx dy \quad (1)$$

with

$$\rho' = \sqrt{(x - \xi')^2 + (y - \eta')^2 + d^2} \quad (2)$$

$R(x, y)$  is the plane reference wave,  $h(x, y)$  is the hologram function and  $\rho'$  is the distance between a point in the hologram plane and a point in the reconstruction plane. An undistorted real image can be produced by using the conjugate reference beam for reconstruction. To reconstruct an undistorted real image in DH it is therefore necessary to insert  $R^*$  instead of  $R$  in Eq. (1) above.

$$\Gamma(\xi, \eta) = \frac{i}{\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) R^*(x, y) \frac{\exp\left(-i \frac{2\pi}{\lambda} \rho\right)}{\rho} dx dy \quad (3)$$

with

$$\rho = \sqrt{(x - \xi)^2 + (y - \eta)^2 + d^2} \quad (4)$$

For a plane reference wave where  $R^* = R = \text{Real amplitude}$ , Eq. (1) and Eq. (3) are equivalent. After calculations (Schnars and Jueptner, 2002) Eq. (3) reduces to

$$\begin{aligned} \Gamma(\xi, \eta) &= \frac{i}{\lambda d} \exp\left(-i \frac{2\pi}{\lambda} d\right) \exp\left[-i \frac{\pi}{\lambda d} (\xi^2 + \eta^2)\right] \\ &\times \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} R^*(x, y) h(x, y) \exp\left[-i \frac{\pi}{\lambda d} (x^2 + y^2)\right] \exp\left[i \frac{2\pi}{\lambda d} (x\xi + y\eta)\right] dx dy \end{aligned} \quad (5)$$

Eq. (5) is called the Fresnel approximation (Goodman, 1968) or Fresnel transformation due to its mathematical similarity with the Fourier Transform. It enables one to reconstruct the undistorted real image.

The magnitude, intensity and phase are calculated by Eq. (6), Eq. (7) and Eq. (8) respectively.

$$M(\xi, \eta) = |\Gamma(\xi, \eta)| \quad (6)$$

$$I(\xi, \eta) = |\Gamma(\xi, \eta)|^2 \quad (7)$$

$$\varphi(\xi, \eta) = \arctan \frac{\text{Im}[\Gamma(\xi, \eta)]}{\text{Re}[\Gamma(\xi, \eta)]} \quad (8)$$

## RECONSTRUCTION BY THE DISCRETE FRESNEL TRANSFORMATION OR THE FOURIER TRANSFORM (FT) METHOD

The digitized version of Eq. (5) (Schnars and Jueptner, 2002) is given by:

$$\begin{aligned} \Gamma(m, n) &= \frac{i}{\lambda d} \exp\left(-i \frac{2\pi}{\lambda} d\right) \exp\left[-i \pi \lambda d \left(\frac{m^2}{N^2 \Delta x^2} + \frac{n^2}{N^2 \Delta y^2}\right)\right] \\ &\times \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} R^*(k, l) h(k, l) \exp\left[-i \frac{\pi}{\lambda d} (k^2 \Delta x^2 + l^2 \Delta y^2)\right] \exp\left[i 2\pi \left(\frac{km}{N} + \frac{ln}{N}\right)\right] \quad (9) \\ &\text{for } m = 0, 1, \dots, N-1 \quad ; \quad n = 0, 1, \dots, N-1 \end{aligned}$$

where  $\lambda$  is the wavelength used,  $d$  is the reconstruction distance,  $\Delta x$  and  $\Delta y$  are the pixel size of the CCD or CMOS in the horizontal and vertical directions respectively and  $N \times N$  is the size at which the hologram function  $h(x, y)$  is sampled.

The relation in Eq. (9) is the discrete Fresnel transform and is one of the main relations used in this paper for numerical reconstruction. In order to calculate the matrix  $\Gamma$ , we must multiply  $R^*(k, l)$  by  $h(k, l)$  and  $\exp\left[-i \pi / \lambda d (k^2 \Delta x^2 + l^2 \Delta y^2)\right]$  and then apply the inverse discrete Fourier transform to this product. For this reason, the method of numerical reconstruction using Eq. (9) is termed as the Fourier transform (FT) method for numerical reconstruction by the authors in this paper. The factor in front of the sum only affects the phase and can be neglected in most applications.

In the FT method of reconstruction, the pixel size of the numerically reconstructed image varies with the reconstruction distance  $d$  and is given by:

$$\Delta \xi = \frac{\lambda d}{N \Delta x} \quad ; \quad \Delta \eta = \frac{\lambda d}{N \Delta y} \quad (10)$$

As a result of this variation of the pixel size of the numerically reconstructed image with the reconstruction distance  $d$ , the magnification of the reconstructed image also varies with the reconstruction distance  $d$  in the FT method.

## RECONSTRUCTION BY THE CONVOLUTION (CV) METHOD

In the convolution approach of reconstruction, the reconstruction wave is defined as the convolution of the product of the reference wave and the digital hologram function with the impulse response function of coherent optical system. Mathematically, the reconstructed wave is defined as:

$$\Gamma(\xi, \eta) = [(R^*(k, l)h(k, l)) \otimes g(k, l)] \quad (11)$$

where  $\otimes$  indicates a two-dimensional convolution and  $g(k, l)$  denotes the numerical impulse response function of coherent optical system which is given by:

$$g(k, l) = \frac{i}{\lambda} \frac{\exp \left[ -i \frac{2\pi}{\lambda} \sqrt{d^2 + \left(k - \frac{N}{2}\right)^2 \Delta x^2 + \left(l - \frac{N}{2}\right)^2 \Delta y^2} \right]}{\sqrt{d^2 + \left(k - \frac{N}{2}\right)^2 \Delta x^2 + \left(l - \frac{N}{2}\right)^2 \Delta y^2}} \quad (12)$$

The convolution theorem can be applied to Eq. (11). According to this theorem the Fourier transform of the convolution of  $(h \cdot R^*)$  with  $g$  is the product of the individual transforms  $\mathfrak{S}(h \cdot R^*)$  and  $\mathfrak{S}(g)$ . So, reconstructed object wave can be calculated by Fourier transforming the product of the digital hologram and the reference wave first. This is followed by multiplying with the Fourier transform of the numerical impulse response function, and taking an inverse Fourier transform of this product. Mathematically, this can be represented as:

$$\Gamma(\xi, \eta) = \mathfrak{S}^{-1} \{ \mathfrak{S}(h \cdot R^*) \bullet \mathfrak{S}(g) \} \quad (13)$$

The Fourier transform of  $g(k, l)$  can be expressed analytically ((Schnars and Jueptner, 2002) :

$$G(n, m) = \exp \left\{ -i \frac{2\pi d}{\lambda} \sqrt{1 - \frac{\lambda^2 \left( n + \frac{N^2 \Delta x^2}{2d\lambda} \right)^2}{N^2 \Delta x^2} - \frac{\lambda^2 \left( m + \frac{N^2 \Delta y^2}{2d\lambda} \right)^2}{N^2 \Delta y^2}} \right\} \quad (14)$$

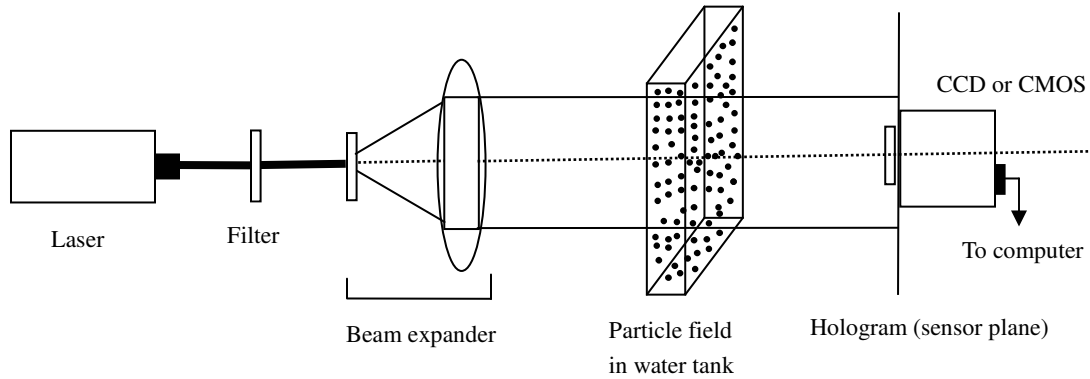
This saves one Fourier transform for reconstruction and Eq. (13) simplifies to:

$$\Gamma(\xi, \eta) = \mathfrak{S}^{-1} \{ \mathfrak{S}(h \cdot R^*) \bullet G \} \quad (15)$$

The authors have used the above Eq. (15) as the CV method for the purpose of numerical reconstruction in this paper. The pixel size of the reconstructed image by the CV method is the same as that of the CCD or CMOS and does not vary with reconstruction distance, i.e.,  $\Delta\xi = \Delta x$  and  $\Delta\eta = \Delta y$ . In the CV method, the magnification of the reconstructed image remains constant with the reconstruction distance and equal to the pixel size of the CCD or CMOS sensor.

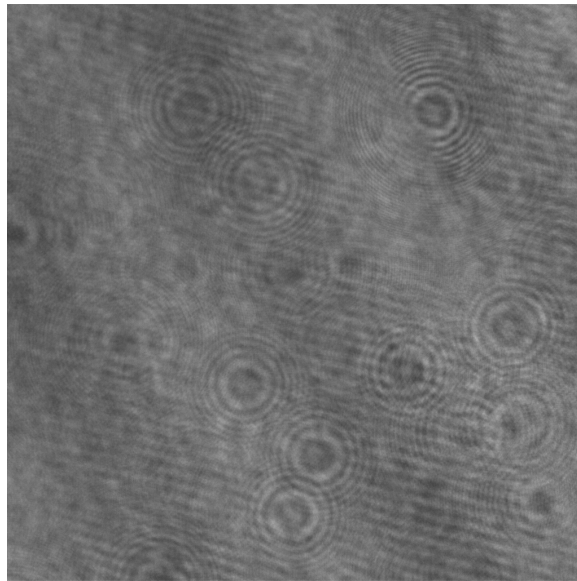
## EXPERIMENTS AND RESULTS

So far, various optical geometries have been proposed for particle analysis using DH. The simplest and most common optical system for particle analysis is using the collimating laser beam; here a collimated beam is used to shine the particle field distribution. The experiment is carried out using an in-line holography set-up as shown in Fig. 3.



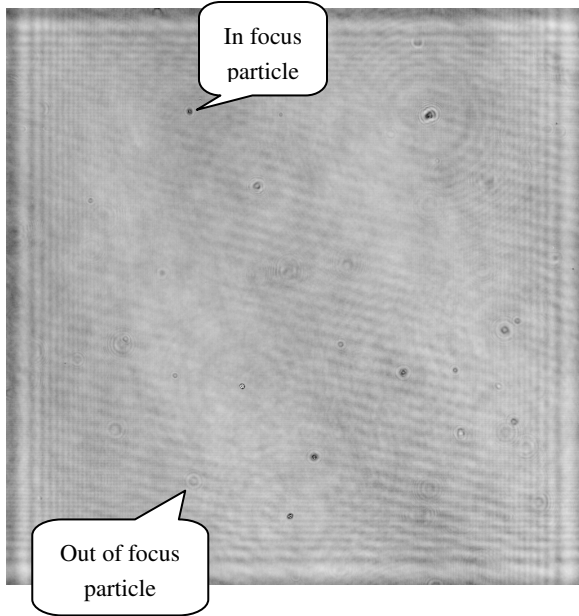
**Fig. 3. Typical digital in-line holography experimental set-up for recording of a particle field**

The water tank of size  $156 \times 81 \times 16 \text{ mm}^3$  filled entirely with water is regarded as the sample volume for this experiment. The micro Orgasol particles of sizes  $\sim 60 \mu\text{m}$  are randomly distributed throughout this sample volume. Spatially filtered He-Ne laser beam of wavelength  $632.8 \text{ nm}$  is used to illuminate the water tank for hologram recording as shown in Fig. 3. Digital holograms of the particles were directly recorded by an 8-bit CMOS camera (Silicon Video 9M001) with  $1024 \times 1024$  pixels of size  $5.2 \mu\text{m}$ . This CMOS camera is kept at a distance of  $42 \text{ mm}$  from the water tank. A typical digitally recorded hologram of micro-particles in 3D is shown in Fig. 4.

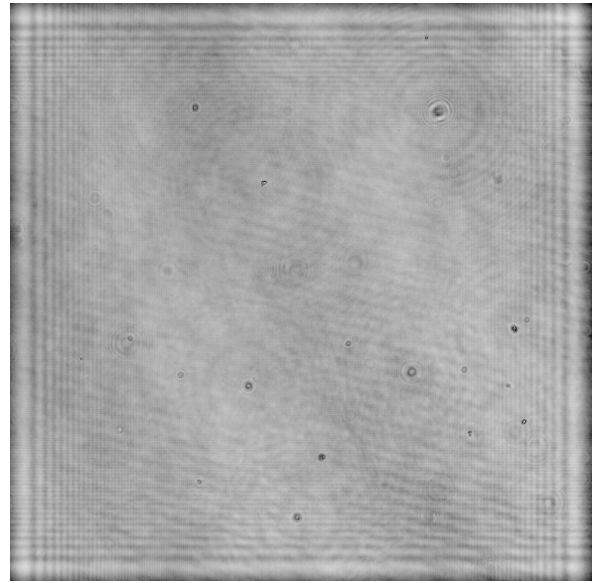


**Fig. 4. Typical digital in-line hologram of particles**

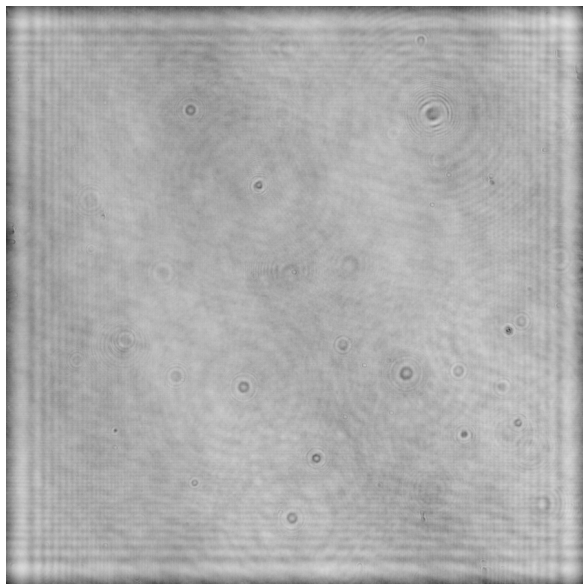
The numerical reconstruction of this hologram is performed using both the FT and CV schemes. Eq. (5) is the main equation used for the numerical reconstruction of the generated hologram based on the FT method. Similarly, Eq. (15) is the main equation used for the numerical reconstruction of the generated hologram based on the CV method. Fig. 5(a) to Fig. 5(c) shows the intensity images of the numerically reconstructed holograms based on the FT method at three different reconstruction distances. Similarly, Fig. 5(d) to Fig. 5(f) shows the intensity images of the numerically reconstructed holograms based on the CV method at three different reconstruction distances.



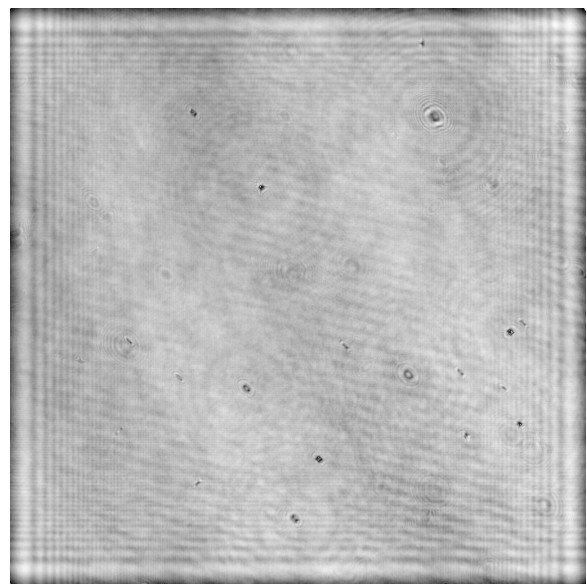
**(a) FT based intensity image at  $d = 44$  mm**



**(b) FT based intensity image at  $d = 47$  mm**

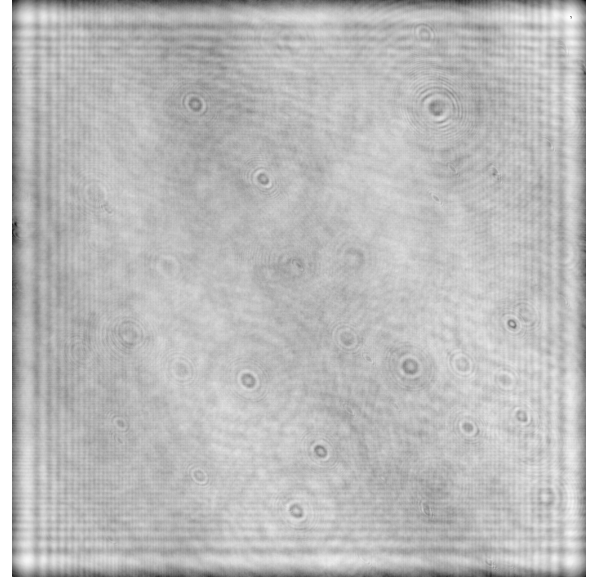
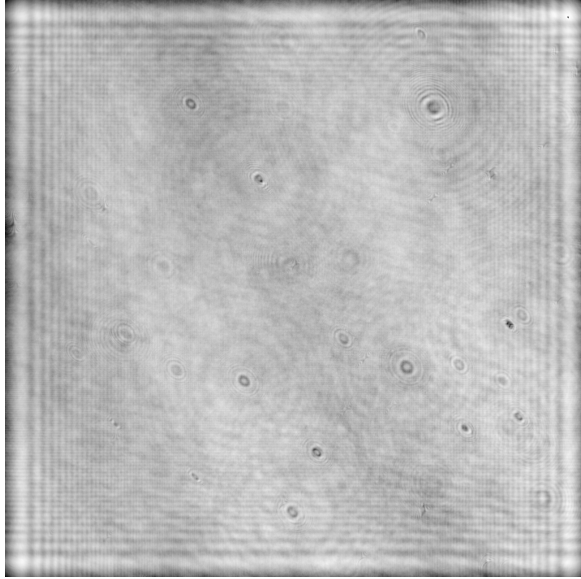


**(c) FT based intensity image at  $d = 50$  mm**



**(d) CV based intensity image at  $d = 44$  mm**





(e) CV based intensity image at  $d = 47$  mm

(f) CV based intensity image at  $d = 50$  mm

**Fig. 5. Numerically reconstructed intensity images of particles based on FT and CV approach**

In all these reconstructed intensity images, the in-focus particles appear as sharp black dots while the out-of-focus particles appear as hollow light concentric circles as depicted explicitly in Fig. 5(a). If we compare all these reconstructed images it can be clearly seen that the particles which appear as focused at certain distance are defocused and disappeared when the reconstruction distance is changed in case of both FT and CV methods. This signifies the possibility of locating each particle individually at different depths using a single recorded hologram.

Furthermore, the reconstructed intensity images in Fig. 5 don't show much difference between the FT and CV numerical methods of reconstruction. However, both these methods have some advantages and disadvantages. The FT method uses only one Fourier transform whereas the CV method uses two Fourier transform as depicted in Eq. (9) and Eq. (15). As a result, FT method of reconstruction is usually faster than its counterpart CV method which is time-consuming. But in FT method, the pixel size of the numerically reconstructed image varies with the reconstruction distance. This variable pixel dimension at the hologram plane and the noise introduced by the zero-diffraction order represent main disadvantageous features of the FT method. On the other hand the CV method offers the reconstructed image usually free of the zero order beams. Another interesting feature of the CV method in comparison to the FT method is the pixel size constancy in the reconstructed images. The pixel size in this case is independent of the holographic arrangement parameters and is equal to the size of CCD or CMOS pixel. But one serious drawback of the CV method is that it suffers from the interference called wrap-around error. In order to avoid this error, the authors have stretched the size of the recorded hologram artificially from 1024 X 1024 to 2048 X 2048 by padding zeros around the original image. Though this increases space and time complexity, the results are free from wrap-around error.

## CONCLUSION

Digital holography is a promising technique for 3D measurement of small particles. In this study, the

authors have successfully reconstructed the hologram of small particles field digitally using a single hologram. The numerical reconstruction of holograms using two different methods namely FT and CV have been presented and compared. In the FT method, the magnification of the reconstructed image varies with the reconstruction distance, while it remains constant and equal to the pixel size of CCD or CMOS sensor in the CV method. As a result, the FT method can also be used for objects larger than the CCD or CMOS array size, but the CV method is used only for small objects. Apart from this, FT though faster in computation than its counterpart CV method suffers from the noise introduced by the zero-diffraction order. In this paper only the numerical reconstruction of particles at different depths is presented but it is still left to test the applicability of digital in-line holography to particle field extraction and tracking from these reconstructed images. The present authors will make further efforts in the future to overcome this limitation by including extraction and tracking of particles from these reconstructed images as their future work.

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