Abnormal Kinetic Energy of Charged Dust Particles in Plasmas

G. Norman, V. Stegailov, and A. Timofeev

Joint Institute for High Temperatures RAS, 125412 Moscow, Russia
Moscow Institute of Physics and Technology (State University), 141700 Dolgoprudny, Moscow Region, Russia

Received 18 September 2009, accepted 08 December 2009
Published online 25 January 2010

Key words Dusty plasmas, kinetic temperature, fluctuating charge, forced resonance, parametric resonance.
PACS 52.27.Lw

A mechanism of the increase of the average kinetic energy of charged dust particles in gas discharge plasmas is suggested. Particle charge fluctuation is the reason for the appearance of forced resonance, which heats vertical oscillations. The energy transfer from vertical oscillations to the horizontal ones is based on the parametric resonance. It arises because of the overlapping of the eigenfrequency range of the horizontal oscillations in a dust particle cluster with the eigenfrequency range of particle vertical oscillations in near-electrode plasmas. The combination of the parametric resonance and the forced resonance explains the high kinetic temperature of dust particles. The estimated frequency, amplitude and kinetic energy are close to the experimental values.

© 2010 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

1 Introduction

Anomalously large kinetic energy of dust particles in gas discharge plasmas attracts a remarkable interest in the study of dusty plasmas [1–3]. It is found that dust particles in gas discharge plasma can acquire kinetic energy about 10 eV and above [4–12]. The value significantly exceeds both the ion and electron temperatures. At such kinetic energies dust particles continue to shape crystalline structure [13–15].

As the phenomenon of an abnormal heating of dust particles in plasma is known since 1996, a number of attempts has been made to find the mechanism of this phenomenon [8, 11, 16–19]. However all of them result in the estimations which are considerably lower than the experimental values. Effect of electric field fluctuations [11, 16] leads to heating of dust particles to the kinetic temperature above the temperature of ambient gas, but on two or more orders of magnitude smaller than the experimental values. Moreover the lack of correlation between fluctuations of the electric field and kinetic temperature of dust component of plasma is experimentally demonstrated [10]. The influence of dust particle charge fluctuations on kinetic temperature is considered on the basis of the Langevin approach, which allows to get the value of two orders of magnitude below the experimental value [8]. Spatial fluctuations of dust particle charges [9, 12, 14, 20] also lead to the increase of kinetic energy of dust particles. However this mechanism is based on a finite charging time of dust particles and gives an estimate of the kinetic temperature many orders of magnitude below the experimental value. Thus, the effect of the abnormally high kinetic energy of dust particles remains unresolved.

This paper presents a mechanism of the increase of the average kinetic energy of dust particles in gas discharge plasma. In this article we use [21–27] to account for characteristics of near-electrode layer of gas discharge and the literature on the theory of forced and parametric resonances [28, 29] to account for the effects of oscillatory motion of a dust particle. The heating of vertical motion of a dust particle is considered in section 2. The heating of horizontal oscillations of system of dust particles in plasma gas discharge is discussed in section 3. Comparison of the suggested theory and experimental data is presented in section 4.
2 Vertical motion of a single dust particle in the near-electrode layer

The dust particle in gas discharge plasma is charged and usually hangs in the electrode layer, where the electric field is strong enough to offset the force of gravity. Important forces acting on dust particles are drag force created by flux of ions, electric field force, dust pressure force, thermophoretic force, friction force in a neutral gas and, finally, gravity force. The friction force prevails over ion drag force, thermophoretic force and some other forces for dust particles with high kinetic energy. So we neglect these forces for this particular problem.

The vertical motion of a single dust particle in near-electrode layer of gas discharge is considered:

$$m\ddot{y} = -mg + Z(y, t)eE(y) - 2m\gamma\dot{y},$$  \hspace{1cm} (1)

where $y$ is dust particle displacement from the equilibrium position on the vertical axis, $m$, $Z$ are mass and average charge number of a dust particle, $e$ is electron charge, $E$ is electric field near the particle, $\gamma$ is coefficient of friction. Electric field near a dust particle in the near-electrode layer of a gas discharge can be represented as

$$E \approx E_0(1 + ety + ety^2/2),$$  \hspace{1cm} (2)

where $E_0$ is the field in the equilibrium position of the dust particle, $et$, $ety$ are normalized expansion coefficients. The dust particle charge in plasma is determined by the flow of plasma components on the mote. So the charge fluctuates over time due to fluctuations of the flow of ions and electrons on the surface of dust particle:

$$Z(y, t) \approx Z_{eq}(y) + Z_0 \cdot \delta q(t),$$  \hspace{1cm} (3)

where $Z_{eq}$, $Z_0$ are average charges of dust particle at height $y$ and at height $y = 0$ accordingly, $\delta q(t) \approx \sum_{i} q_i \sin(\omega_i t + \phi_i)$ is Fourier series expansion of fluctuating term of dust particle charge, normalized to the average charge of dust particle, $q_i$, $\omega_i$, $\phi_i$ are coefficients, frequencies and phases of harmonic expansion. Electron and ion densities vary greatly with height in the near-electrode layer of a gas discharge. It leads to the dependence of a dust particle equilibrium charge on the height:

$$Z_{eq}(y) \approx Z_0(1 + q'ty),$$  \hspace{1cm} (4)

where $q'$ is the expansion coefficient. Taking into account expressions (2)-(4) the electric force is represented as

$$Z(y, t) \cdot e \cdot E(y) \approx Z_0 e E_0(1 + q'ty + \delta q(t))(1 + ety + ety^2/2) \approx Z_0 e E_0 \cdot f(y) + Z_0 e E_0 \cdot \delta q(t),$$  \hspace{1cm} (5)

where the last item depends only on time. The force of gravity is balanced in equilibrium by the electric force:

$$mg = Z_0 e E_0.$$

(6)

The classical equation of an oscillator with friction and external force is obtained by substitution expressions (5), (6) in the equation (1) and linearization:

$$\ddot{y} + 2\gamma\dot{y} + \Omega_y^2 y \approx g \cdot \delta q(t),$$  \hspace{1cm} (7)

where frequency of free oscillations is determined by expression:

$$\Omega_y = \sqrt{-g(\epsilon t + q')}.$$  \hspace{1cm} (8)

Fluctuations of the dust particle charge are characterized by a wide spectrum. We can select harmonic of fluctuating charge with a frequency close to the eigenfrequency of system oscillations:

$$\ddot{y} + 2\gamma\dot{y} + \Omega_y^2 y \approx g \cdot q_k \sin(\Omega_y t) = F(t),$$  \hspace{1cm} (9)

In this case forced resonance of exciting force with the eigenoscillations of the system is occurred [26-27]. It leads to an increase in the average kinetic energy of the system. Solution of equation (9) allows us to obtain formulas for estimating frequency $\Omega_y$, amplitude $A_y$ and average kinetic energy $K_y$ of dust particle oscillations.

$$A_y \approx g \cdot q_k / 2\Omega_y \gamma.$$  \hspace{1cm} (10)
\[ K_y = \frac{\langle m v^2 \rangle}{2} \approx m \left( g \cdot \frac{q_k}{2 \gamma} \right)^2 / 2. \]  

(11)

The account of non-linear terms of expression (5) leads to occurrence of the multiple frequencies \( \Omega_y, 2\Omega_y, \ldots \) in a spectrum of oscillations of dust particle.

This section considers the first two upper arrows in Fig. 1. External source supports gas discharge. Dust charge fluctuations lead to occurrence of the exciting force by the mechanism described. The energy pumping by exciting force into vertical motion of dust particles becomes possible due to the overlapping of the frequency ranges (Fig. 2) of exciting force and vertical eigenoscillations of dust particle. So, the inflow of kinetic energy of dust particle is explained by charge fluctuations of dust particles. Energy loss is explained by friction with neutral gas. The balance between energy loss and energy inflow determines the amplitude and the kinetic energy of dust particles. Thus, this mechanism explains the heating of the vertical oscillations of dust particles in plasma and allows obtaining basic characteristics of vertical motion of dust particles. However, the described mechanism does not explain the abnormal high kinetic temperature of the horizontal motion.

Fig. 1 Scheme of energy transfer from the gas discharge to the movement of dust particles. (Online colour: www.cppjournal.org)

Fig. 2 Frequency ranges of various oscillatory processes in dusty plasmas: 0 – oscillation of dust particles in laboratory experiment; 1 – charge fluctuation of a dust particle; 2 – eigenfrequencies of particle vertical oscillations in near-electrode plasma; 3 – eigenfrequencies of particle horizontal oscillations in a field-trap; 4 – eigenfrequencies of plasma-dust cluster, obtained by molecular dynamics.

### 3 Horizontal motion of the dust particle interacting with other dust particles.

We take into account forces of gravity, friction and interaction between dust particles and electric force. The trap force retains dust particles from scattering in a horizontal plane.

\[ \begin{align*}
\ddot{x} &= \sum F_{\text{inter}} + F_{\text{trap}} + F_{\text{friction}} + \ldots \\
\ddot{y} &= \sum F_{\text{inter}} + F_{\text{elecr}} + F_{\text{friction}} + F_{\text{grav}} + \ldots
\end{align*} \]

(12)

The interparticle interaction is simulated by Yukawa potential \( U_{ij}(r_{ij} - r_i) \approx Q^2 e^{-\kappa r_{ij}} / r_{ij} \). The trap keeping dust particles from scattering is described by a parabolic potential. The remaining forces acting on the particle were described in the section about the vertical movement of a single particle. The forces acting on dust particles are expanded in a Taylor series. The system of equations describing the motion of dust particles is obtained by leaving only significant terms of the expansion:

\[ \begin{align*}
\ddot{x} &= -a_1 x + a_2 x z + a_3 x^3 + a_4 x z^2 + \ldots \\
\ddot{y} &= -b_1 y + b_2 y^2 + b_3 y^3 + b_4 y x^2 + \ldots + F(t)
\end{align*} \]

(13)
where \( a_j, b_j, j \in N \) are coefficients of the force expansion in Taylor series. Nonlinearity of (13) leads to splitting of horizontal and vertical eigenfrequencies. The overlapping of these frequency sets and the main items of (13)

\[
\begin{align*}
\ddot{x} &\approx -a_1 x + a_4 x z^2 + \ldots \\
\ddot{y} &\approx -b_1 y + b_4 x y^2 + \ldots
\end{align*}
\]  

(14)

reveal the appearance of phenomenon [28, 29] of parametric resonance. It forms the mechanism of transferring energy from the vertical oscillations to horizontal ones. Numerical estimation shows the equality of coefficients \( a_4 = b_4 \), so the second parametric resonance appears. It heats the vertical oscillations due to the horizontal. This process slows down the heating of the horizontal oscillations and leads system to balance:

\[
A_y^2 - A_x^2 \approx \lambda \Omega_y / b_4 > 0.
\]  

(15)

The amplitude of the vertical oscillations turns out to be larger than the amplitude of horizontal vibrations.

The general scheme of heating is shown in Fig. 1. External source supports gas discharge, which in turn provides the charge fluctuations of dust particles. Dust charge fluctuations lead to occurrence of the exciting force by mechanism described. The energy pumping by exciting force into vertical motion of dust particles becomes possible due to the overlapping of the frequency ranges (Fig. 2) of exciting force and vertical eigenoscillations of dust particle. Large amplitude of dust particle oscillations leads to occurrence of nonlinear terms in the expansion of the forces acting on the particles. The crystalline order of the plasma-dust system, nonlinear effects, the form of system (13), the overlapping of the frequency ranges of vertical and horizontal oscillations indicate the parametric resonance. It supports the exchange of energy between the horizontal and vertical oscillations. Note that the pumping of the horizontal oscillation occurs at a frequency of vertical oscillations, and the frequency of vertical oscillations is mainly determined by the parameters of the near-electrode layer. The balance between the energy loss of dust particles and energy inflow determines the amplitude and the kinetic energy of dust particles.

4 Comparison with experiment

We have two free parameters in the model developed: electric field gradient in near-electrode layer, amplitude of the fluctuations of the charge. Also in developed model we have three parameters to be compared with experiment: frequency, amplitude and kinetic temperature of dust particle oscillations in plasma.

The frequency \( \Omega_y = \sqrt{-g(e\epsilon + q\eta)} \) of dust particle oscillations in near-electrode layer explains the small width of the range and the characteristic values of the vertical oscillatory frequencies of dust particles obtained from experimental data. The theory of near-electrode layer allows to make a rough estimate of the parameter \( e\epsilon + q\eta \). Thus, we obtain the frequency range \( \Omega_y \approx 20 \div 170 \text{ s}^{-1} \). Experimental data on the frequencies of dust particle oscillations fall into estimated range.

The vertical oscillation amplitude \( A_y \approx g \cdot q_k / 2\Omega_{y} \gamma \) has power dependence on the neutral gas pressure \( A_y \propto p^{-1.5} \) for the typical conditions of laboratory experiments on dusty plasma. Approximation of experimental data [7] (Fig. 3) confirms the gained dependence and gives the dependence of the oscillation amplitude on pressure.
the discharge power $A_d(p, W) \approx (−7, 5 + 33 \cdot W^{−0.3}) \cdot (p_d)^{−1.5}$, where $W$ is discharge power. Numerical evaluation of the kinetic temperature of dust particles is also close to the experimental values: $T_d \approx 50 \text{ eV}$[2], $T_d \approx 17 \text{ eV}$[6], $T_d \approx 1 \div 10 \text{ eV}$[3].

Prevalence of vertical amplitude over horizontal has also proved to be true for the experimental data [7].

## 5 Conclusions

Abnormal kinetic energy (temperature) of dust particles in gas discharge plasma is derived from the theory taking into account charge fluctuations, features of near-electrode plasma and description of the dust particle motion near the equilibrium position in dust crystal in terms of the theory of forced and parametric resonances.

a) The oscillation frequency $\Omega_d$ of dust particle mainly depends on gradient of electric field (2) and gradient of dust particle charge (4) as given in (8).

b) The spectrum of vertical oscillations is likely to consist of two main peaks at frequencies $\Omega_d$ and $2\Omega_d$.

c) The dependencies of amplitude and kinetic temperature of dust particle oscillations on the amplitude of dust particle charge fluctuations, frequency of particle oscillations, particle mass and drag coefficient are obtained.

d) The predominance of vertical oscillations over horizontal ones is established using obtained coupling between them.

e) The oscillation frequency of plasma-dust crystals is determined by the parameters of the near-electrode layer of gas discharge, rather than by the interaction potential between dust particles for the case of strongly heated dust particle oscillations.

### Acknowledgements

The authors acknowledge fruitful discussions with S I. Popel concerning charge fluctuation and I.V. Schweigert on forced oscillation. This work is partially supported by the RAS programs 12, OE12 and the Federal Special Program under contract 02.740.11.0236. A.V. Timofeev acknowledges the support of the Dynasty Foundation.

### References


