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THEORY AND GEOMETRY COMPARISON AMONG INCLINED FREE - OVER SILL - REPELLED HYDRAULIC JUMPS

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ABSTRACT

In this study, which is based on recent laboratory measurements by the author and further elaboration of them, a comparison is presented between theory (onedimensional flow equations) and geometry (free surface water profiles), concerning the free, the over a sill and the repelled inclined (angle φ, $0^o \leq \varphi \leq 9^o$) hydraulic jumps (Fig.1), showing that all these profiles are systematic. The comparison among the jump profiles is based on a number of previous papers by the author and show that all corresponding profiles are lowering when angle φ is increasing, and that for larger inclination angles the respective free and over-sill profiles become almost identical. The results may be useful to the hydraulic engineer when selecting the most convenient type of jump for his technical design needs.

Keywords : Free Jump. Jump Over a Sill. Repelled Jump.

INTRODUCTION

The steady inclined hydraulic jump has a considerable interest when designing open channels and small dam or sluice gate outflows. Although the jump has various forms, three particular jumps are examined and compared here, all from the point of view of theory (onedimensional equations) and geometry - especially in relation to their water free surface profiles.

Figs. 1a, 1b and 1c, show the pertinent basic characteristics of the above hydraulic jumps, the free jump, the jump over a sill and the repelled jump correspondingly. The free jump has a length L and two conjugate depths d_1, d_2 (uniform flow cross sections), in a rectangular channel with an inclination angle φ (and slope $J_o = \sin\varphi$), while the discharge (per unit width) is q and the typical water depth is d at a distance x. The main parameter is the Froude number $Fd_1 = q/g^{1/2} \cdot d_1^{3/2}$, which is larger than 1 (supercritical flow), while $D=d_2/d_1$.

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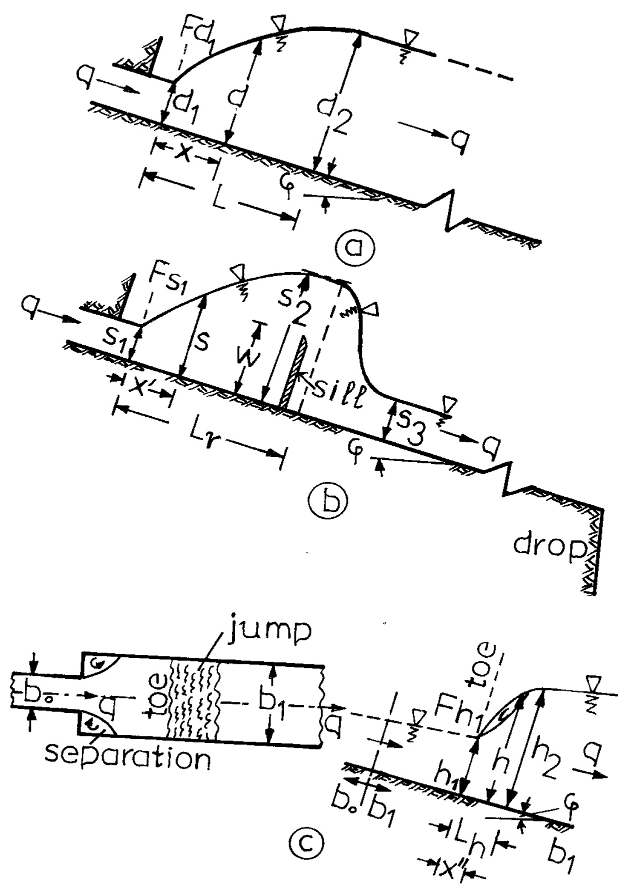

Figure 1. Basic flow characteristics.

The jump over a sill (w) is more complicate. It has a minimum upstream water depth s_1 (uniform flow cross section), a maximum depth s_2 (no uniform cross section and not always over the sill – but including w), and it ends up to a uniform flow cross section with water depth s_3, while $Fs_1 = q/g^{1/2} \cdot (s_1)^{3/2} (>1)$. At any distance x' the upstream water depth is s, while the entire length of the ascending jump is Lr. The upstream profile of the jump over a sill has usually two shapes, one shown in Fig 1b (similar to the profile of the free jump) and another one (not shown here) which is a steady wave – like jump. In this study only the profiles of Fig. 1b are examined and compared to the profile of Fig. 1a, since they have similar shapes. The dimensionless ratio of s_1, s_3, is $S=s_3/s_1$.

Figure 1c schematically shows the repelled jump characteristics. The rectangular channels in series, with widths bo and b_1 and expansion ratio $r = b_o / b_1 (< 1)$, have a common inclination angle φ and slope J_o=sinφ. The discharge per unit width (b_1) is q, the conjugate jump depths are h_1 and h_2 - with their ratio $R=h_2/h_1(>1)$, while the inclined length of the jump between h_1 and h_2 is L_h. In the present flow case the jump appears - with its mean free surface profile and roller - to be entirely created in the b_1 channel at a small distance from the end of two symmetrical separation zones. The toe of this jump (at depth h_1) is not far from the separation zones, while its tailwater (depth h_2) is lying at a horizontal distance $L_h \cdot \cos\varphi$. The typical water depth is h at a distance x'' (from h_1), while the most important parameters of this jump are the expansion ratio r and the Froude number at cross section 1, $Fh_1 = q/g^{1/2} \cdot (h_1)^{3/2} (> 1)$. The flow separation has a strong effect on h_1, h_2, h, R, L_h, quantities.

In all jumps the flow is fully turbulent since the pertinent Reynolds numbers have high enough values, while any comparison is meant for $F_{d1}= F_{s1}= F_{h1}= Fr_1$.

THE ONEDIMENSIONAL EQUATIONS

The continuity equations along corresponding flows (x, x', x'') are respectively

$$\left.\begin{aligned} q &= V_1 \cdot d_1 = V_2 \cdot d_2 , \\ q &= V_1' \cdot s_1 = V_2' \cdot s_2 , \\ q &= V_1'' \cdot h_1 = V_2'' \cdot h_2 . \end{aligned}\right\} \qquad (1)$$

The momentum equations

The jump over a sill

Since the jump over a sill is more complicate the onedimensional momentum equation for this jump is first examined between s_1 and s_3,

$$P_x + W_t \cdot J_o - G_x = \rho \cdot q^2 \cdot [(1/s_3) - (1/s_1)]$$

where, per unit channel width,

$P_x = 0.5 \cdot \gamma \cdot \left[(s_1)^2 - (s_3)^2\right] \cdot \cos\varphi$ = total pressure force along x',

W_t = total water weight under the flow profile = $K_w \cdot P_x$,

$G_x = N_x + F_x = K_G \cdot P_x$,

N_x = force exerted by the sill along x',

F_x = tractive force from the boundaries,

(K_W, K_G) = dimensionless W_t and G_x force coefficients,

γ = specific water weight.

If the Froude number Fr_1 and

$$S = s_3/s_1 ,\ K_w \cdot P_x ,\ K_G \cdot P_x ,$$

are introduced, then the equation

$$S^2 \cdot (T \cdot \cos\varphi) + S \cdot (T \cdot \cos\varphi) - 2 \cdot (Fr_1)^2 = 0 , \qquad (2)$$

where

$$T = 1 + K_w \cdot J_o - K_G ,$$

is received.

Eq. (1) may suitably be solved

$$S = s_3/s_1 = 0.5 \cdot \left[\left(1 + 8 \cdot \frac{(Fr_1)^2}{T \cdot \cos\varphi} \right)^{1/2} - 1 \right] \qquad (3)$$

where s_1 (Fr_1), s_3 and S, K_w, K_G, should experimentally be measured.

The free jump

If there is a jump without sill (w=0-free jump), N_x=0, G_x=F_x, s_1=d_1, s_3=s_2=d_2, L_r =L, x' =x, W_t=water weight between d_1 and d_2, D=d_2/ d_1 , eq. (3) is adjusted to

$$D = 0.5 \cdot \left[\left\{ 1 + 8 \cdot \frac{Fr_1^2}{(1 + K_w \cdot J_o - K_G) \cdot \cos\varphi} \right\}^{1/2} - 1 \right], \qquad (4)$$

while for horizontal channels ($K_w \cdot J_o = 0$, F_x=0, K_G=0, cosφ=1),

$$D = d_2/d_1 = 0.5 \cdot \left[\left(1 + 8 \cdot Fr_1^2 \right)^{1/2} - 1 \right], \qquad (5)$$

which is the well known conjugate depths' equation for the classical free hydraulic jump.

The repelled jump

For the repelled jump eq. (3) is similarly adjusted to

$$R = h_2/h_1 = 0.5 \cdot \left[\left\{ 1 + 8 \cdot \frac{(Fr_1)^2}{(1 + K_w \cdot J_o - K_G) \cdot \cos\varphi} \right\}^{1/2} - 1 \right], \qquad (6)$$

while for horizontal repelled jumps $(K_w \cdot J_o = 0$, F_x=0, cosφ=1), R(=D) is given again by eq. (5). h_1 (Fr_1), h_2 and R, K_w, K_G, should experimentally be measured, since they all are strongly affected by the separation zones. This separation cannot be predicted by the onedimensional momentum equation.

Based on the previous comparisons it can be seen that, although the differences, the above equations are interconnected, i.e. one may come from the others. W_t (K_w) may be found after the profiles' determination (integration).

PROFILE DETERMINATION METHOD

In order to dimensionalize the profile results suitable terms are used for all jumps :

Jumps over a sill

The dimensionless profiles in terms of

$$\bar{s} = (s - s_1)/(s_2 - s_1) \text{ vs } \bar{x}'' = x''/L_r ,$$

have been experimentally determined by Demetriou, 2007, as being described by the equation

$$\bar{s} \cong \left(7.48 - 4.76 \cdot e^{J_o}\right) \cdot (\bar{x}'') - \left(6.48 - 4.76 \cdot e^{J_o}\right) \cdot (\bar{x}'')^{1.5} \qquad (7)$$

for φ=0°-3°-6°-9°-12°-14° and Fr_1 up to 9, while $0 \le \bar{x}'' \le 1$.

The free jumps

A similar equation was also experimentally determined by Demetriou, 2006,

$$\bar{d} = (d - d_1)/(d_2 - d_1) \cong (3.37 - 8.11 \cdot J_o) \cdot (\bar{x}) - (2.37 - 8.11 \cdot J_o) \cdot (\bar{x})^{1.5} \qquad (8)$$

where $\bar{x} = x/L$ ($0 \le \bar{x} \le 1$), $0° \le φ \le 16°$, $2 \le Fr_1 \le 19$.

The repelled jumps

Another equation was also experimentally determined, by Demetriou et al, 2003-2006, for the repelled jumps,

$$\bar{h} = (h - h_1)/(h_2 - h_1) \cong \alpha \cdot (\bar{x}'') - (\alpha - 1) \cdot (\bar{x}'')^{1.5} \qquad (9)$$

where $\alpha = (-0.141 \cdot φ + 3.37) \cdot r^{(0.026φ + 0.67)}$, $0.5 \le r \le 1$ φ in degrees within the field φ=0°-2°-4°-6°-8° (which was slightly extrapolated here to φ=9°), $2.0 \le Fr_1 \le 6.0$, $\bar{x}'' = x''/L_h$ ($0 \le \bar{x}'' \le 1$).

PROFILES' COMPARISON

Next figures present a profiles' comparison among the three ascending parts of the jumps.

Fig. 2 for φ=0°, shows the free jump profile (solid line), the jump over a sill profile (dashed line) and 4 indicative profiles of repelled jumps with r=0.5-0.6-0.8-1.0 correspondingly (dashed – dotted lines). The last profiles are going up when r becomes larger, while for r=1 the repelled jump profile coincides with the free jump profile (upper limit). The jump over a sill profile lies under the free jump profile and over the r=0.6 repelled jump profile. As a general rule the jump over a sill profile lies in- between the repelled and the free jump profiles.

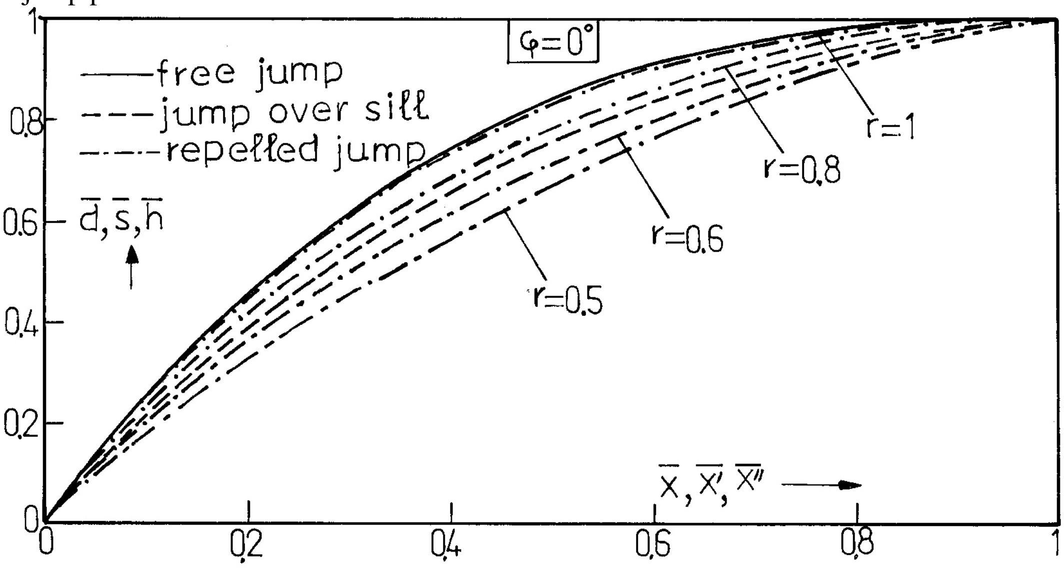

Figure 2. Profiles for φ=0°.

All jump profiles give $\bar{s}=\bar{d}=\bar{h}=0$ for $\bar{x}=\bar{x}'=\bar{x}''=0$, i.e. $s=s_1$, $d=d_1$, $h=h_1$, correspondingly, while for $\bar{x}=\bar{x}'=\bar{x}''=1$ $\bar{s}=\bar{d}=\bar{h}=1$, i.e. $s=s_2$, $d=d_2$, $h=h_2$, respectively. Since all jump lengths (L_r, L, L_h) are Froude number functions, all profiles are $\varphi(J_o)$, $\bar{x},\bar{x}',\bar{x}''$, and Fr_1 dependent, while the repelled jump profile is also r dependent.

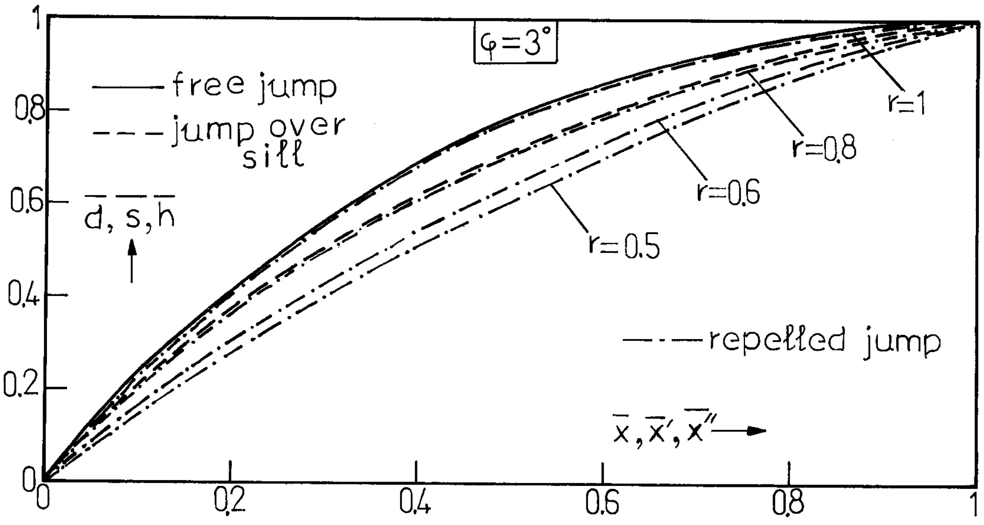

Figure 3. Profiles for φ=3°.

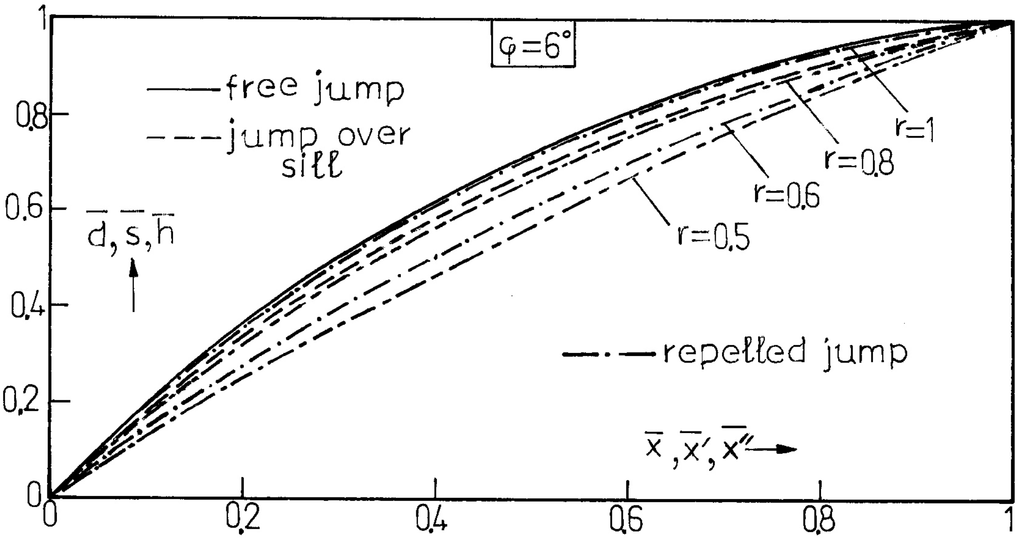

Figure 4. Profiles for φ=6°.

Figs. 3 (for φ=3°), 4 (for φ=6°) and 5 (for φ=9°) show similar profiles' behaviors, while the water weight W_t under these jump profiles is increasing from the repelled jump profiles (smaller r) to the jump over a sill profiles and to the free jump profile (r=1). In the latter case W_t becomes identical to the free jump water weight. The entire behavior is normal and very much as expected, in view of similarities and differences among the three jumps, while the general level of profile curves is lowering when angle φ is increasing from φ=0° to φ=9°. Also, when angle φ is increasing the jump over a sill profiles are approaching corresponding free jump profiles and show a tendency to coincide with them for φ=9°, i.e. one may expect that for φ>9° both profiles will be identical.

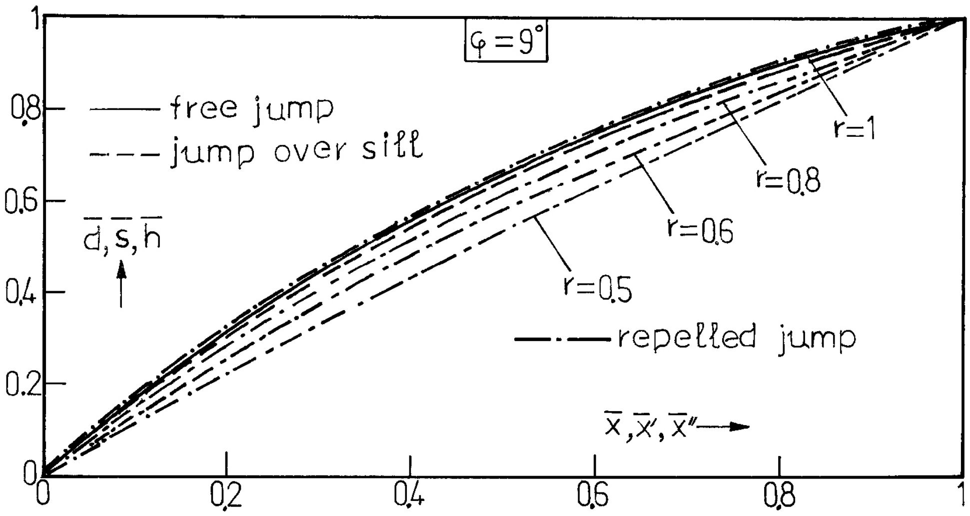

Figure 5. Profiles for φ=9°.

From the point of view of jump profiles, the repelled jump appears to have (for φ=const.) small water depths ($\overline{h} < \overline{s}$ - especially for r=small). This is an advantage – since lower side walls may be constructed – and is combined with larger energy losses along this jump (separation). For small φ angles the jump over a sill has also smaller water depths than the free jump depths ($\overline{s} < \overline{d}$), but this advantage is eliminated for larger angles φ ($\overline{s} \cong \overline{d}$). Finally for r→1 the repelled jump depths are approaching the free jump depths ($\overline{h} \cong \overline{d}$), while for r→0.5, $\overline{h} < \overline{d}$. The entire profiles' similarity does not mean that all three jumps are approaching a total common structure, since all the rest of quantities (such as lengths, conjugate depths' ratios, etc.) are quite different among them – especially in the cases of jump over sill (w), which is the only jump due to an obstacle along the flow, and of repelled jump, which is the only jump created within a non prismatic channel (r).

CONCLUSIONS

In this study, which is based on older laboratory measurements and further elaboration of them, a comparison is presented between flow theory (onedimensional equations) and geometry (free surface water profiles), concerning the free, the over a sill and the repelled inclined (angle φ with 0°≤φ≤9°) hydraulic jumps. The main conclusions are : (1) All profiles are very systematic when compared among them. (2) The general level of profile curves is lowering when angle φ is increasing. (3) The profiles are $\varphi(J_o)$, $\overline{x}, \overline{x}', \overline{x}''$, and Fr_1 dependent, while the repelled jump profiles are also r dependent. (4) The jump over a sill profiles lie between the free jump and the repelled jump profiles, while they both show a tendency to coincide for larger angles φ. (5) For r=1 the repelled jump profiles are identical to corresponding free jump profiles. (6) The onedimensional equations describing the three flow behaviors are interconnected. Although, because of flow separation in the case of repelled jump, the experimental results have a great significance. The present results may be useful to the hydraulic engineer when selecting the most convenient type of jump for his technical design needs.

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APPENDIX I. NOTATION

The following symbols are used:

q, discharge

F_{d1}, F_{s1}, Froude numbers

d, s, water depths

P_x, W_t, G_x, quantities in the onedimensional equation

$D=d_2/d_1$, conjugate depths ratio

$\bar{d}$, $\bar{h}$, dimensionless elements

$R=h_2/h_1$, conjugate depths ratio