

*Критические точки некоторых  
металлов, найденные на основе их  
связи с параметрами линии  
единичного фактора сжимаемости*

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## Линия единичного фактора сжимаемости (линия Бачинского, Z-line)

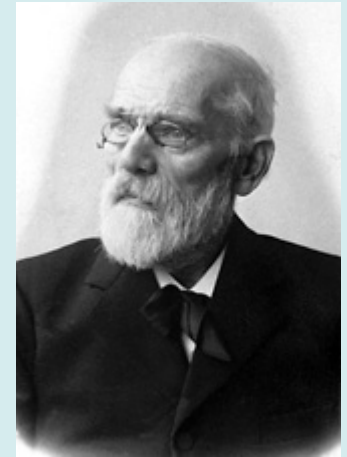
$$Z = \frac{P}{\rho T} = 1$$

Уравнение ван дер Ваальса

$$Z = \frac{P}{\rho T} = 1 + \frac{27\rho(\rho/3 + 8T/27 - 1)}{8T(3 - \rho)}$$

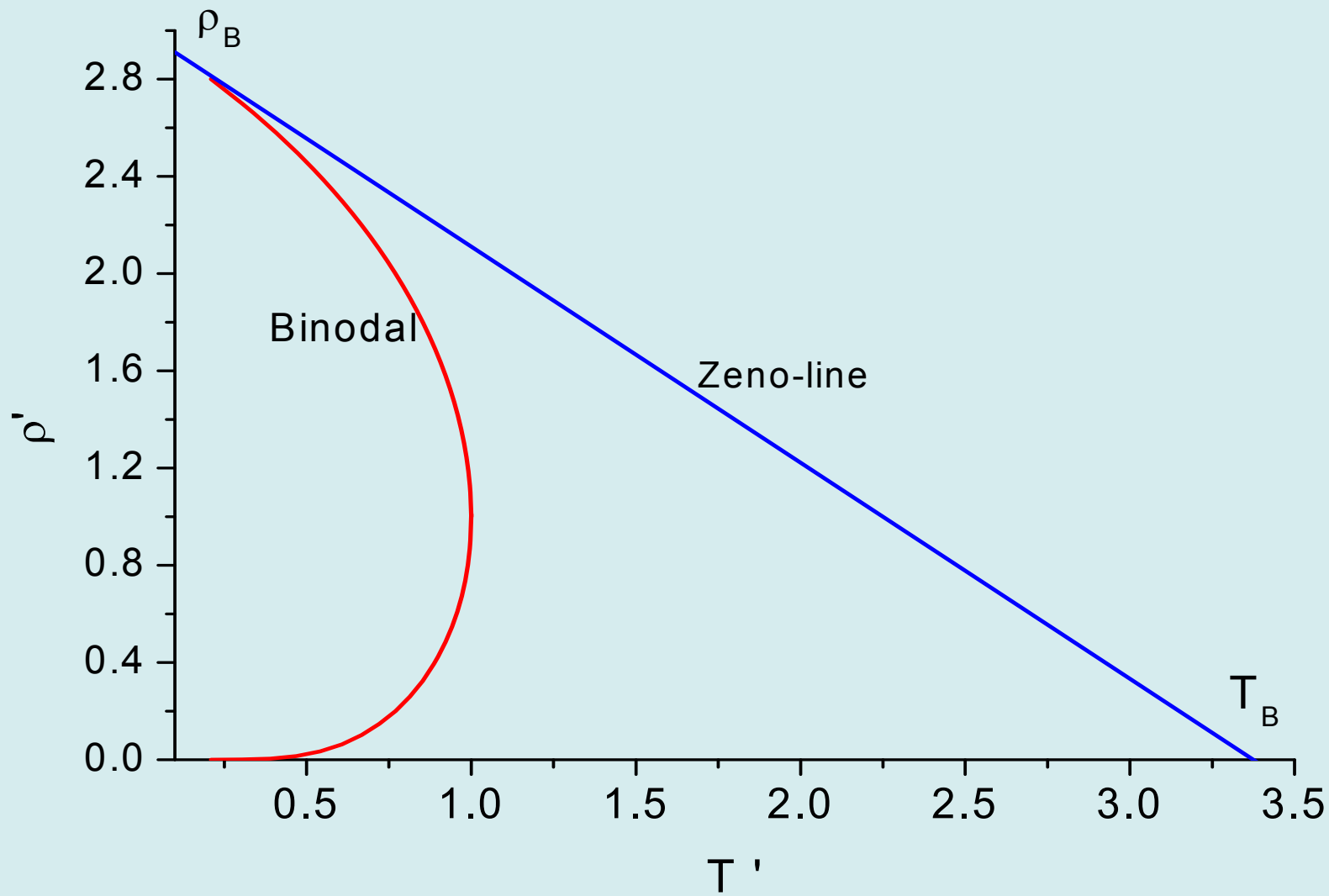
Уравнение Z=1 линии

$$\frac{T}{T_B} + \frac{\rho}{\rho_B} = 1 \quad ; \quad T_B = 27/8 \quad ; \quad \rho_B = 3$$

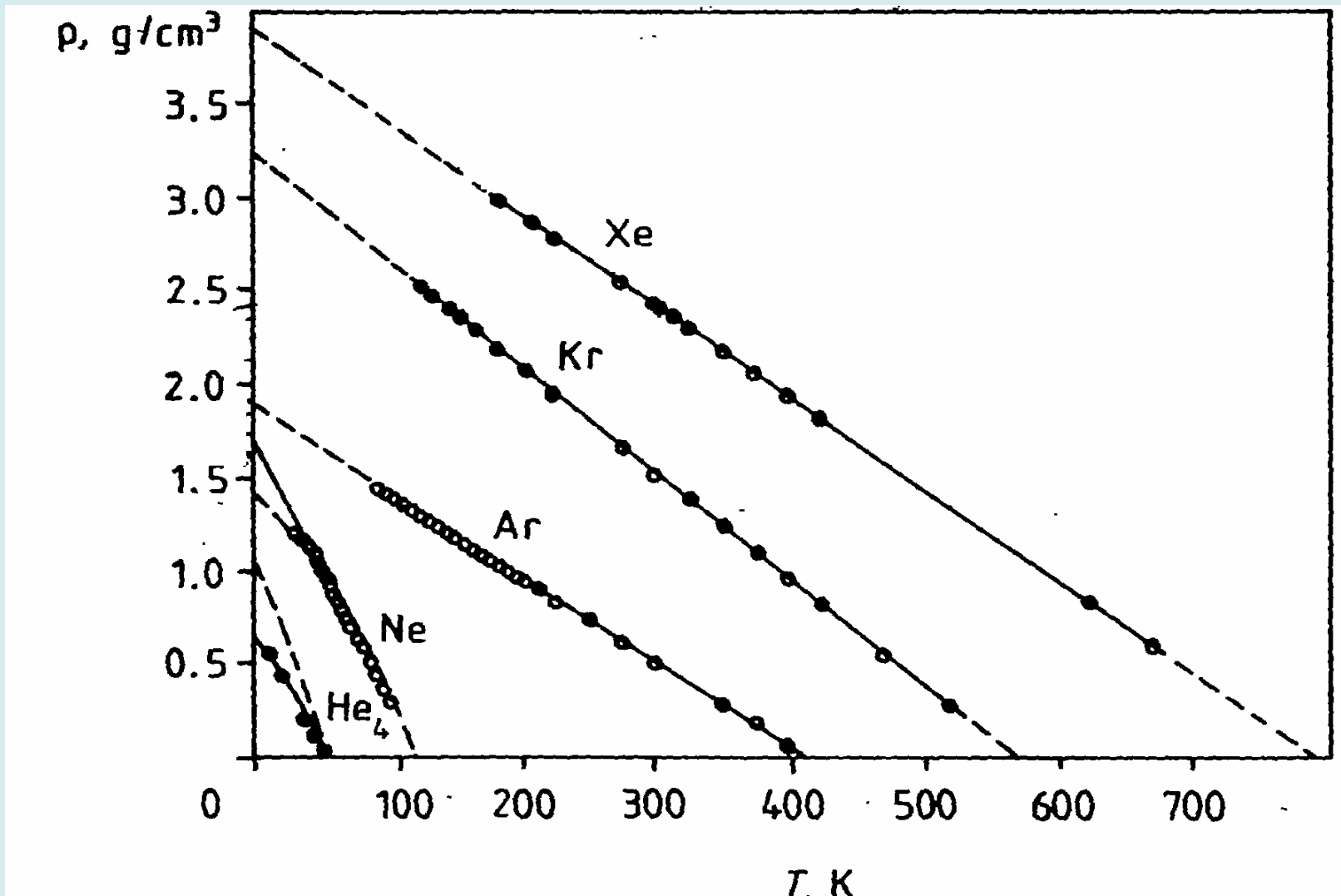


A. Bachinskii, Ann. Der Phys. (1906)

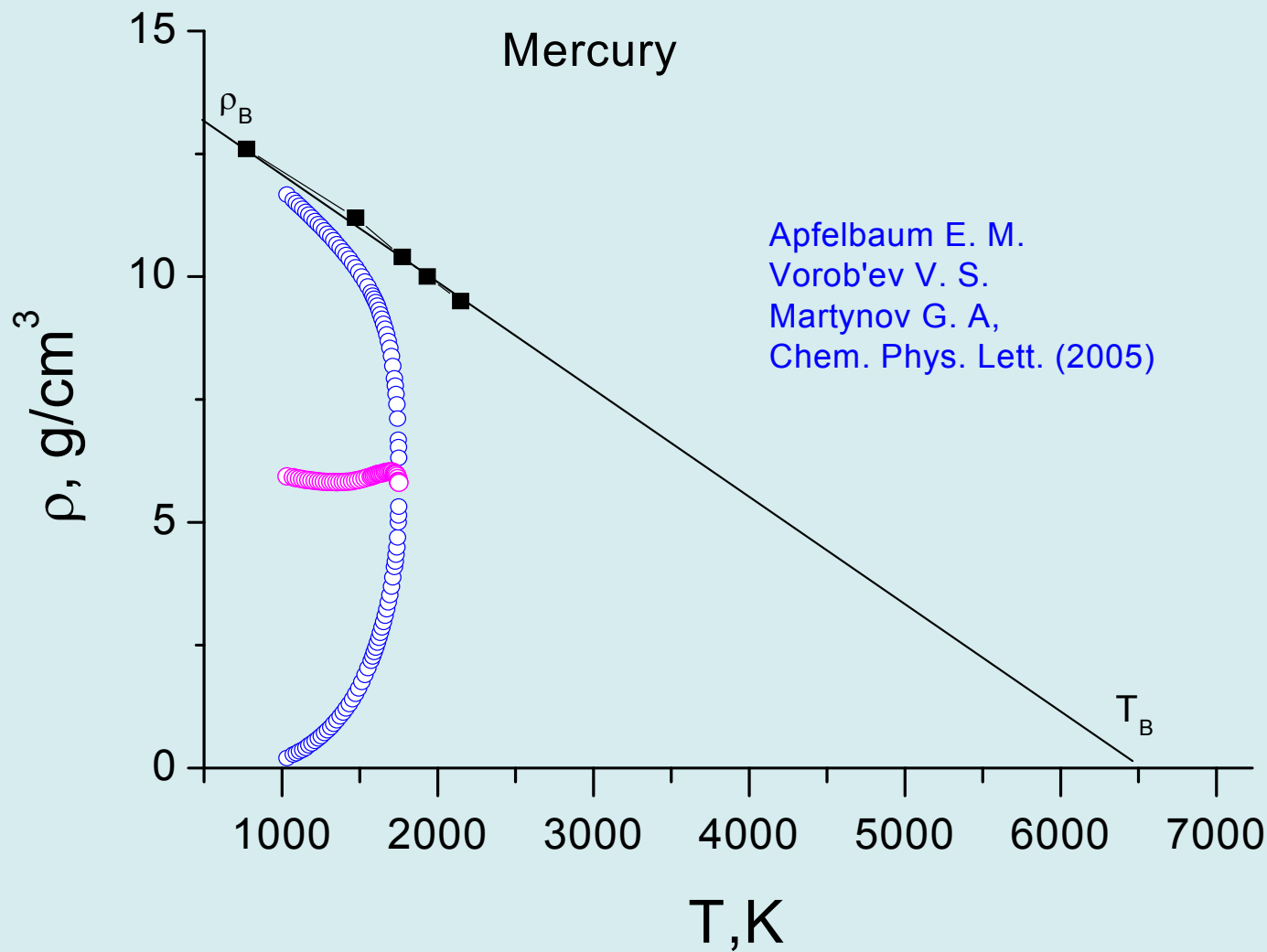
# *vdW binodal and Z-line*

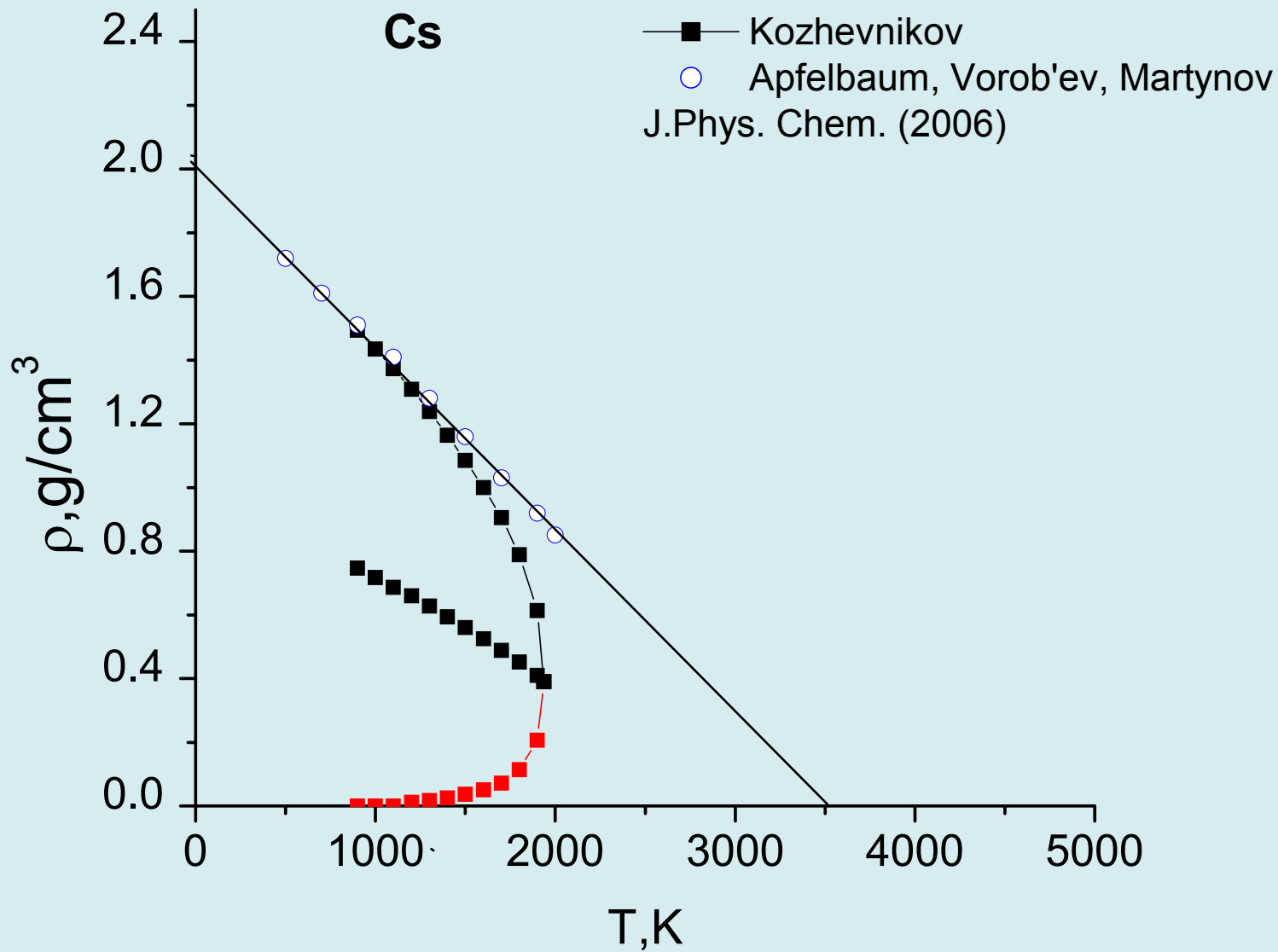


**V. A. RABINOVICH. ET. AL., THERMOPHYSICAL PROPERTIES OF NEON, ARGON, KRYPTON AND XENON (HEMISPHERE, BERLIN-NEW YORK, 1988).**

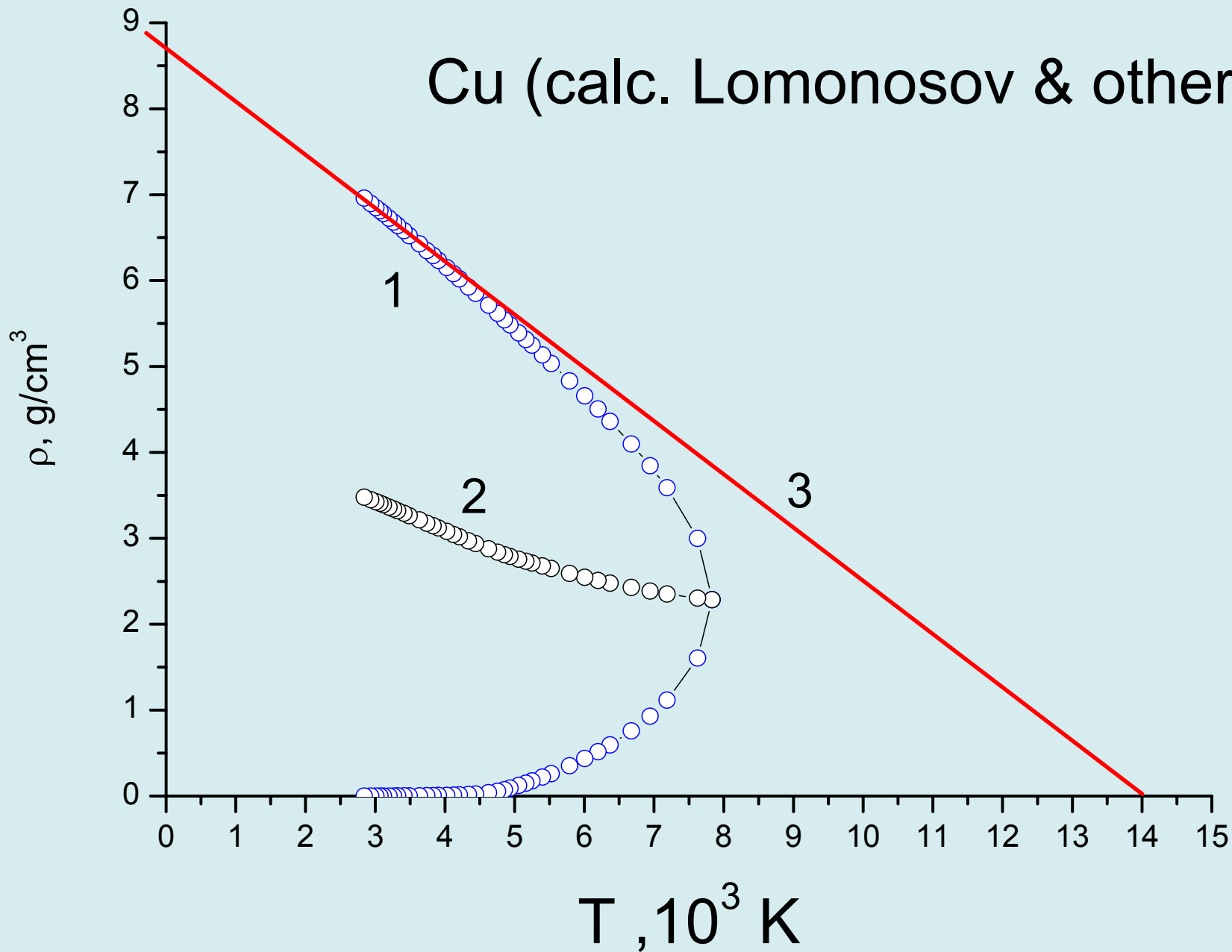


The linearity for Z-line has been confirm also **HYDROCARBONS, WATER, CARBON DIOXIDE, METHANE** and for many others

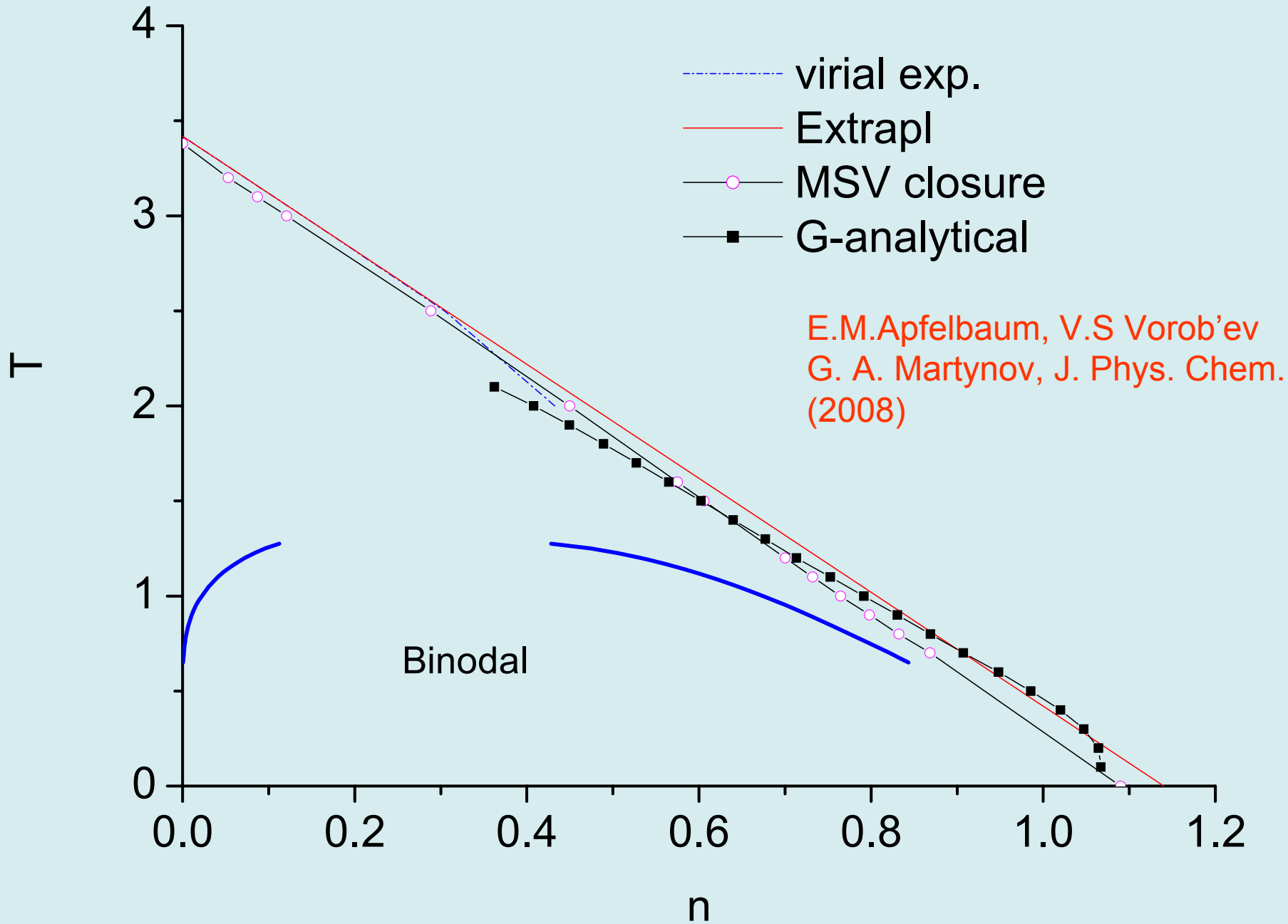




# Cu (calc. Lomonosov & others)







## Liquid branch of binodal

$$\rho(T) = \rho_c + \alpha\tau + \beta\tau^{1/3} \quad \tau = 1 - T/T_c$$

Guggenheim (1945), Reid (1977), Филиппов (1984)

$$\rho(T)_{T \rightarrow 0} \rightarrow \rho_B (1 - T/T_B)$$

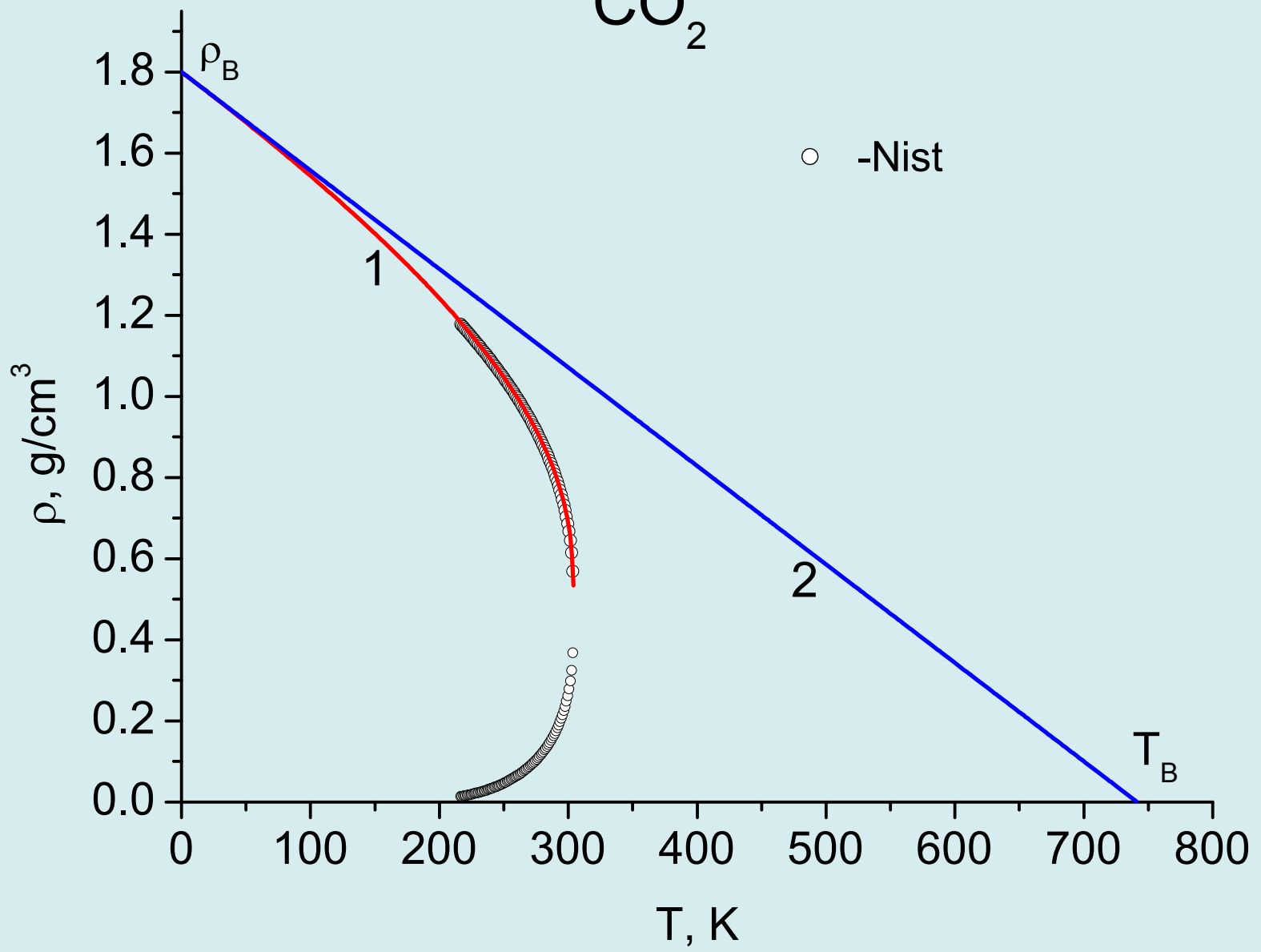
Жидкая ветвь бинодали (Apfelbaum, Vorob'ev J. Phys. Chem.2008)

$$\rho(T) = \rho_c + \frac{1}{2} \left[ \rho_c - \rho_B + 3\rho_B \frac{T_c}{T_B} \right] \tau + \frac{3}{2} \left[ \rho_B - \rho_c - \rho_B \frac{T_c}{T_B} \right] \tau^{1/3}$$

Thus, if  $T/T_c \rightarrow 1$  (critical point), then  $r = r_c$ , and, if  $T \rightarrow 0$ , then  $r = r_B$ .

Besides, Eq.(2) transforms into Eq. (1) if  $T/T_c \ll 1$

CO<sub>2</sub>



$\rho_c m / \rho_B k T_B$ $P_c m / \rho_c k T_c$	Critical parameters				Z-line parameters		Similary parameters			
Model, Subs.	$T_c, K$	$\rho_c,$ $g/cm^3$	$P_c, atm$	$Z_c =$ $P_c m / \rho_c k T_c$	$T_B, K$	$\rho_B,$ $g/cm^3$	$S_2$	$P_c m / \rho_B k T_B$	$T_c / T_B$	$\rho_c / \rho_B$
L-J <sup>1)</sup>	1.305	0.314	0.127	0.308	3.418	1.14	0.076	0.033	0.383	0.285
vdW <sup>1)</sup>	1	1	1	3/8 = 0.375	27/8 = 3.375	3	5/81 =00. 061	1/27 = 0.037	0.296	0.333
Ar	150.7	0.536	48.63	0.286	392.84	1.87	0.078	0.031	0.383	0.286
Ne	44.49	0.482	26.79	0.3 0.34* <sup>1)</sup>	118.85	1.63	0.077	0.033	0.374	0.296
Kr	209.5	0.909	55.25	0.289	537.98	3.24	0.077	0.032	0.389	0.280
Xe	289.7	1.100	58.42	0.286	740.02	3.95	0.077	0.031	0.391	0.278
NH <sub>3</sub>	405.4	0.225	113.30	0.251	935.92	0.95	0.076	0.026	0.433	0.236
CO <sub>2</sub>	304.1	0.468	73.77	0.271	741.4	1.80	0.077	0.029	0.41	0.26
Ethane	305.3	0.207	48.72	0.275	779.35	0.74	0.079	0.03	0.39	0.28
Ethene	282.3	0.214	50.42	0.278	714.12	0.78	0.078	0.03	0.395	0.274
Fluorine	144.4	0.593	51.72	0.273	385.06	2.01	0.080	0.03	0.375	0.295
Hexane	507.8	0.233	30.34	0.263	1235.84	0.90	0.078	0.028	0.41	0.259
Methane	190.6	0.163	45.99	0.282	498.06	0.57	0.078	0.031	0.383	0.286
N <sub>2</sub>	126.2	0.313	33.96	0.286	327.62	1.10	0.078	0.031	0.385	0.284
O <sub>2</sub>	154.6	0.436	50.43	0.285	401.14	1.53	0.078	0.031	0.385	0.285
Propene	365.6	0.223	46.65	0.286	894.42	0.86	0.075	0.032	0.409	0.259
R13	302.0	0.583	38.79	0.277	763.31	2.13	0.078	0.03	0.395	0.274

	Critical parameters				Z-line parameters	Similary parameters				
Model, Subs.	$T_c, K$	$\rho_c, g/cm^3$	$P_c, atm$	$Z_c = P_c m / \rho_c k T_c$	$T_B, K$	$\rho_B, g/cm^3$	$S_2$	$P_c m / \rho_B k T_B$	$T_c / T_B$	$\rho_c / \rho_B$
R22	369.3	0.524	49.9	0.268	907.89	1.99	0.078	0.029	0.407	0.263
R32	351.3	0.424	57.82	0.243	823.92	1.74	0.079	0.025	0.426	0.244
Cs	1924	0.39	94	0.198	4114.47	1.96	0.075	0.018	0.468	0.199
Li	3223	0.120	689	0.17	7165.14	0.545	0.082	0.017	0.45	0.22
Rb	2017	0.29	124.5	0.22	4126.85	1.6	0.069	0.019	0.489	0.18
K	2178	0.18	148	0.178	4656.3	0.89	0.078	0.017	0.478	0.20
Na	2503	0.206	256	0.137	5394.40	1.0	0.083	0.013	0.464	0.206
Hg	1751	5.8	1650	0.387	6552.60	14.40	0.065	0.042	0.267	0.403
Water	647.3	0.32	221.2	0.228	1268.0	1.2	0.105	0.03	0.51	0.267
H <sub>2</sub>	33.15	0.0312	12.964	0.30 0.37*	99.84	0.092	0.079	0.041	0.332	0.338
He <sup>4</sup>	5.195	0.0696	2.2746	0.3 0.387*	19.46	0.173	0.071	0.03	0.257	0.407
He <sup>3</sup>	3.34	0.0385	1.15	0.321 0.47*	18.55	0.078	0.06	0.03	0.18	0.49
Al	6378	0.45	1074	0.12	12888	2.57	0.076	0.01	0.495	0.175
Cu	6000	1.8	2265	0.19	12740	9.05	0.061	0.025	0.471	0.186
W	12387	4.92	7448	0.27	29131	20.1	0.078	0.027	0.425	0.245
U	7000	3.3	1712	0.21	14030	19.3	0.077	0.02	0.499	0.170
Zr	15200	1	421	0.031	29330	6.58	0.073	0.024	0.52	0.15

# Empirical similarity relations

$$T_c / T_B + \rho_c / \rho_B = S_1 \approx 0.67$$

$S_1, S_2, S_3$  слабо зависят от вещества

$$S_2 = \frac{\rho_c T_c - P_c}{\rho_B T_B} \quad ; \quad S_3 = \frac{P_c}{\rho_B T_B} \quad \Rightarrow \quad P_c = \rho_c T_c - S_2 \rho_B T_B$$

$$x_1 = \rho_c / \rho_B \quad ; \quad x_2 = T_c / T_B$$

$$x_{1,2} = \frac{S_1}{2} \left[ 1 \pm \sqrt{1 - \frac{4S_2}{S_1^2(1 - Z_c)}} \right]$$

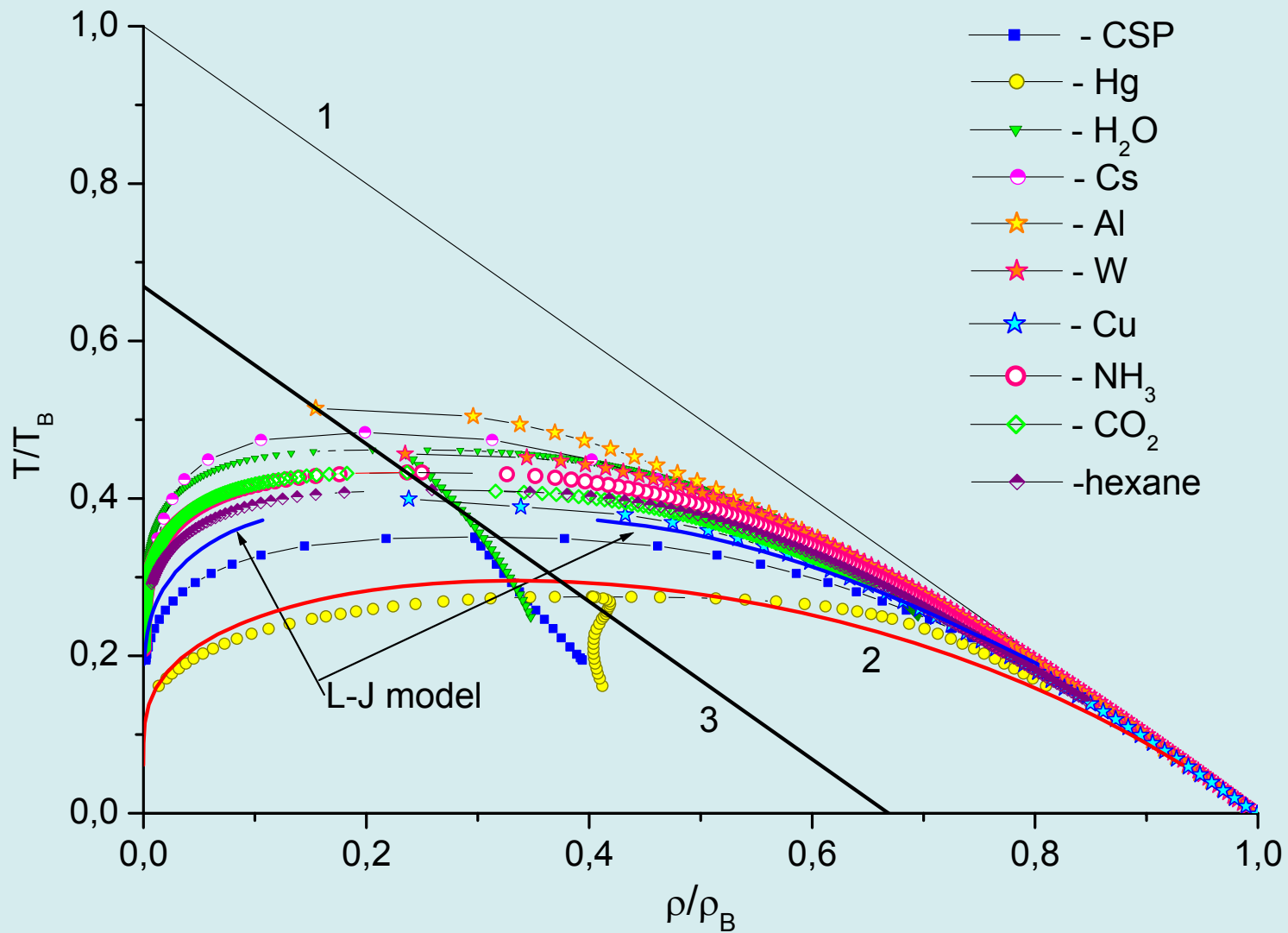
$$Z_c \leq 0.32$$

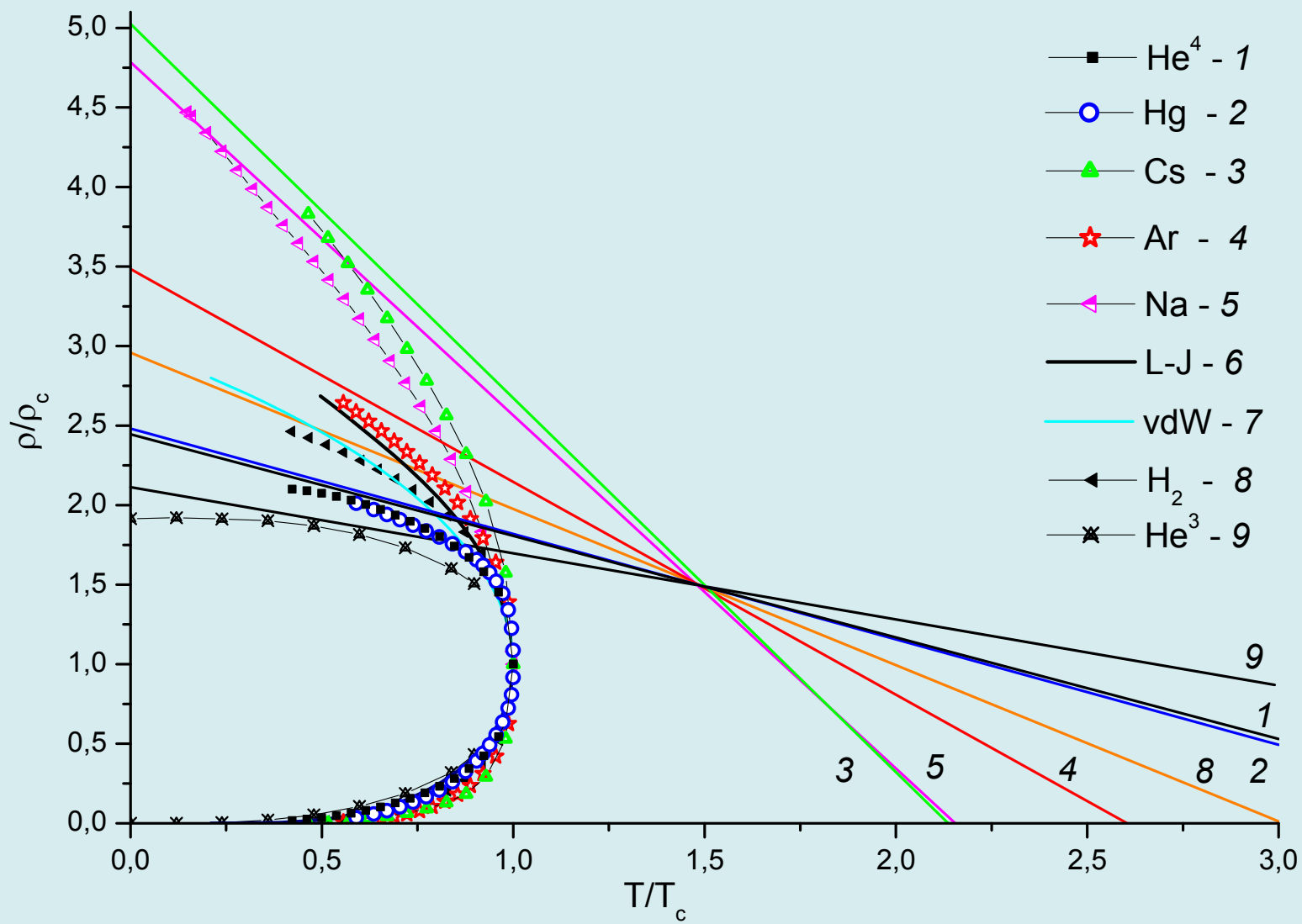
Все классические вещества

$$x_{1,2} = \frac{S_1}{2} \left[ 1 \pm \sqrt{1 - \frac{4S_3}{S_1^2 Z_c}} \right]$$

$$Z_c \text{ or } Z^* \geq 0.37$$

Ртуть и квантовые жидкости







## Quantum Liquids

$$d_c = (T_c / P_c)^{1/3} \text{ характерная длина}$$

$$Z_c = P_c / \rho_c T_c = 1 / (\rho_c d_c^3)$$

В квантовом случае

$$d_c^* \Rightarrow \left( \frac{T_c}{P_c (1 + aB)} \right)^{1/3} \quad (\text{Garrabos, J. Physique, 1985})$$

De Boer parameter

$$B = h / \sigma \sqrt{2\pi m D}$$

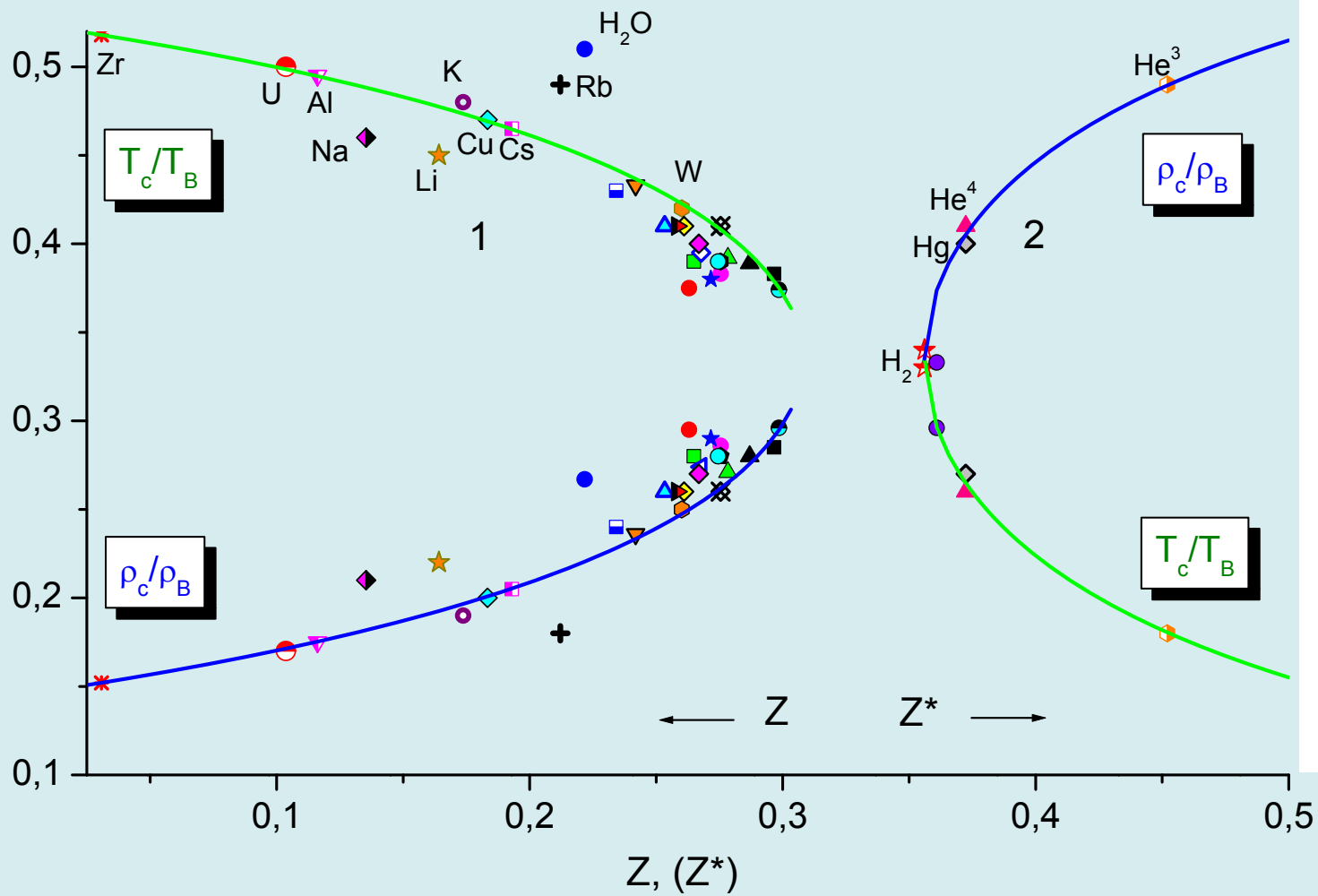
## Quantum Liquids

De Boer parameter  $B = h / \sigma \sqrt{2\pi m D}$

	<i>Ar</i>	<i>Ne</i>	<i>D<sub>2</sub></i>	<i>H<sub>2</sub></i>	<i>He<sup>4</sup></i>	<i>He<sup>3</sup></i>
<i>B</i>	0.069	0.217	0.452	0.78	0.99	1.76
<i>Z<sub>c</sub></i>	0.289	0.3	0.3	0.3	0.3	0.321
<i>Z*</i>	0.293	0.319	0.34	0.37	0.386	0.48

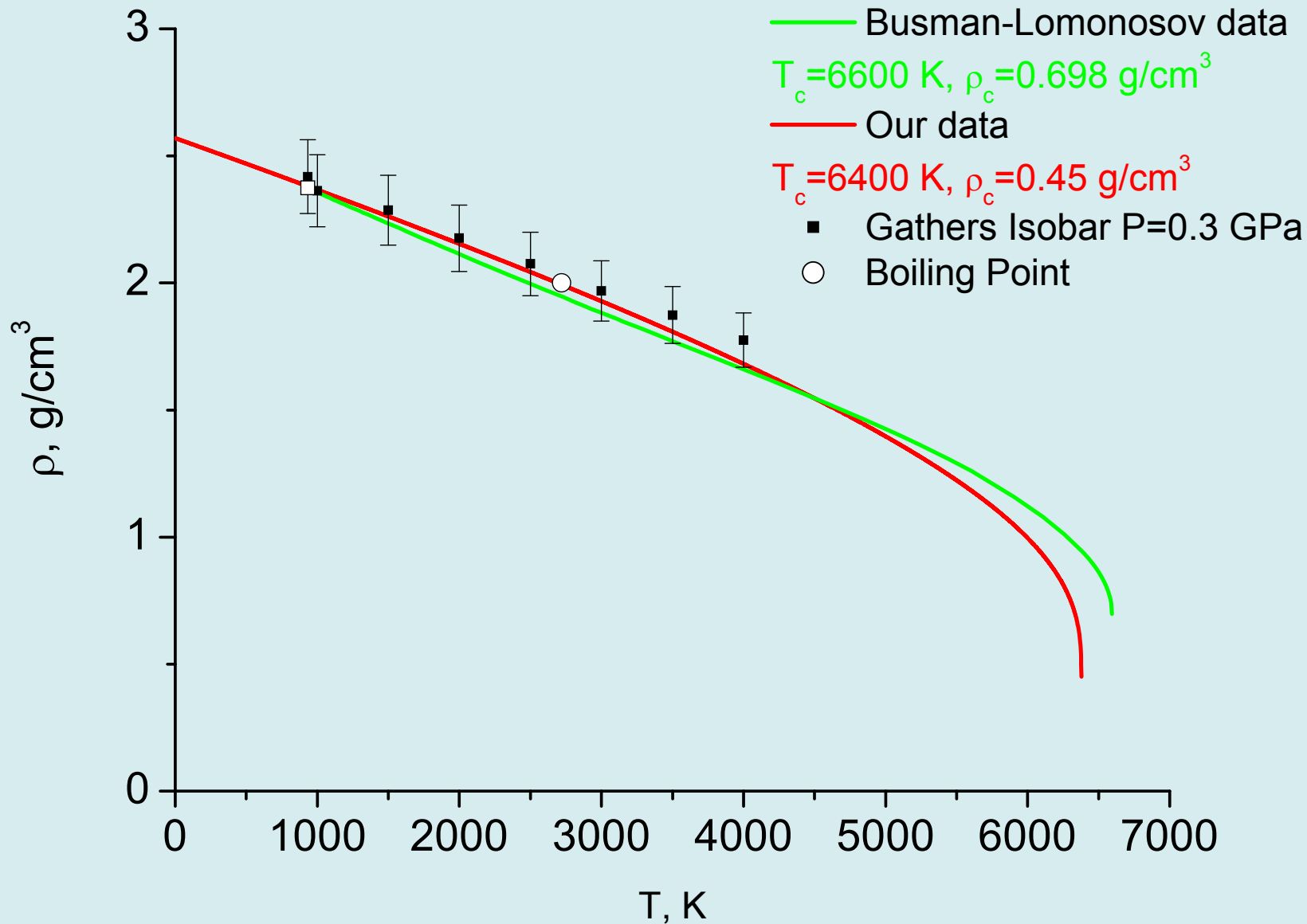
Effective factor compressibility  $Z^* = Z_c (1 + aB)$

$$Z_c(\text{Hg}) = Z^*(\text{He}^4).$$

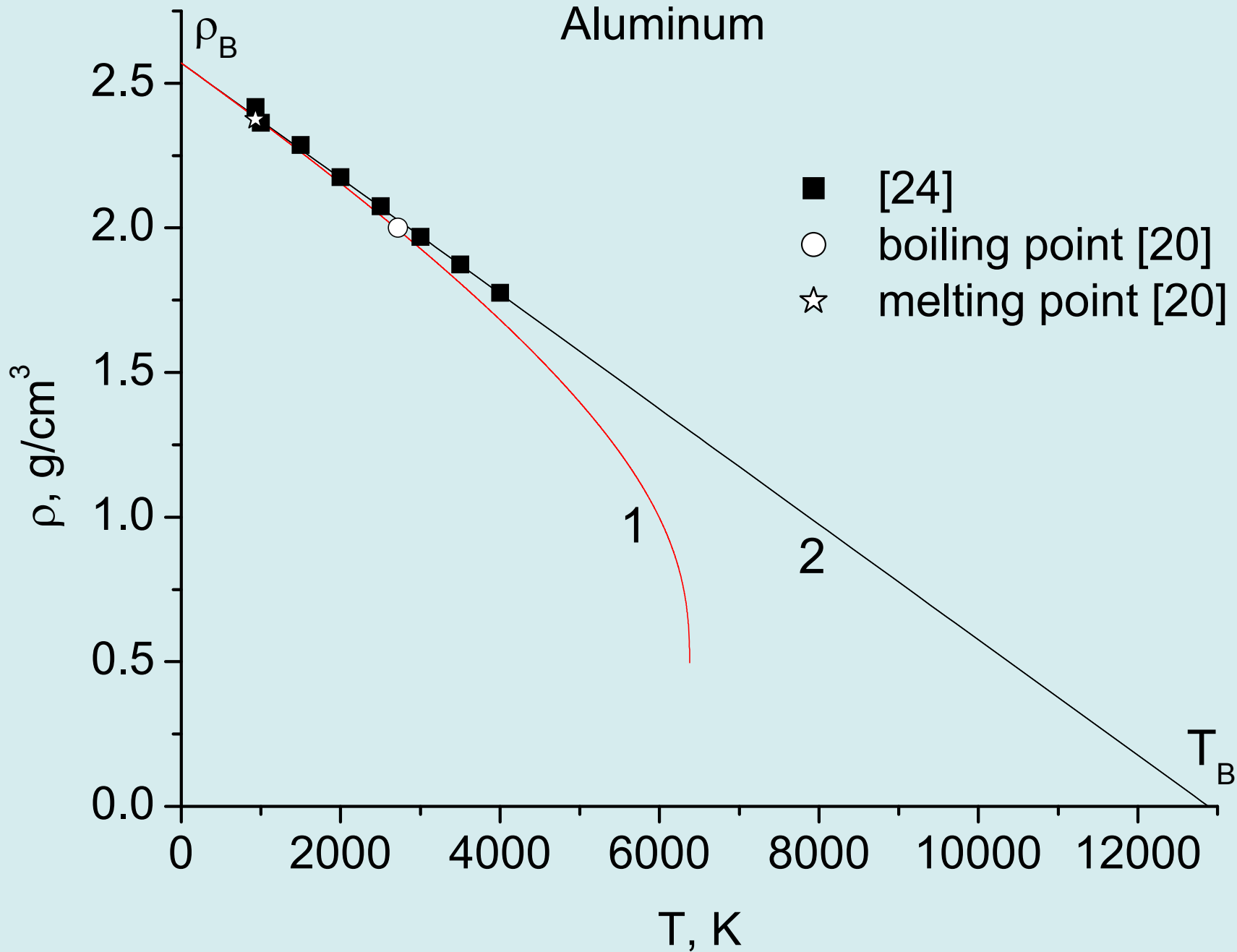


- L-J
- vdW
- Ar
- Ne
- ▲ Kr
- ▲ Xe
- ▼ NH<sub>3</sub>
- ◆ CO<sub>2</sub>
- Ethane
- ◇ Ethene
- Fluorine
- ▲ Hexane
- ★ Methane
- N<sub>2</sub>
- O<sub>2</sub>
- ⊗ Propene
- ◆ R13
- ▶ R22
- R32

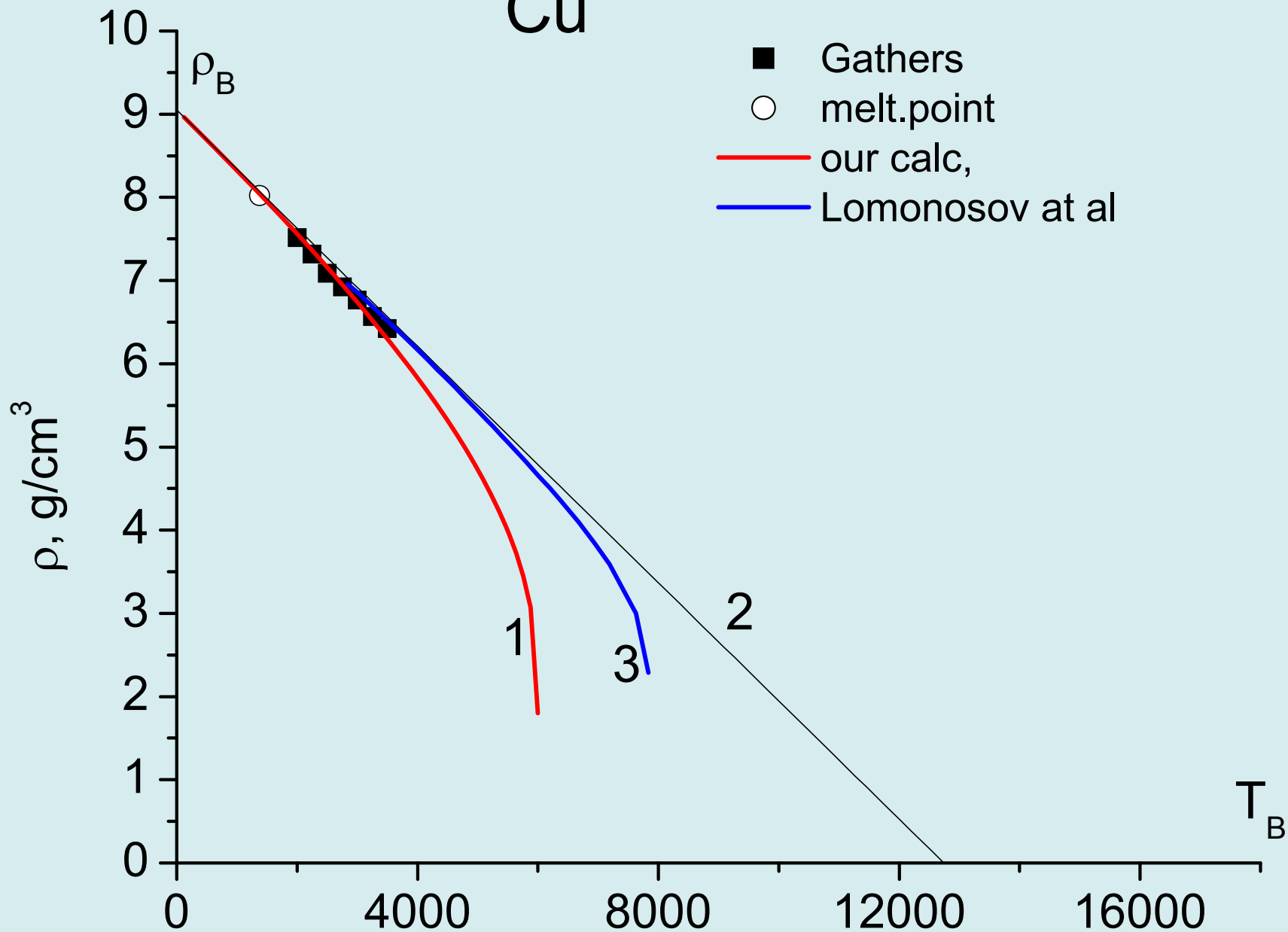
# Aluminum Liquid Binodal



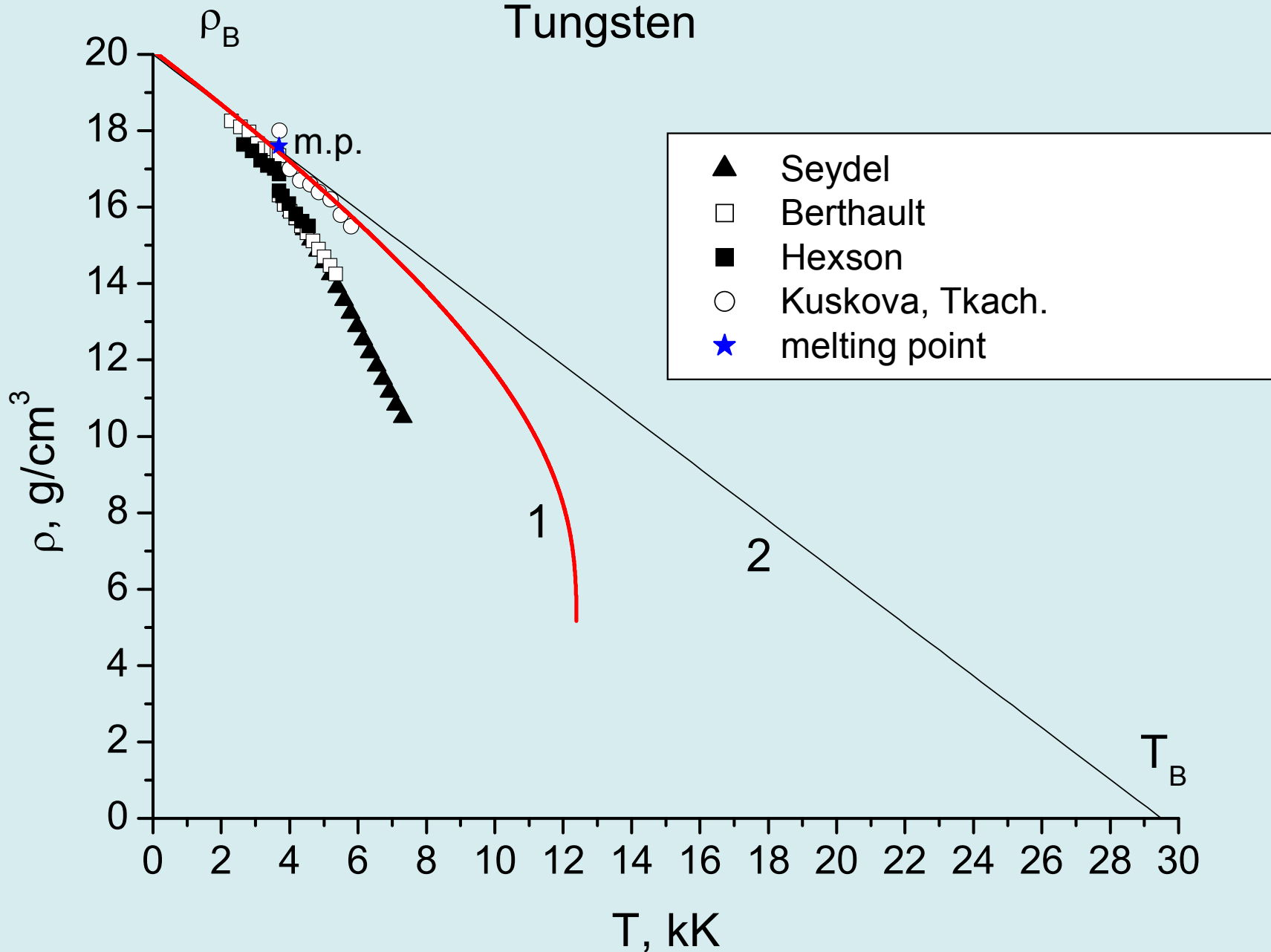
# Aluminum



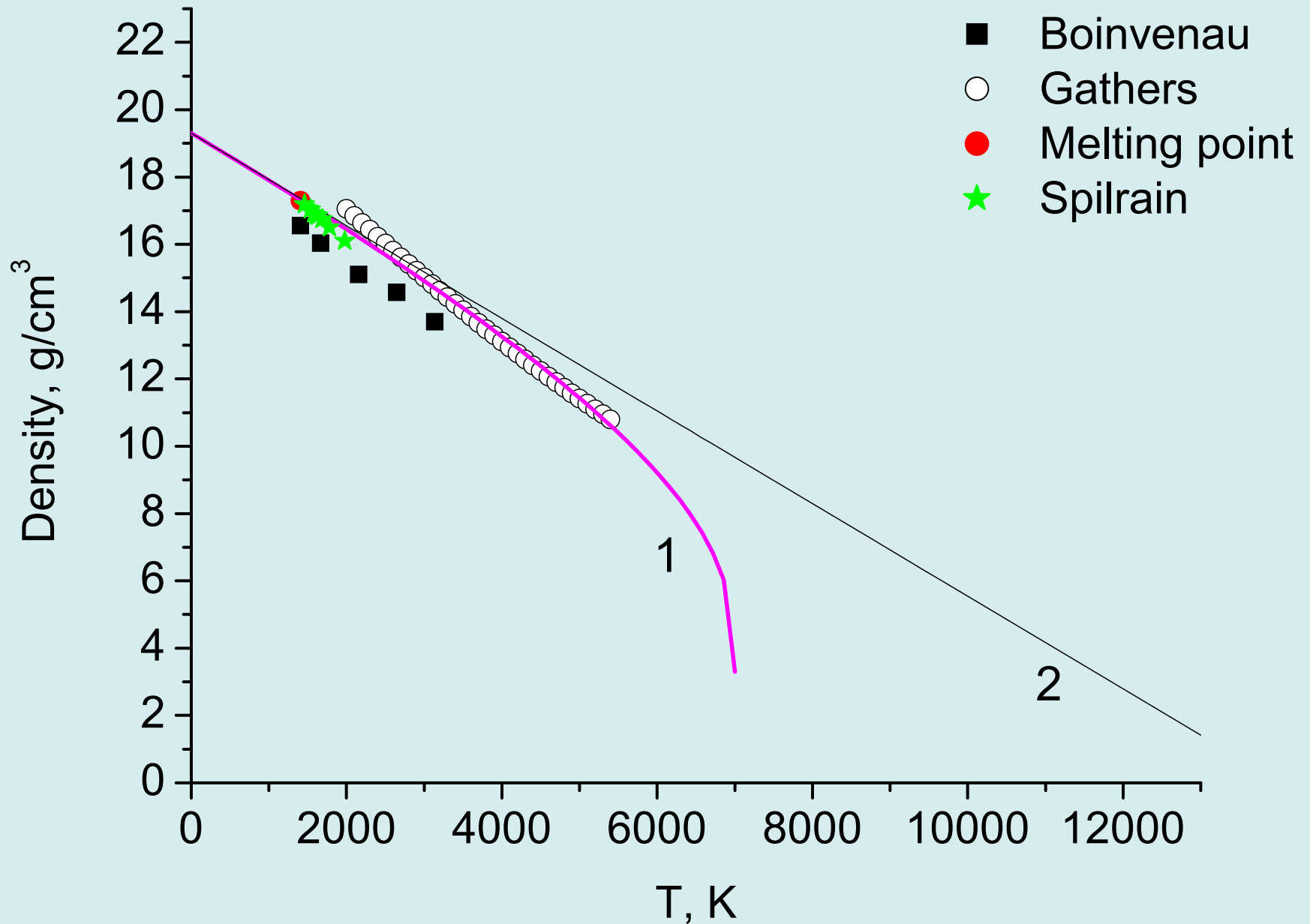
# Cu



# Tungsten



# Uranium





# Zirconium

○ Korobenko, Savvatimskii (2003) P=1 atm

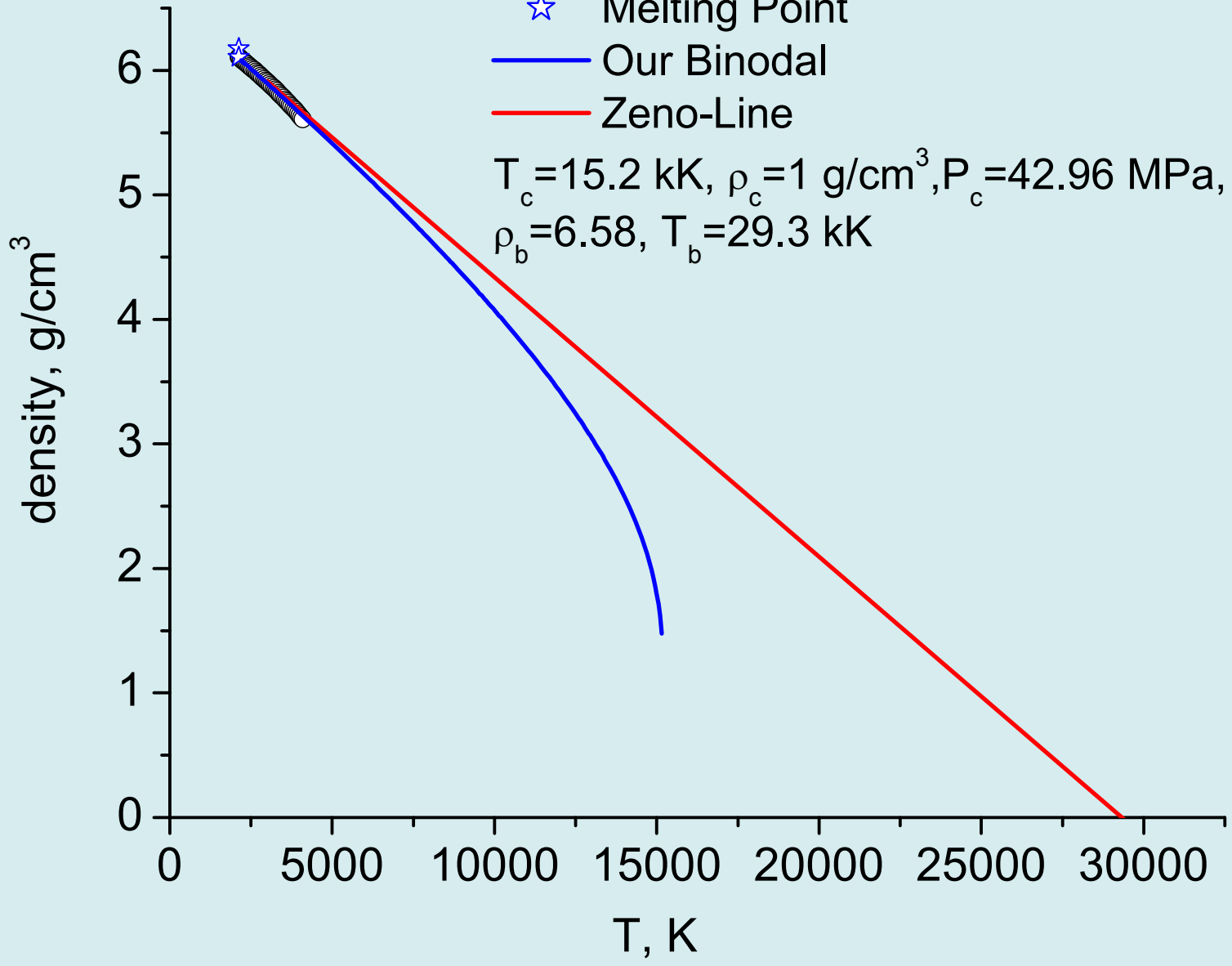
☆ Melting Point

— Our Binodal

— Zeno-Line

$T_c = 15.2 \text{ kK}$ ,  $\rho_c = 1 \text{ g/cm}^3$ ,  $P_c = 42.96 \text{ MPa}$ ,  $Z_c = 0.031$

$\rho_b = 6.58$ ,  $T_b = 29.3 \text{ kK}$



<b>Metal</b>	$T_c$ , K	$\rho_c$ , g/cm <sup>3</sup>	Method	Ref.	$T_B$ , K	$\rho_B$ , g/cm <sup>3</sup>	$P_c$ , atm	$Z_c$
<b>Al</b>	<b>6378</b> 8860 8000	<b>0.45</b> 0.28 0.64	<b>this work</b> scaling Extrapol.	- Lik Fort.	<b>12890</b>	<b>2.57</b>	<b>1074</b> 4680 4470	<b>0.12</b> 0.60 0.28
<b>Cu</b>	<b>7093</b> 7620 8390	<b>1.95</b> 1.4 2.4	<b>this work</b> scaling extrapol	- Lik Fort.	<b>15600</b>	<b>8.6</b>	<b>4500</b> 5770 7460	<b>0.19</b> 0.39 0.28
<b>W</b>	<b>12390</b> 12500 14000	<b>4.92</b> 4.52 4.71	<b>this work</b> estimate estimate	- <b>Boin.</b> <b>Seyd.</b>	<b>29130</b>	<b>20.1</b>	<b>7450</b> 1100 5000	<b>0.27</b> 0.43 0.13
<b>U</b>	<b>7000</b> 9000 11630	<b>3.3</b> 2.6 5.3	this work estimate extrapol	Lik. Fort	<b>14030</b>	<b>19.3</b>	<b>1710</b> 5000 6100	<b>0.11</b> 0.60 0.28
<b>Zr</b>	<b>15200</b>	<b>1</b>	<b>this work</b>		<b>29330</b>	<b>6.58</b>	<b>421</b>	<b>0.031</b>

**Спасибо за внимание**