НЕЛИНЕЙНЫЕ ИЗОТЕРМИЧЕСКИЕ ВОЛНЫ

В ВЫРОЖДЕННОЙ ЭЛЕКТРОННОЙ ПЛАЗМЕ

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NON-LINEAR ISOTHERMAL WAVES

IN DEGENERATE ELECTRON PLASMAS

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Background:

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$$p = \frac{mV_F^2}{3n_0^2} n^3$$

1D-waves
in 1D-Fermi cold gas

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$$p = \frac{mV_F^2}{5n_0^{2/3}} n^{5/3}$$

1D-waves
in 3D-Fermi cold gas

Equation of state of warm 3D-Fermi gas

$$n(\mu, T) = \frac{(2mkT)^{3/2}}{2\pi^2\hbar^3} \int_0^\infty \frac{z^{1/2}dz}{\exp(z-\eta)+1}$$

$$p(\mu, T) = \frac{2}{3}\frac{(2mkT)^{5/2}}{4m\pi^2\hbar^3} \int_0^\infty \frac{z^{3/2}dz}{\exp(z-\eta)+1}$$

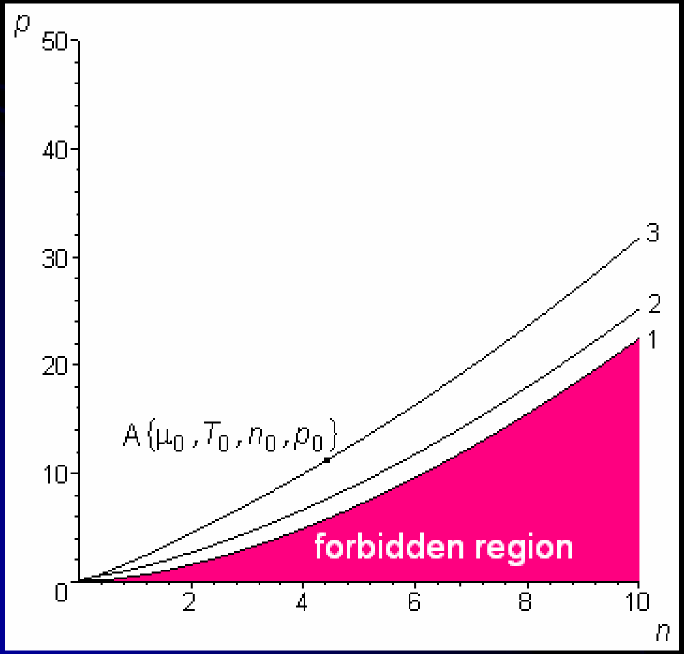
$$z = \varepsilon/kT \qquad \eta = \mu/kT$$

Integralless parametric form of the equation of state

$$n(\mu,T) = -\frac{(2mkT)^{3/2}}{2\pi^2\hbar^3}\Gamma\left(\frac{3}{2}\right)\mathrm{Li}_{3/2}\left(-\exp\frac{\mu}{kT}\right) = -\frac{(mkT)^{3/2}}{2^{1/2}\pi^{3/2}\hbar^3}\mathrm{Li}_{3/2}\left(-\exp\frac{\mu}{kT}\right)$$

$$p(\mu,T) = -\frac{2}{3}\frac{(2mkT)^{5/2}}{4m\pi^2\hbar^3}\Gamma\left(\frac{5}{2}\right)\mathrm{Li}_{5/2}\left(-\exp\frac{\mu}{kT}\right) = -\frac{(mkT)^{5/2}}{2^{1/2}\pi^{3/2}m\hbar^3}\mathrm{Li}_{5/2}\left(-\exp\frac{\mu}{kT}\right)$$

Normalized isotherms



Polylogarithm

$$\mathrm{Li}_\nu(x) = \sum_{k=1}^{\infty}\frac{x^k}{k^\nu} \qquad \frac{d}{dx}\mathrm{Li}_\nu(x) = \frac{1}{x}\mathrm{Li}_{\nu-1}(x)$$

- *Weisstein E.W.* // MathWorld – A Wolfram Web Resource. http://mathworld.wolfram.com/Polylogarithm.html .
- *Lewin L.* Polylogarithms and associated functions. NY – Oxford: North Holland. 1981.
- *Пыхтеев Г.Н., Мелешко И.Н.* Полилогарифмы, их свойства и методы вычисления. Минск: Изд-во БГУ. 1976.

Basic equations

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nu) = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} = \frac{e}{m}\frac{\partial \varphi}{\partial z} - \frac{1}{mn}\frac{\partial p}{\partial z}$$

$$\frac{\partial^2 \varphi}{\partial z^2} = 4\pi e(n - n_0)$$

Gradient of pressure term

$$\frac{1}{n}\frac{\partial p}{\partial z} = \frac{\partial \mu}{\partial z} - \frac{\mu}{kT}\frac{\partial T}{\partial z}$$

Wave frame

$$\xi = z - Vt \qquad \frac{\partial}{\partial t} = -V\frac{d}{d\xi} \qquad \frac{\partial}{\partial z} = \frac{d}{d\xi}$$

System of ODE

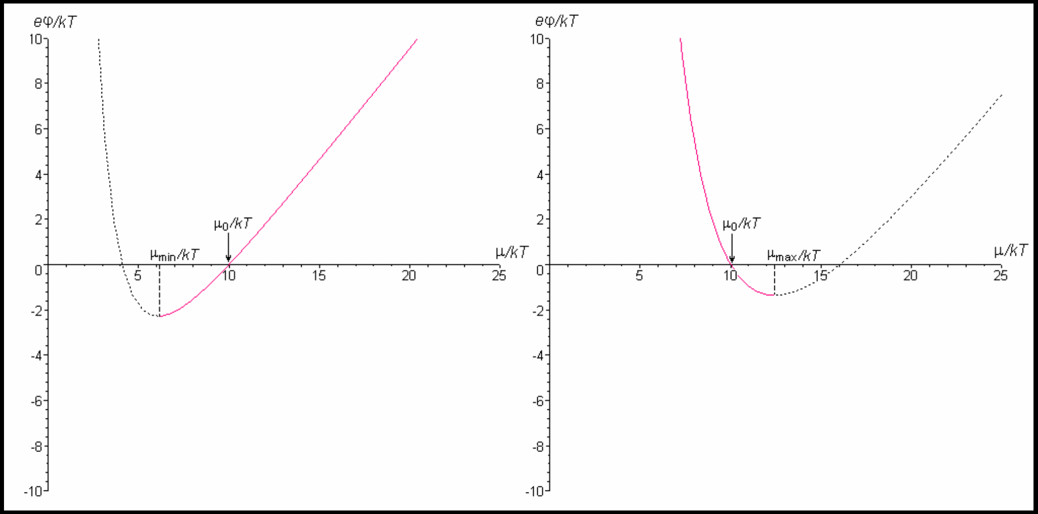
$$-V\frac{dn}{d\xi} + \frac{d}{d\xi}(nu) = 0$$

$$-V\frac{du}{d\xi} + u\frac{du}{d\xi} = \frac{e}{m}\frac{d\varphi}{d\xi} - \frac{1}{m}\frac{d\mu}{d\xi}$$

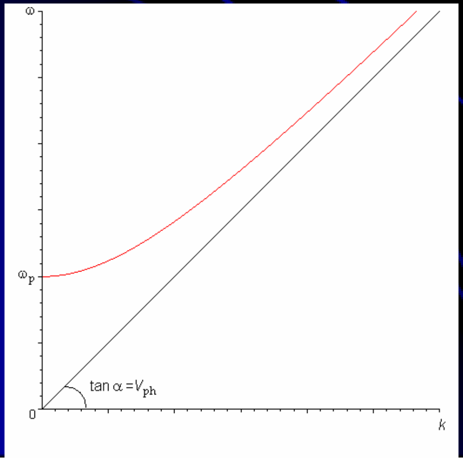
$$\frac{d^2\varphi}{d\xi^2} = 4\pi e(n - n_0)$$

Law of conservation of energy

$$e\varphi = (\mu - \mu_0) + \frac{mV^2}{2}\left\{\left[\frac{\mathrm{Li}_{3/2}(-\exp\mu_0/kT)}{\mathrm{Li}_{3/2}(-\exp\mu/kT)}\right]^2 - 1\right\}$$



Dispersion of wave



Minimal phase velocity

$$V_{ph} = \sqrt{\frac{kT}{m}\,\frac{\mathrm{Li}_{3/2}\left(-\exp\frac{\mu_0}{kT}\right)}{\mathrm{Li}_{1/2}\left(-\exp\frac{\mu_0}{kT}\right)}}$$

$V_{ph} = \sqrt{2\mu_0/3m}$ at $\mu_0/kT \to \infty$

$V_{ph} = \sqrt{\frac{kT}{m}}\sqrt{-\frac{1}{\sqrt{2}}\frac{\zeta(3/2)}{\zeta(1/2)}} \approx 1.1246856\sqrt{\frac{kT}{m}}$ at $\mu_0/kT \to 0$

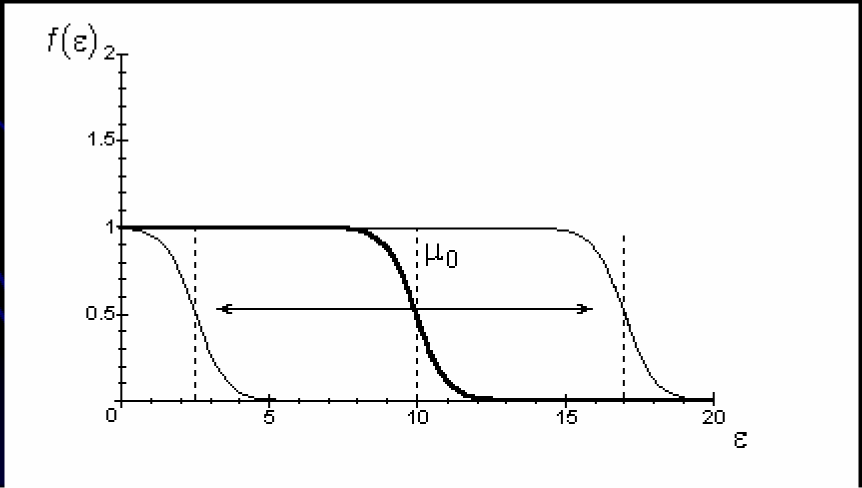
The main non-linear equation

$$\left[1-\frac{mV^2}{kT}\frac{\mathrm{Li}_{3/2}^2(-\exp\mu_0/kT)\mathrm{Li}_{1/2}(-\exp\mu/kT)}{\mathrm{Li}_{3/2}^3(-\exp\mu/kT)}\right]\frac{d^2\mu}{d\xi^2}+$$

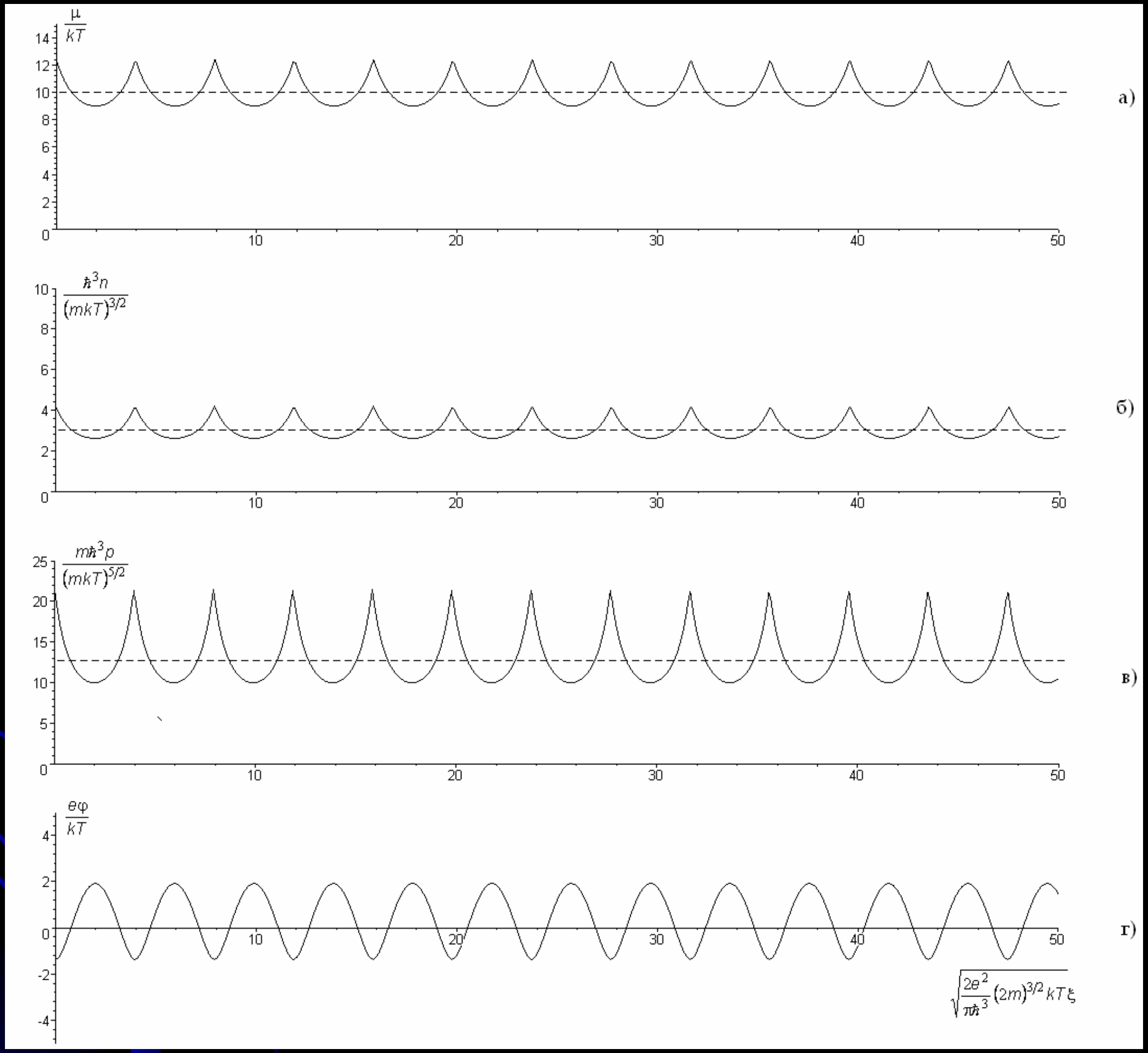
$$+\frac{mV^2}{(kT)^2}\frac{\mathrm{Li}_{3/2}^2(-\exp\mu_0/kT)}{\mathrm{Li}_{3/2}^3(-\exp\mu/kT)}\left[3\frac{\mathrm{Li}_{1/2}^2(-\exp\mu/kT)}{\mathrm{Li}_{3/2}(-\exp\mu/kT)}-\mathrm{Li}_{-1/2}(-\exp\mu/kT)\right]\left(\frac{d\mu}{d\xi}\right)^2=$$

$$=4\pi e^2\frac{(2mkT)^{3/2}}{2\pi^2\hbar^3}\Gamma\left(\frac{3}{2}\right)\left[\mathrm{Li}_{3/2}(-\exp\mu_0/kT)-\mathrm{Li}_{3/2}(-\exp\mu/kT)\right]$$

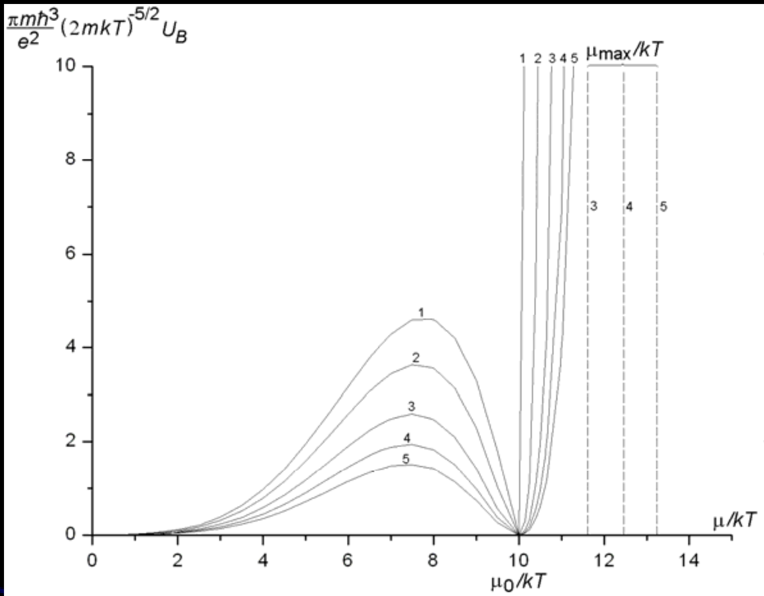
The equation is solvable in quadrature and it may be analyzed by a modified pseudopotential method!

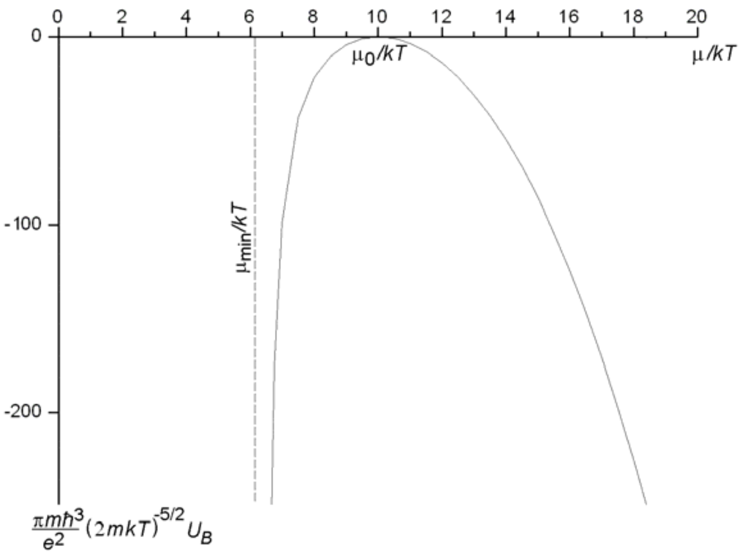


Example of solution
(a wave score)



Pseudopotential analysis





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Thank you!