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THE CRITICAL ANALYSIS OF MODELS OF TURBULENCE OF LAMINAR-TURBULENT TRANSITION AND A ROLE OF DIFFUSION BY PRESSURE FLUCTUATIONS

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ABSTRACT

The most representative moment low-Reynolds turbulence models are characterized by a limited accuracy of the laminar-turbulent transition prediction (Abe-Kondoh-Nagano, Abe-Jang-Leschziner, Abid, Jones-Launder, etc.). Given GIPS turbulence model (1. addition of $\partial pv/\partial y$ diffusion by pressure fluctuations with constant C_{pv} to k turbulence energy transition equation; 2. algebraic model for turbulent Reynolds stress models with two constant c_1 and c_2; 3. hypothesis of "complete" generation of turbulence on Heisenberg) allowed to increase the accuracy of a) prediction of the plate flow-past zone of the laminar-turbulent transition; b) calculation of c_f friction in a plane channel at low Reynolds numbers.

Keywords: Flucome 2009, Low-Re model, pressure-velocity, laminar-turbulent transition, DNS

INTRODUCTION

Laminar-turbulent transition prediction still remains a stumbling rock for the most part of moment turbulence models in spite of apparent advances in the calculation of complex turbulent flows. But calculations of the laminar-turbulent transition on various turbulence models just prove the fact of their poorness - see works by (Savill,2002), (Unger,1999) etc.. The emergence of "accurate" solutions by the direct numerical calculation of Navier-Stokes equations (DNS) allowed to concentrate the modification of turbulence models on the precision of f_μ damping function behavior in the turbulent viscosity expression, on the direct tailoring of turbulent diffusive factors in the equation of k turbulence energy transition (Hwang-Lin work), on the modification of the behavior of source members of ε diffusion transition equation (Jaw-Hwang work). However this method also failed to clarify the versatility of "new" dependencies. In his work Bradberg (Bradberg, 2001) compares DNS solution for a flat channel with 11 most representative Low-Re turbulence models. However the correctness of Bradberg 1D-model looks rather doubtful since our experience in such calculations shows that the more fancy empirical functions cumber the model, the longer and more torturous is the way to define the flow when calculating the channel with the initially uniform velocity profile at the entry (the example is known to all «calculators» - $k\text{-}\varepsilon\text{-}v'^2$ Durban model).

The success of the modification of well-known Lam-Bremhorst model just by the addition of the model of diffusion by pressure fluctuations into the equation of the turbulence energy transition

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(Golovnev&Platov, 2008), which has significantly improves the laminar-turbulent transition prediction, lack of a similar (to the transfer) testing of turbulence models for flows with low Reynolds number on the channel size (Rem), appearance of DNS solutions for channels with low Reynolds number (Reτ) (works by our Japanese colleagues) made us think that any dependencies for fμ damping function that contain only functions of Reynolds turbulent numbers in various forms, that is $Re_t=k^2/(\nu \cdot \varepsilon)$, $Re_k=k^{1/2}y/\nu$, y+, etc.), tuned especially for a certain Rem Reynolds number, are not versatile. Bradberg came to the same conclusion, analyzing the low accuracy of calculations by Hwang-Lin model. We think that any turbulence model which contains the distance to the nearest wall in any form is doomed to non-versatility. We applied Heisenberg idea that in a certain range of Reynolds numbers the turbulence may be generated … by viscous Newton stresses ($\tau_{ij}=\mu \cdot \partial U_i/\partial x_j$ or $uv=\mu \cdot \partial U/y$ for a thin layer). Formally in many monographs such a form of k turbulence generation representation by turbulent and molecular viscosity is given as

$$\lambda_k = P_k / \varepsilon = \left[(\mu + \mu_t) \times \partial U / \partial y \right] / \varepsilon$$

But according to "the traditions of Prandtl's analysis of small values" the "molecular" generation is neglected due to its smallness at high Re_t (Reynolds) turbulence values. Because of poor knowledge of German authors of given article "forgot" to omit this "small" member «on Prandtl» and with the use of a well-known algebraic turbulence model for stresses on LRR Launder-Reece-Rodi model, but with the generation of a "summary" model managed to get an adequate form of behavior of $f_\mu=(uv+\cdot\varepsilon+)/(C\mu \cdot k^{+2)}$ damping function.

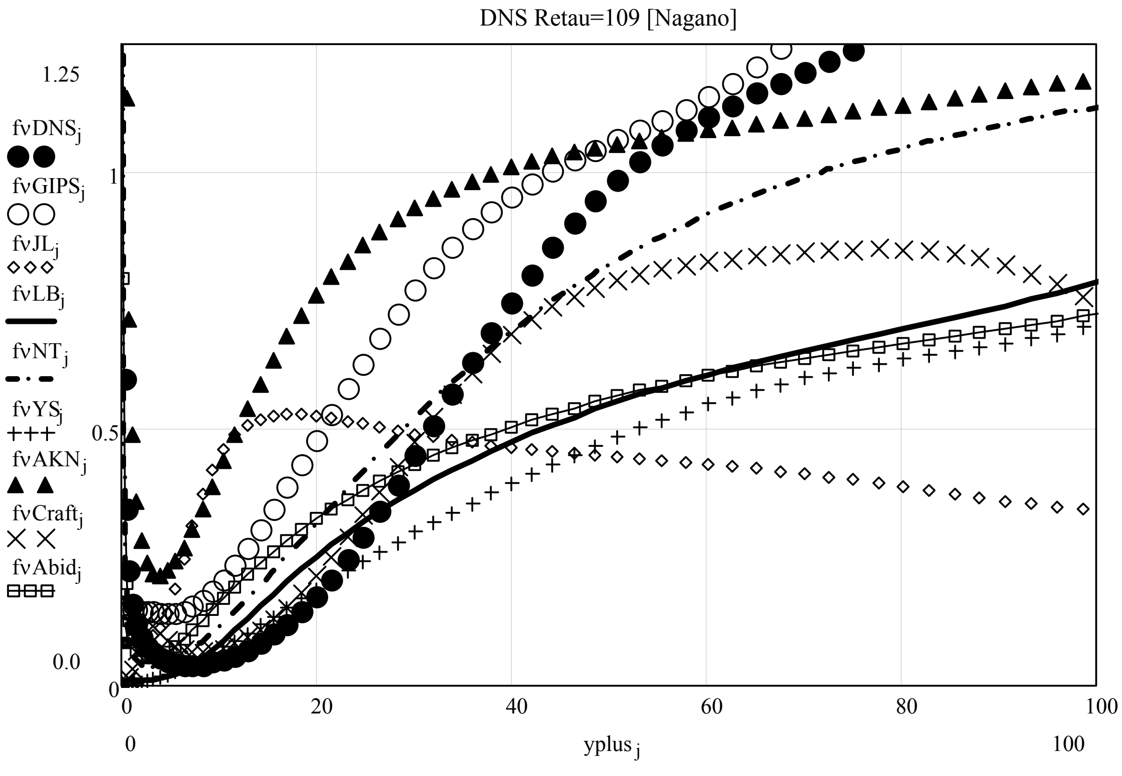

Fig. 1. fμ damping function on DNS (Re_τ=109, Nagano) and turbulence models (GIPS – model of the authors: JL – Jones-Launder, LB – Lam-Bremhorst, NT – Nagano-Tagawa, YS – Yang-Shih, AKN – Abe-Kondoh-Nagano, Craft – Craft-Launder-Suga, Abid – Abid)

Given work compares the results of calculation of the flow in a flat channel at low Reynolds numbers and the laminar-turbulent transition on plate according to GIPS turbulence model proposed by the authors and moment turbulence models by other authors.

GIPS universal turbulence model

GIPS (GolovnevIgorPlatovSergey) model proposed by the authors differs from well-known ones by the introduction of a model member for the diffusion by pressure fluctuations (in the equation of the turbulence energy transition) and the form of the complete turbulence generation in the algebraic model of Reynolds stress transition. A system of convective transition equations (continuity and fluctuation with the turbulent viscosity concept) given (just for good layout) in 2D representation (flat task)

$$\frac{\partial(\rho U)}{\partial x}+\frac{\partial(\rho V)}{\partial y}=0$$

$$\frac{\partial(\rho UU)}{\partial x}+\frac{\partial(\rho VU)}{\partial y}=\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial U}{\partial y}\right)-\frac{\partial \mathrm{p}}{\partial x}$$

is concluded by k-ε 2-parameter model (Craft,1996):

$$\frac{\partial(\rho Uk)}{\partial x}+\frac{\partial(\rho Vk)}{\partial y}=\frac{\partial}{\partial y}\left(\left\{\mu+\frac{\mu_t}{\sigma_k}\right\}\frac{\partial k}{\partial y}\right)+\mu_t\left(\frac{\partial U}{\partial y}\right)^2-\rho\varepsilon+D_k+\frac{\partial\overline{\mathrm{pv}}}{\partial y} \tag{1}$$

$$\frac{\partial(\rho U\varepsilon)}{\partial x}+\frac{\partial(\rho V\varepsilon)}{\partial y}=\frac{\partial}{\partial y}\left(\left\{\mu+\frac{\mu_t}{\sigma_\varepsilon}\right\}\frac{\partial \varepsilon}{\partial y}\right)+\left[C_{\varepsilon 1}\,\mu_t\left(\frac{\partial U}{\partial y}\right)^2-C_{\varepsilon 2}\rho f_2\varepsilon\right]\cdot T_k+E_\varepsilon \tag{2}$$

where:

$D_k=-2\mu\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2$;

$T_k=\frac{\varepsilon}{k}$;

$E_k=0.0022\mu_t\left(2\frac{k}{\varepsilon}\left|\frac{\partial U}{\partial y}\right|\right)\frac{k^2}{\varepsilon}\left(\frac{\partial^2 U}{\partial^2 y}\right)^2$

$C_{\varepsilon 1}=1.44$; $C_{\varepsilon 2}=1.92$; $\sigma_k=1$; $\sigma_\varepsilon=1.3$;.

$f_2=1-0.3e^{-Re_t^2}$

Turbulent viscosity is determined by an algebraic model for Reynolds stresses

$$\mu_t=\rho\times C_\mu\times f_\mu\times(k^2/\varepsilon)=\mu\times C_\mu\times f_\mu\times \mathrm{Re}_t \tag{3}$$

Complete LRR model (Launder-Reece-Rodi,1975),

$$\frac{u'^2}{k}=\frac{2}{3}\times\left[1+\lambda_\Sigma\times\frac{\beta_{LRR}+2\times(1-\alpha_{LRR})}{C_{1LRR}+\lambda_\Sigma-1}\right] \qquad \frac{v'^2}{k}=\frac{2}{3}\times\left[1-\lambda_\Sigma\times\frac{\beta_{LRR}+2\times(1-\alpha_{LRR})}{C_{1LRR}+\lambda_\Sigma-1}\right] \tag{4}$$

$$C_\mu\times f_\mu=\frac{2}{3}\times\left[\frac{-\beta_{LRR}\times(u^2/k)+\gamma_{LRR}+(1-\alpha_{LRR})\times(v^2/k)}{C_{1LRR}+\lambda_\Sigma-1}\right] \tag{5}$$

and C_1=1.5; C_2=0.40; $\alpha_{LRR}=(C_2+8)/11$ $\beta_{LRR}=(8\times C_2-2)/11$ $\gamma_{LRR}=(30\times C_2-2)/55$

The innovation of our model lies in the introduction of two new members:
A) to express the turbulence energy for the generation member in algebraic expressions we use Heisenberg hypothesis (Heisenberg, 1924) of the generation both by viscous Newton and Reynolds turbulent stresses

$$\lambda_\Sigma=P_\Sigma/\varepsilon=[(\mu+\mu_t)\times\partial U/\partial y]/\varepsilon \tag{6}$$

B) turbulent diffusion by triple moments and pressure fluctuations (kv and vp) in the equation of k turbulence energy transition are being modeled separately.
Triple correlation of velocity is given in a common gradient presentation

$$\frac{\partial}{\partial y}\left[\overline{\mathrm{vk}}\right]=\frac{\partial}{\partial y}\left[\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right] \tag{6}$$

but for a member with pressure fluctuations our GIPS model is used

$$\frac{\partial\overline{\mathrm{vp}}}{\partial y}=C_{pv}\frac{\partial(k\cdot U)}{\partial y} \tag{7}$$

We used equation (7) earlier, though not with proposed here algebraic model – (4-5) ratios, but by simple additive (without changing original constants and functions) inclusion of Lam-Bremhorst turbulence into the model; and this inclusion, according to the work (Golovnev&Platov,2008), has significantly extended and improved the possibility to predict the laminar-turbulent transition on the model considered as basic one for SOLID WORKS complex of 3D visual simulation.
Here are some details of our ideas about the presentation of the diffusion by pressure fluctuations (7).
Experiments (Leontyev, 1972) and DNS digital modeling results allow to consider approximately that R_{PV} correlation factor

$$R_{PV} = \overline{pv}/\left(\sqrt{p^2}\sqrt{v^2}\right) \tag{8}$$

is constant across the shifted layer, that is, formally

$$p'v' \equiv pv = R_{pv} \times \sqrt{p^2} \times \sqrt{v^2} \tag{9}$$

For non-sheared turbulence in $\partial U / \partial y \approx 0$ thin sheared layer (for example, on the channel axis) the turbulence is close to the isotropic one, then pressure fluctuations (Kraichnan,1956): $p \approx A \times u'^2$, (where A proportionality factor depends on Re_t turbulent Reynolds number (often is taken as A=0.5), then the expression for pv becomes similar to Shishov model (Leontyev&Shishov&Roganov, 1979):

$$pv = C_{pvS} \times \left(\sqrt{u^2}\right)^2 \times \sqrt{v^2} = C_{pvS} \times \left(u^2 \times v\right) \tag{10}$$

and just becomes “a part” of the common gradient representation of the turbulent diffusion while C_{pvS} factor is added to C_{uuv} model expression factor of u^2v triple correlation.

Few but correct measurements of pressure fluctuations in various turbulent shear currents (flows) of Kawamura (1960), Kobashi (1957), Kono (1982), Kuroda-Matsuzawa-Ogawa-Inoue (1983) show that Eu’=p’/(ρu’U) Euler fluctuation number is slightly changing across the shear layer (in channels, jets, wakes and boundary layers) and may be caused by a cascade-type mechanism of turbulence energy transition to the dissipation taking energy from the operation of strength (pressure) fluctuations of the mean flow and further on from “large” eddies to “minor” ones. Rotta described this physically as the dissipation of velocity fluctuations at moles of pressure fluctuations. Note that this linear dependence just follows from known Laplace equation for pressure fluctuations in case u '<<U.

Simplifying and terminating the turbulence model at “k-ε” level on Launder and supposing $u'^2 \approx A_u * k$ and $v'^2 \approx A_v * k$ proportionalities (where A_u and A_v are constant) we get an extremely simple expression for pv:

$$pv = C_{pv} \times k \times U \tag{15}$$

where C_{pv} is some “total” factor received, according to Launder method, by simple numerical optimization. For GIPS model we have found C_{PV}=0.003 factor as an optimal one.

Calculations were performed in the parabolic 2D stationary installation on the program similar to GENMIX (Spalding,1977) one. Number of the network nodes in across-track direction n = 80.

Boundary conditions

At a solid wall: U=V=0; k_w=0; ε_w=0

At external edge:

A) for a channel $\frac{\partial U}{\partial y} = 0$; $\frac{\partial k}{\partial y} = 0$;$\frac{\partial \varepsilon}{\partial y} = 0$

б) for a boundary layer at a plate (U_∞=const) with Tu external turbulence similar to Unger work (Unger,2002) on power laws of turbulence degeneration:

$$Tu^{-2} = C \times \left(x + x_0\right)^m ;$$

$$k = 1.5\left(Tu \cdot U\right)^2 = 1.5 \cdot C^2 \cdot U_\infty^2 \cdot \left((x + x_0)/d\right)^{-2/n}$$

$$\varepsilon(x) = \frac{3}{2} \cdot \frac{2}{n} \cdot C^2 \cdot \frac{U_\infty^2}{d} \cdot \left((x + x_0)/d\right)^{-2/n-1}$$

with degeneration power index n = 1.2 and factor C = 0.833

Calculation of turbulent channel flow at very low Reynolds numbers

DNS data from the work (Tsukahara&Seki&Kawamura, 2006) for calculations are given in Table No. 1

Table 1

Re□	Rem	Rec	u□	Cf	ρ	μ		h	Uc	Um		Uin
64	1860	1200	0,10	0,00569	1,18	1,84E-05	#	0,01	1,866	1,446	#	1,45
70	2020	1270	0,11	0,00608	1,18	1,84E-05	#	0,01	1,975	1,570	#	1,57
80	2320	1430	0,12	0,00626	1,18	1,84E-05	#	0,01	2,224	1,804	#	1,78
110	3290	1960	0,17	0,00630	1,18	1,84E-05	#	0,01	3,048	2,558	#	2,46
150	4620	2720	0,23	0,00608	1,18	1,84E-05	#	0,01	4,229	3,592	#	3,39
180	5730	3360	0,28	0,00574	1,18	1,84E-05	#	0,01	5,224	4,455	#	4,17

Flat channel calculation was performed as developing up to stabilization with a thin input laminar boundary layer and turbulence in the flow kernel on the channel axis equal to 3%. Such an approach, differing from Bredberg testing (Bredberg, 2002), allows to track the dynamics of the flow in a long flat channel (L>500h):

Table 2

Reт	64	64	70	70	80	80	110	110	150	150	180	180
turbulence model	cf	Δ,%	cf	Δ,%	cf	Δ,%	cf	Δ,%	cf	Δ,%	cf	Δ,%
DNS [Tsukahara]	0,0057		0,0061		0,0063		0,0063		0,0061		0,0057	
Abe-Jang-Leschziner	0,0076	25,4	0,0075	19,3	0,0073	14,7	0,0069	8,7	0,0065	6,4	0,0062	7,9
Abe-Kondoh-Nagano	0,0069	17,8	0,0069	11,7	0,0068	7,8	0,0065	2,8	0,0061	0,9	0,0059	2,7
Abe-K-N(rev.) +∂pv/∂y	0,0059	3,6	0,0061	0,1	0,0062	-1,5	0,0064	1,6	1,0000	99,4	0,0058	0,7
Abid	0,0029	-98,2	0,0046	-30,9	0,0052	-20,4	0,0055	-13,7	0,0055	-10,6	0,0054	-6,3
Chen-Patel, "1k"	0,0083	31,8	0,0081	25,1	0,0078	19,3	0,0070	9,9	0,0064	4,5	0,0060	4,7
Fan-Lakshmira-Barnett	0,0029	-98,2	0,0048	-27,1	0,0052	-20,1	0,0055	-14,7	0,0054	-12,0	0,0053	-7,9
Gatski-Speziale II	0,0029	-94,2	0,0026	-130,2	0,0053	-18,6	0,0050	-26,7	0,0051	-19,7	0,0051	-13,4
Golovnev-Platov GIPS	0,0057	0,9	0,0063	4,2	0,0062	-0,5	0,0062	-1,1	0,0060	-2,1	0,0060	4,3
Hwang-Lien	0,0097	41,2	0,0094	35,0	0,0091	30,9	0,0084	24,6	0,0077	20,6	0,0073	21,8
Hwang-Lien(rev.) +∂pv/∂y	0,0067	14,6	0,0064	4,3	0,0076	18,1	0,0069	9,2	0,0070	13,1	0,0063	8,3
Iacovides-Launder, "1k"	0,0088	35,4	0,0086	29,1	0,0082	23,7	0,0074	14,8	0,0067	9,6	0,0064	9,6
Jaw-Hwang	0,0080	28,6	0,0078	22,5	0,0078	19,4	0,0073	13,7	0,0067	9,4	0,0063	9,2
Jaw-Hwang(rev.) +∂pv/∂y	0,0060	5,0	0,0068	10,3	0,0068	7,7	0,0065	2,5	0,0059	-2,4	0,0059	1,9
Jones-Launder	0,0058	2,6	0,0059	-3,0	0,0059	-5,4	0,0059	-6,4	0,0057	-6,0	0,0056	-2,9
Lam-Bremhorst	0,0073	21,7	0,0072	15,5	0,0071	11,2	0,0067	6,0	0,0060	-2,1	0,0061	6,2
Lam-Bremhorst +∂pv/∂y	0,0062	7,8	0,0068	10,5	0,0067	6,0	0,0063	0,6	0,0059	-3,1	0,0058	1,5
Launder-Sharma	0,0029	-98,2	0,0026	-130,2	0,0023	-172,2	0,0017	-281,8	0,0052	-17,4	0,0052	-11,0
Nagano-Hishida-Asano	0,0049	-16,1	0,0050	-22,3	0,0050	-24,9	0,0049	-28,0	0,0048	-28,1	0,0046	-24,5
Nagano-Tagawa	0,0053	-7,5	0,0053	-14,9	0,0053	-18,8	0,0052	-22,1	0,0050	-22,4	0,0048	-18,8
Norris-Reynolds, "1k"	0,0060	-27,4	0,0092	18,2	0,0089	17,2	0,0081	14,3	0,0074	11,7	0,0070	17,4
Rodi, "1k"	0,0090	36,9	0,0088	30,9	0,0084	25,7	0,0076	17,2	0,0069	12,2	0,0066	12,4
Wilcox LRN, "k-ω"	0,0089	35,7	0,0083	26,6	0,0083	24,3	0,0076	16,7	0,0070	12,7	0,0063	8,3
Yang-Shih	0,0072	20,8	0,0070	12,7	0,0070	10,5	0,0065	2,6	0,0060	-1,5	0,0058	1,4
Yang-Shih + ∂pv/∂y	0,0060	5,3	0,0062	2,0	0,0061	-3,0	0,0063	-0,6	0,0058	-4,7	0,0053	-8,7

where +∂pv/∂y symbol means additive inclusion of the model of diffusion by pressure fluctuations in k turbulence energy transition, and (rev.) symbol means the change of “original” values of coefficients in k-ε expression on standard $C_{\varepsilon 1}$=1.44; $C_{\varepsilon 2}$=1.92; σ_k=1; σ_ε=1.3 ones.

For the lowest Reynolds number - Reτ=64:

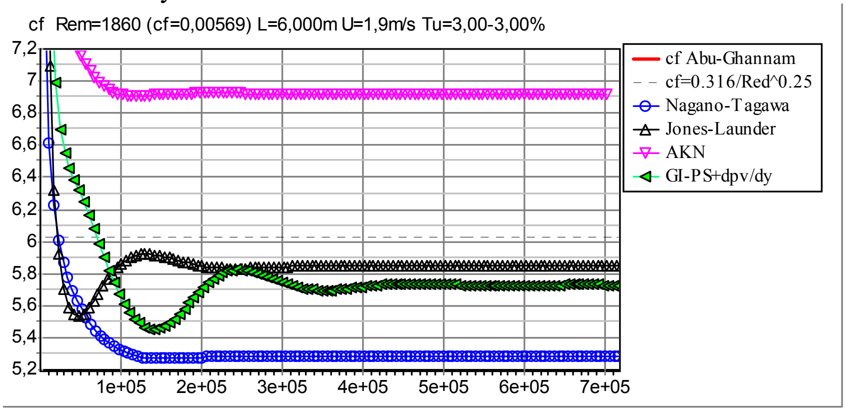

Fig. 2. Calculation of c_f friction in the channel (bottom axis – Re on the axial coordinate)

For average Reynolds number - Reτ=110:

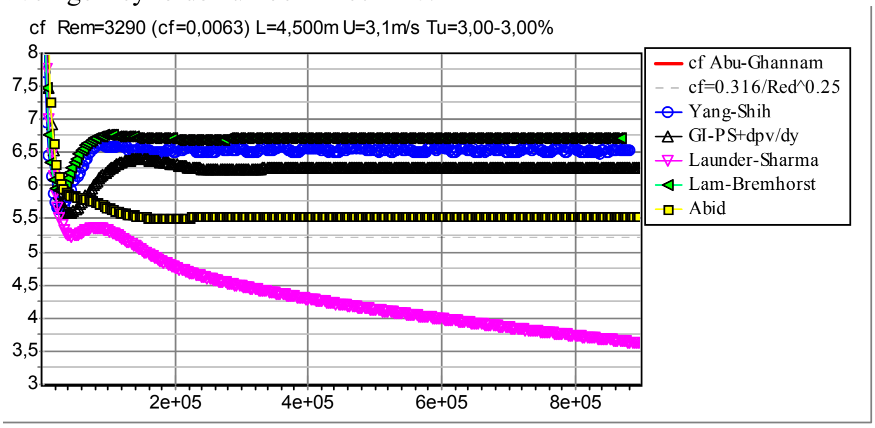

Fig. 3. Calculation of c_f friction (on Launder-Sharma model – degeneration into the laminar flow)

For small Reynolds number - Reτ=70:

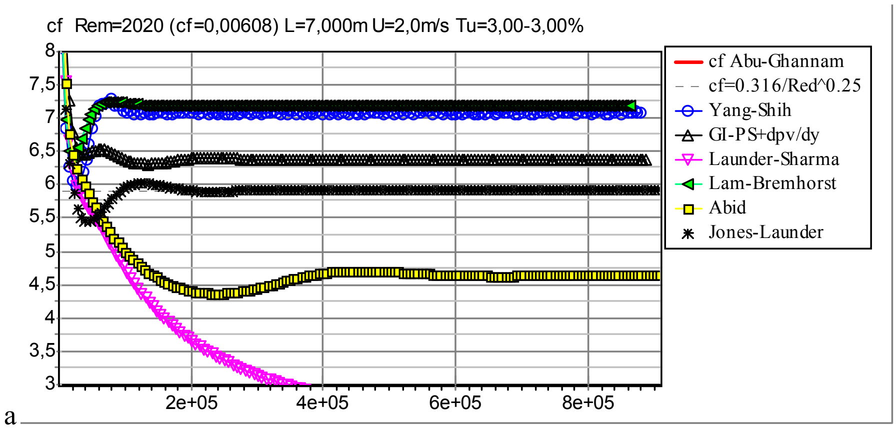

Fig. 4. Calculation of c_f friction (on Launder-Sharma model – degeneration into the laminar flow)

Calculation of the laminar-turbulent transition

Unger thesis (Unger, 1999) shows the poorness of Lam-Bremhorst model (Lam&Bremhorst, 1981) of the laminar-turbulent transition. Simple additive inclusion of our model of the diffusion by pressure fluctuations (Golovnev,Platov,2008) has significantly changed the ability of this model to predict c_f friction of the laminar-turbulent transition, as may be seen from the Figure (for Tu≈2% case):

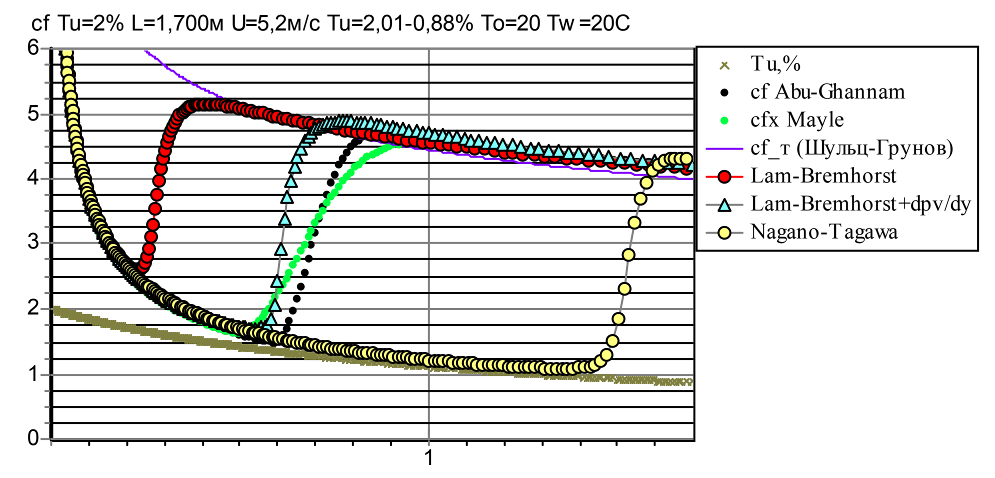

Fig. 5. Calculation of c_f friction of the laminar-turbulent transition on the plate

Comparison here is made by known correlative dependencies of Abu-Ghannam, Mayle and calculations of the representative turbulence model (Nagano&Tagawa, 1990) for which it occurred possible also to improve the transition prediction:

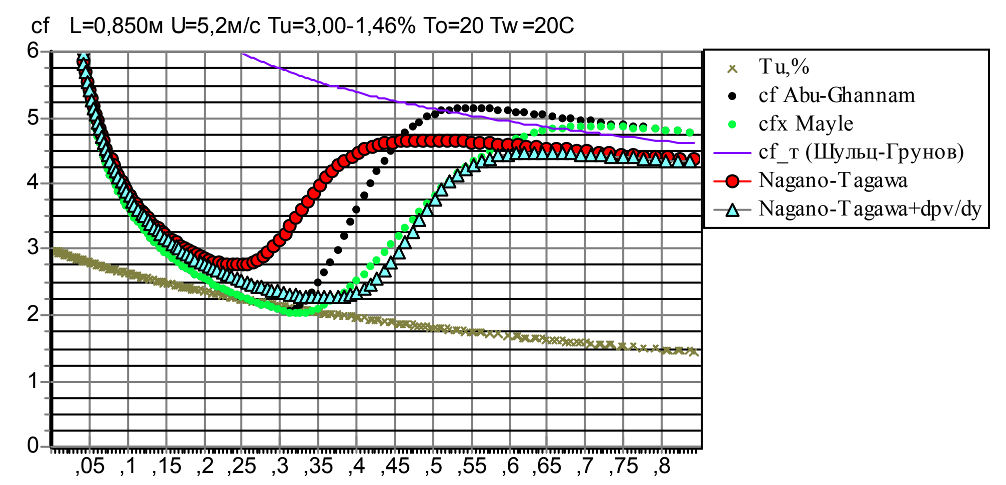

Fig. 6. Calculation of c_f friction of the laminar-turbulent transition on the plate

This additive inclusion of the member of the diffusion by pressure fluctuations allowed even … to re-standardize “fancy” constants of certain models. For example, Abe-Jang-Leschziner constants of the initial algebraic model of Reynolds stresses (Lardeau&Leschziner&Li, 2004) that is $C_{\varepsilon 1}$=1.45; $C_{\varepsilon 2}$=1.83;

σ_k-1.2; σ_ε=1.5 have regained "common" values that is $C_{\varepsilon1}$=1.44; $C_{\varepsilon2}$=1.92; σ_k=1.0; σ_ε=1.3 at C_{PV}=-0.008; accuracy of the transition prediction has been significantly improved what can be seen in the Figure:

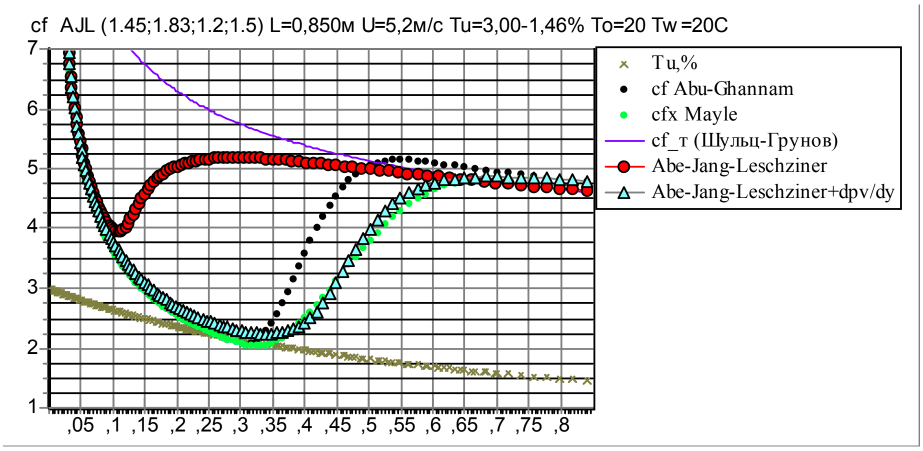

Fig. 7. Calculation of c_f friction of the laminar-turbulent transition on the plate

And here, at last, are the results of the heat exchange calculation (Stanton number) on GIPS model (at Pr_t=0.90) according to experimental data (Blair&Werle, 1980):

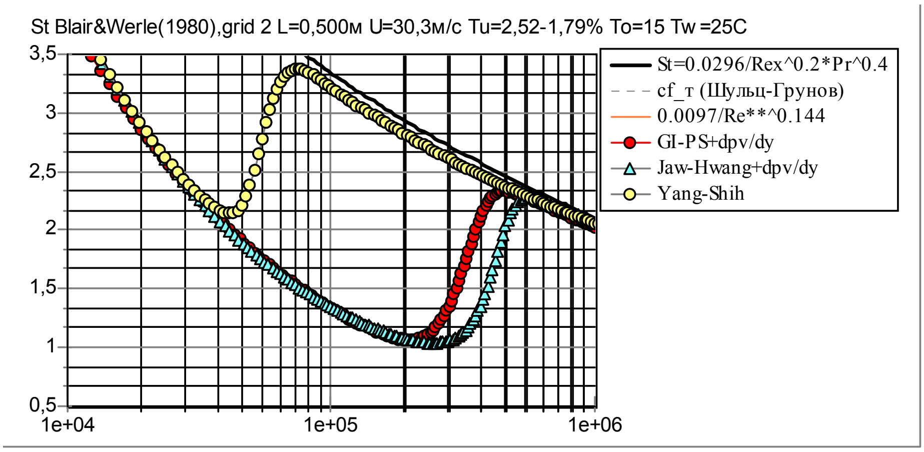

Fig. 8. Calculation of c_f friction of the laminar-turbulent transition on the plate

from where it is easy to see that the model (Yang&Shih, 1993) renders much earlier heat exchange intensification than our GIPS model and modified by us Jaw-Hwang model (with re-standardized factors and addition of the model of the diffusion by pressure fluctuations).

Conclusion

Thus, proposed by us GIPS model may be used as a point-to-point method of laminar-turbulent transition calculations without any experimental data (and duly correlations) and *ad hoc* functions in any range of Reynolds numbers (from viscous-laminar to developed turbulent).

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APPENDIX I. NOTATION

The following symbols were used …

c_f	skin-friction coefficient, $\tau_w/(\rho U_c^2/2)$
C_{pv}	coefficient of model "pressure-velocity"
h	½ height of channel
f_μ, f_v	wall-damping function, f_μ=(uv+•ε+)/(Cμ•k$^{+2)}$
k	turbulence energy, $(u^2+v^2+w^2)/2$
c	covariation $p^{'}v^{'}$
Re_c, Re_m, Re_τ	Reynolds number (Uc, Uτ, h), Um, 2h
Tu	turbulence intensity
u, v	fluctuation of velocity
U, V	average velocity
U_c	axis velocity
U_m	average velocity in channel
u_iu_j	stress tensor
x, y	coordinates
ε	dissipation rate of turbulence energy
μ, μ_t	molecular and turbulent viscosity
τ_w	skin-friction
(rev.)	Modification of k-ε models into "standard" values of variables in ε dissipation equation ($C_{\varepsilon 1}$=1.44; $C_{\varepsilon 2}$=1.92; σ_k=1; σ_ε=1.3) + ∂pv/∂y