Master Equation with Two Times: Diffusion in External Field

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Generalized master equation for diffusion involving two times, introduced in [1,2], applies to the problem of diffusion in a time-dependent (in general inhomogeneous) external field $\mathbf{F}(\mathbf{r},\tau)$. We consider the case of the quasi Fokker-Planck approximation, when the probability transition function for diffusion (PTD-function) $W(\mathbf{u}, t-\tau; \mathbf{F}(\mathbf{r},\tau))$ does not possess a long tail in coordinate space and can be expanded as the function of instantaneous displacements. For relatively weak external fields the functional W can be linearized in the external field

$$W(\mathbf{u}, t-\tau; \mathbf{F}(\mathbf{r},\tau)) = W_0(u, t-\tau) + W_1(u, t-\tau)(\mathbf{u}\cdot\mathbf{F}(\mathbf{r},\tau)). \tag{1}$$

Then we arrive to a simple generalization of diffusion equation:

$$\frac{\partial f(\mathbf{r},t)}{\partial t} = \frac{d}{dt}\int_0^t d\tau \left[L(t-\tau)\nabla(\mathbf{F}(\mathbf{r},\tau)f(\mathbf{r},\tau)) + B_0(t-\tau)\Delta f(\mathbf{r},\tau)\right], \tag{2}$$

containing the retardation factors $L(t-\tau)$ and $B_0(t-\tau)$ (connected by integral relations with W_1 and W_0, respectively) [3]. For constant in time $L \equiv -b$ (b is mobility) and $B_0 \equiv D$ Eq. (2) (D is diffusion coefficient) is reduced to the usual diffusion equation:

$$\frac{\partial f(\mathbf{r},t)}{\partial t} = D\Delta f(\mathbf{r},t) - b\nabla\left(\mathbf{F}(\mathbf{r},t)f(\mathbf{r},t)\right). \tag{3}$$

The results of the present report gives the opportunity to consider a wide class of the problems of normal and anomalous transport in external fields for the systems with chemical reactions, including plasma.

[1] S.A. Trigger, G.J.F. van Heijst, P.P.J.M. Schram, *Physica A*, 347, 77 (2005) [2] S.A. Trigger, G.J.F. van Heijst, P.P.J.M. Schram, *J. of Physics: Conference Series*, 11, 37 (2005) [3] S.A. Trigger, Generalized Master Equation with Two Times: Diffusion in External Field, http://arXiv: condensed-mattter/0608060, July 2006, submitted to PRE.