Laser wakefield acceleration of supershort electron bunches in guiding structures

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Expected potential

of laser – plasma acceleration of electrons

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Electric field of plasma wave (*with phase velocity ~ c,* $\lambda_p = 2\pi c/\omega_p$):

$$E_P\,[\mathrm{V/m}] \approx 10^2\,\alpha\,(\,n_e\,[\mathrm{cm^3}]\,)^{1/2} \propto \gamma_g^{-1} = \omega_p/\omega_0$$

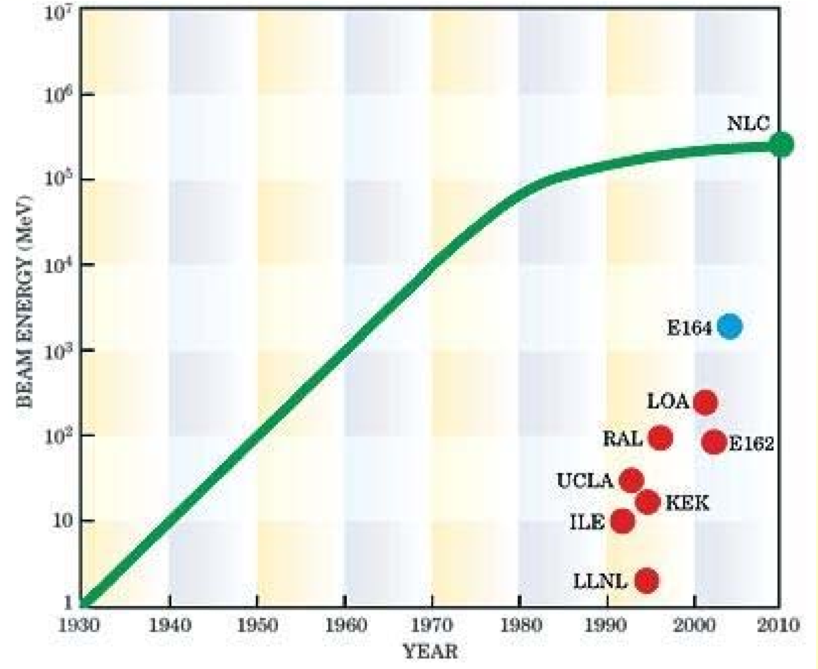
$\alpha = \delta n / n_0$ – ***plasma wave amplitude; at*** $\alpha = 0.3 \div 1.0$, $n_e = 10^{17} \div 10^{18}$ cm^{-3}:

$$E_P = 10 \div 100\ \mathrm{GV/m}$$

maximum of accelerating gradient in traditional accelerators (RF linac):

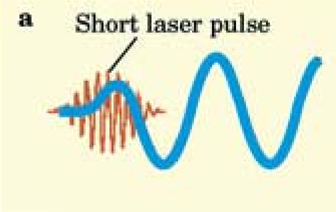
$E_{RF} \sim 10 - 100$ MV/m

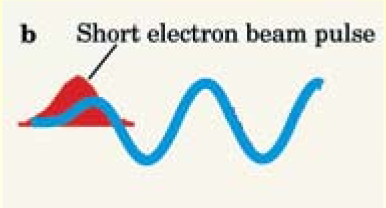
Exponential growth of "the Livingston curve" began tapering off around 1980



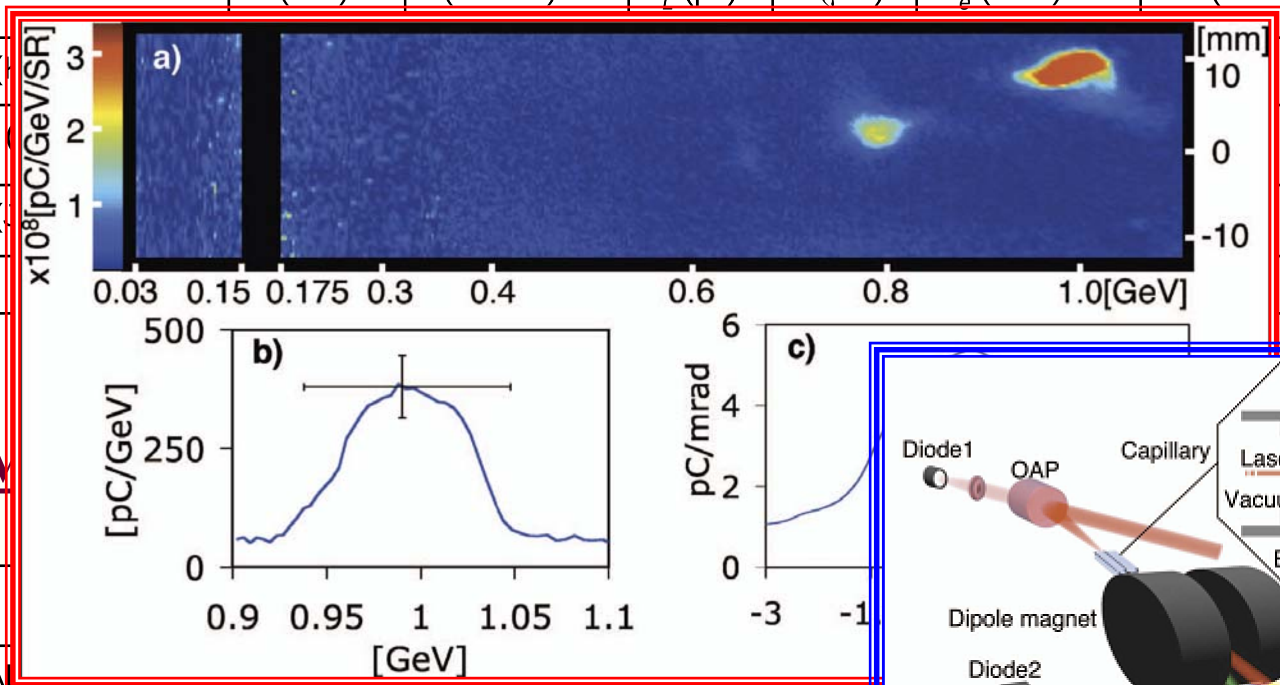
Parameters and results of some experiments

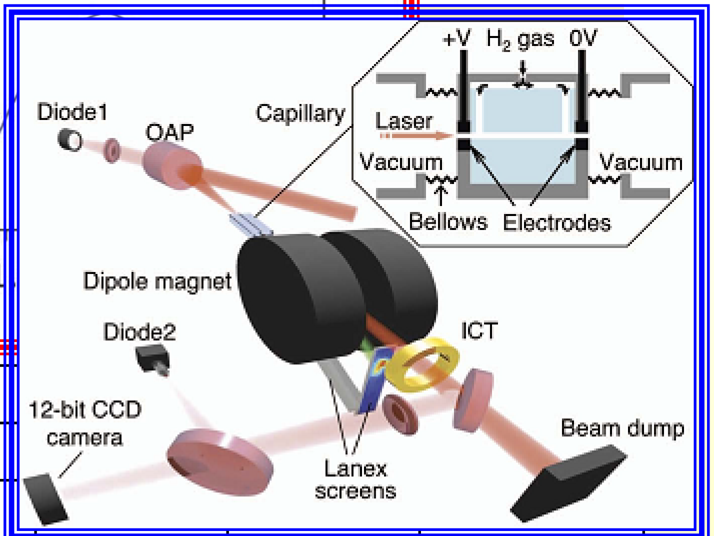
for **standard LWFA** scheme $c\tau_L \approx \lambda_p/2$





	P (TW)	I (W/cm^2)	τ_L (ps)	λ (μm)	N_e (cm^{-3})	ΔE (MeV)	E_{ac} (GV/m)
Japan (							0.7
France (							1.5
Japan (							20
France							**200**

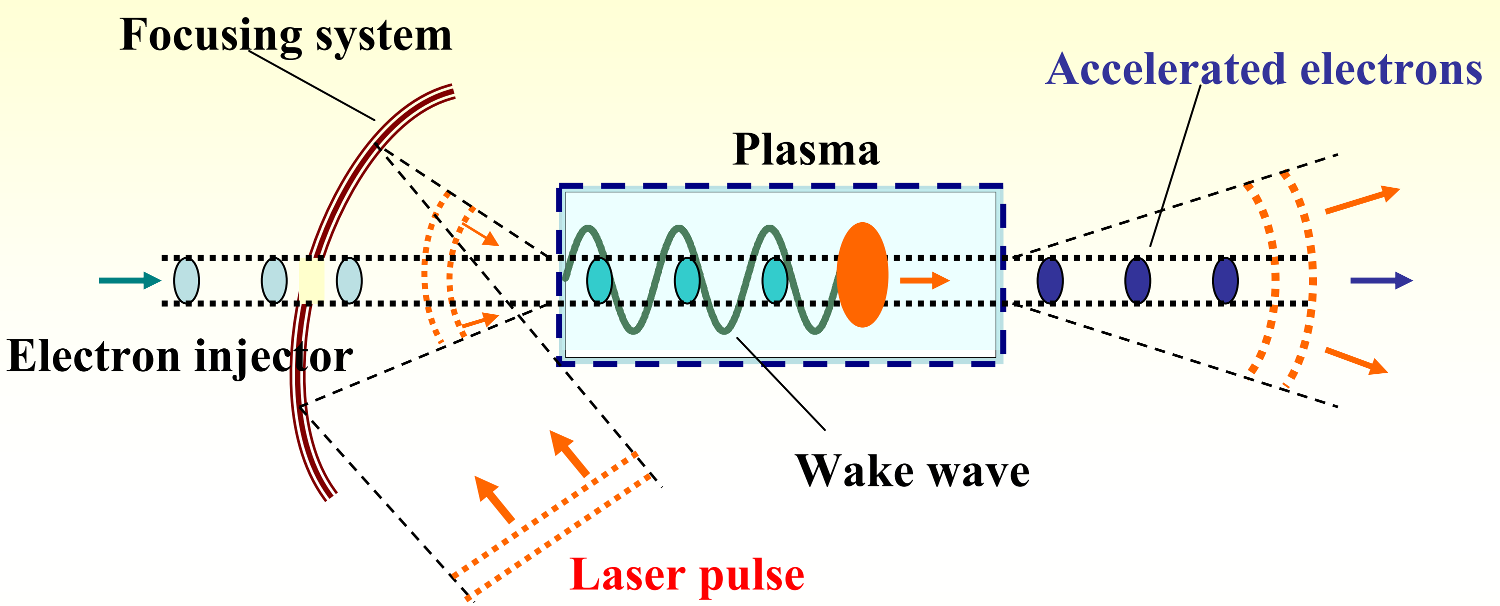


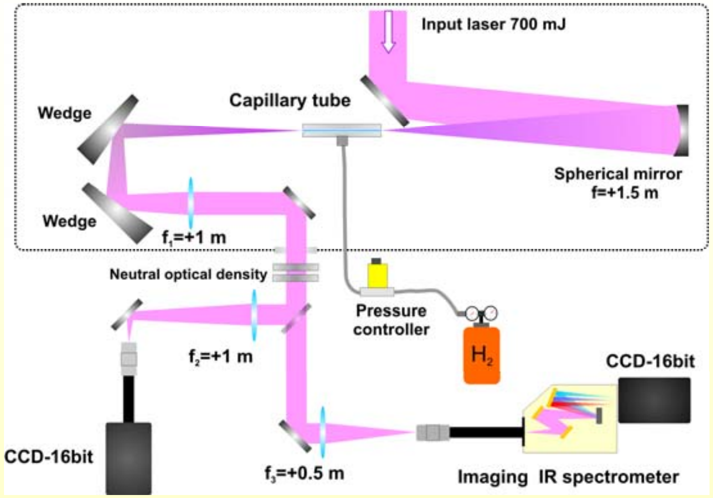


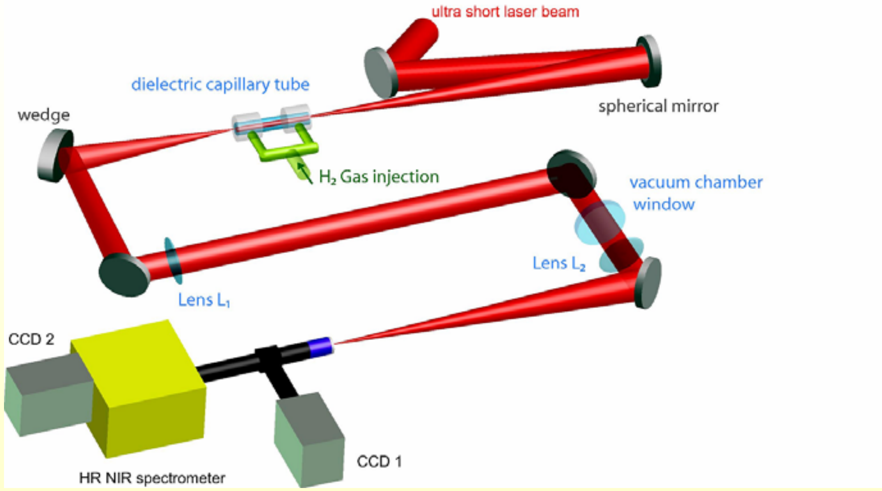
for **Self-M**

UK (RAL)							
France (CEA)	5	10^{19}	0.3	1.05			
USA (CUOS)	4.5	10^{18}	0.4	1.05			
USA (NRL)	2.5	4×10^{18}	0.4	1.05			
Japan (KEK)	3	10^{17}	1.0	1.05	10^{19}	17	30

Scheme of one cascade of the laser wake-field accelerator







B. Cros, et al. Schematic view of the experiment **at the Lund Laser Center**

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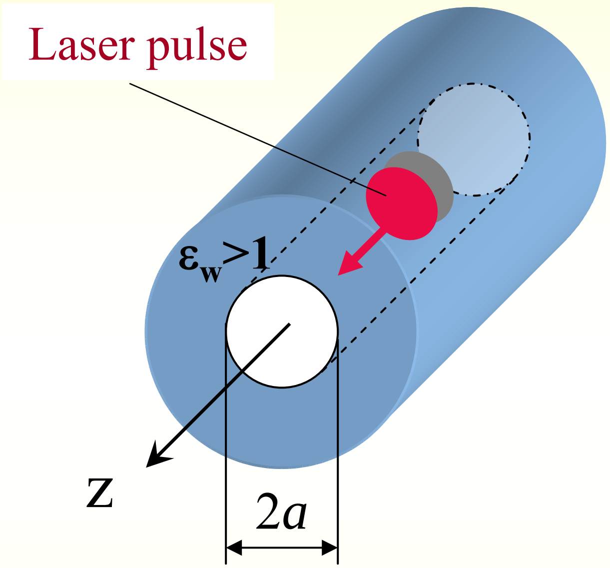
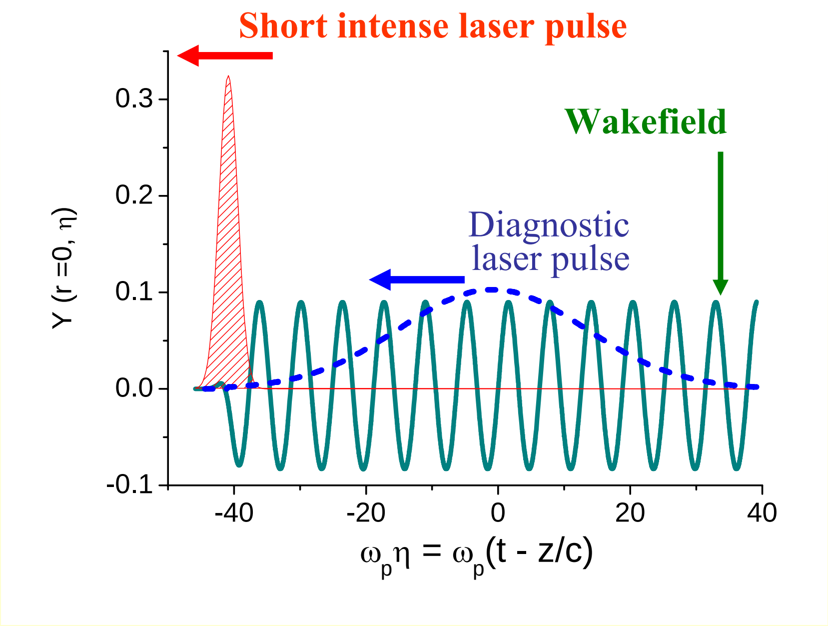
Wakefield generation by guided laser pulses
➢ *spectroscopic diagnostics of the wakefield*

$$\tilde{E}(r)=\sum_{n=1}^{N_m} C_n J_0(k_{\perp n} r), \quad k_{\perp n} = \frac{u_n}{a} - i\frac{u_s}{k_{w\perp} a^2}$$

$$C_n = \frac{2}{[a J_1(u_n)]^2}\int_0^a E(r) J_0(u_n r/a)\, r\, dr, \quad J_0(u_n)=0$$





for the Gaussian laser pulse
$E(r) = E_0 \exp(-r^2/r_0^2)$
Energy coupling to the main mode
98% at $r_0/a = 0.645$

Loading effect in LWFA of short e-bunches

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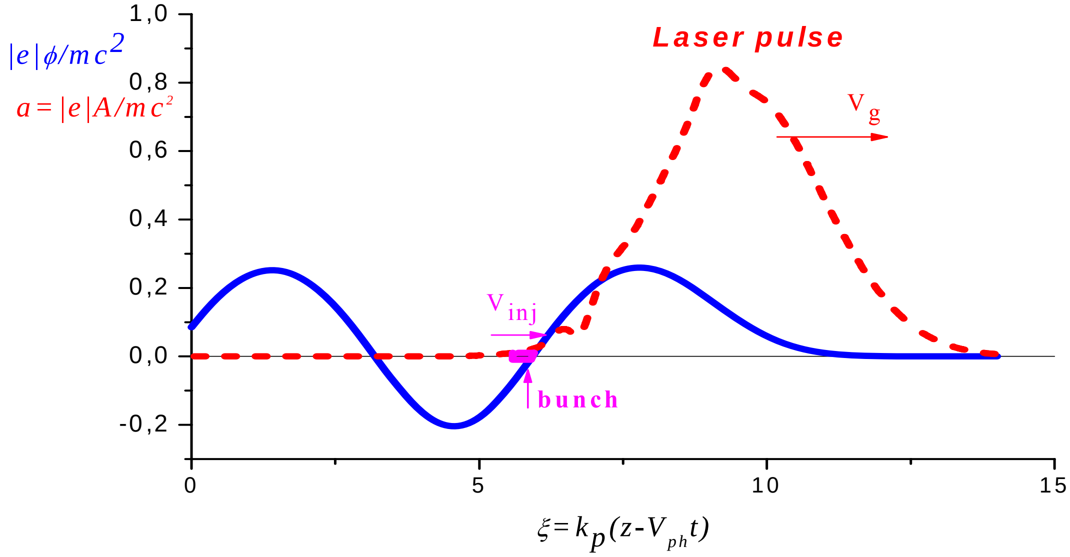
Scheme of short e-bunch injection into LWFA

➢*Laser and plasma parameters*

Parameters of the laser pulse and electron bunch

$a_0 = \frac{|e|E_L}{mc\omega} = 1.0$ $\gamma_{ph} = \omega/\omega_p = 50$ $E_{inj} = 80\,mc^2$ $L_b = 0.1 k_p^{-1}$



$$E_{max} \approx 2\gamma_{ph}^2 \varphi_{max}$$

$$|\Delta E| \approx 2\gamma_{ph}^2 k_p L_{b0} \left\{ \frac{d\phi(\xi_{inj})}{d\xi} \right\}$$

$$\Delta E / E_{max} \simeq k_p L_b \simeq 10\%$$

for $L_b \approx$ 1mkm (3fs !)

The wake field of a cylindrical electron bunch moving with the velocity V(t)

$$\delta\varphi_b \equiv \frac{e}{mc^2}\delta\Phi_b = \frac{n_b}{n_0}\left[1 - \mathrm{I}_0(\rho)\mathrm{K}_1(\rho_b)\rho_b\right](1-\cos\varsigma)$$

$$\varsigma = k_p\left[z - \int_0^t V(t')dt'\right]$$

$$\text{where } \rho < \rho_b = k_p R_b, \quad -k_p L_b \le \varsigma \le 0, \quad k_p = \omega_p / c$$

For a wide electron bunch $R_b >> k_p^{-1}$ ***and*** $r_\perp < R_b,\ k_p(R_b - r_\perp) > 1$

the wake field of electron bunch can be approximated by 1-D distribution :

$$\delta\varphi_b \equiv \frac{e\delta\Phi_b}{mc^2} = \frac{1}{2}\frac{n_b}{n_0}\varsigma^2$$

where ζ=0 corresponds to the leading front of the bunch,
and $\varsigma = -k_p L_b < 1$ corresponds to the trailing edge

Loading effect doesn't influence substantially the maximum energy of accelerated electrons under condition

$$\frac{n_b}{n_0} k_p L_b << \varphi_{max}$$

An electron motion in the laser and e-bunch wake fields

$$\frac{dq}{d\tau}=\frac{\partial}{\partial \bar{z}}\left(\varphi+\delta\varphi_b\right)$$

$$\left[E/mc^2-\beta_{ph}q-\varphi\right]_{\xi_{inj}}^{\xi}=\frac{n_b}{n_0}(\xi-\xi_{inj})\varsigma$$

$$where\; q=P/mc,\quad \tau=\omega_p t,\quad \bar{z}=k_p z$$

The energy spread at the end of acceleration

$$\frac{\Delta E}{mc^2}=2\gamma_{ph}^2 k_p L_b\left\{\left(\frac{d\varphi}{d\xi_{inj}}-\frac{d\varphi}{d\xi}\right)+\frac{k_p L_b}{2}\left(\frac{d^2\varphi}{d\xi^2}-\frac{d^2\varphi}{d\xi_{inj}^2}\right)-\frac{n_b}{n_0}(\xi-\xi_{inj})\right\}$$

Optimization of bunch acceleration

The energy spread of the bunch has a minimum at the condition:

$$\frac{d\varphi}{d\xi_{inj}} - \frac{d\varphi}{d\xi} + \frac{k_p L_b}{2}\left(\frac{d^2\varphi}{d\xi^2} - \frac{d^2\varphi}{d\xi_{inj}^2}\right) - \frac{n_b}{n_0}(\xi - \xi_{inj}) = 0$$

The optimal bunch density for a minimum energy spread :

$$n_b = \frac{n_0}{\xi_{\max} - \xi_{inj}}\left\{\frac{d\varphi}{d\xi_{inj}} + \frac{k_p L_b}{2}\frac{d^2\varphi}{d\xi_{\max}^2}\right\}$$

The minimal energy spread for optimal bunch density:

$$\frac{|\Delta E_{\min}|}{mc^2} = \gamma_{ph}^2 \frac{(k_p L_b)^2}{4}\left|\frac{d^2\varphi}{d\xi_{\max}^2}\right|$$

N.E. Andreev and S.V. Kuznetsov, IEEE Trans. on Plasma Sci., vol. 36, No.4. pp. 1765-1772, 2008

Nonlinear plasma response – Effective Potential

The nonlinear relativistic plasma response can be expressed through a single scalar function (potential) Φ :

$$\frac{\nu}{\gamma} = \frac{\nu_0 + \Delta_\perp \Phi}{\Phi + \delta\Phi_S} \qquad \delta\Phi_S = -\frac{1}{\nu_0}\int_{+\infty}^{\xi} \frac{\partial \nu_0}{\partial \xi'}\left(\Phi - 1 + \frac{|\boldsymbol{a}|^2}{4} - \frac{\mu}{4}\mathrm{Re}\left(\boldsymbol{a}^* \cdot \boldsymbol{a}^*\right)\right) d\xi'$$

$$\Phi = \gamma - q_z + \int_{+\infty}^{\xi} \frac{S_0}{\nu}\left(q_z - \frac{|\boldsymbol{a}|^2}{2} - \frac{\mu}{4}\mathrm{Re}\left(\boldsymbol{a}^* \cdot \boldsymbol{a}^*\right)\right) d\xi' \equiv \gamma - q_z - \delta\Phi_S$$

The electric and magnetic fields in plasma can be also expressed through the potential Φ:

$$E_z = \frac{\partial \Phi}{\partial \xi}, \qquad E_r - B_\varphi = \frac{\partial \Phi}{\partial \rho},$$

For a wide (in comparison with the plasma skin depth $1/k_p$) laser pulse the equation for the potential can be linearized with respect to the small parameter $|\Phi - 1|/(k_p L_\perp)^2$

$$\left\{(\Delta_\perp - \nu_0)\frac{\partial^2}{\partial \xi^2} - \frac{\partial \ln \nu_0}{\partial \rho}\frac{\partial^3}{\partial \rho\, \partial \xi^2} + \nu_0 \Delta_\perp\right\}\Phi - \frac{\nu_0^2}{2}\left[1 - \frac{1 + |\boldsymbol{a}|^2}{\left(\Phi + \delta\Phi_S\right)^2}\right] = \nu_0\left[\Delta_\perp \frac{|\boldsymbol{a}|^2}{4} - \boldsymbol{N_b}(\xi, \rho, z)\right]$$

The acceleration of relativistic electrons of the witness *e*-beam pulse in the wakefield

$$\frac{dP_z}{d\tau} = \frac{\partial}{\partial \xi}\Phi + F_z \qquad F_z = -\frac{1}{\gamma}\frac{\partial}{\partial \xi}|\boldsymbol{a}|^2 / 4$$

$$\frac{d\mathbf{P}_r}{d\tau} = \frac{\partial}{\partial \boldsymbol{\rho}}\Phi + \mathbf{F}_r \qquad \mathbf{F}_r = -\frac{1}{\gamma}\frac{\partial}{\partial \boldsymbol{\rho}}|\boldsymbol{a}|^2 / 4$$

and

$$\frac{d\xi}{d\tau} = \frac{P_z}{\sqrt{1 + P_z^2 + P_r^2}} - 1 \qquad \frac{d\boldsymbol{\rho}}{d\tau} = \frac{\mathbf{P}_r}{\sqrt{1 + P_z^2 + P_r^2}}$$

where P_z, $\mathbf{P}_r = \{P_x, P_y\}$ are components of momentum, longitudinal and perpendicular to the axis *OZ*, of an accelerating electron normalized to *mc*, $\tau = \omega_{p0} t$ $\zeta = \xi + \tau$

Computer simulation and comparison with analytic predictions

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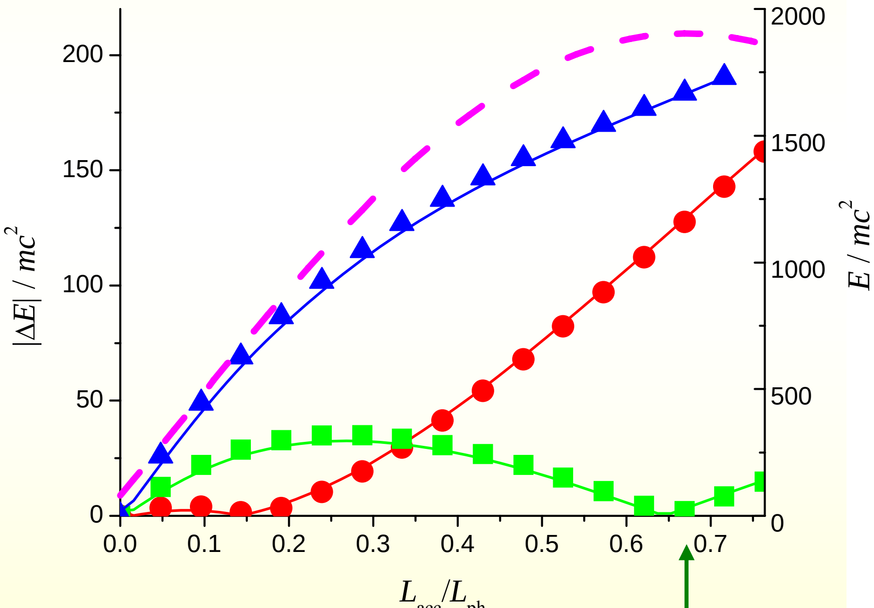
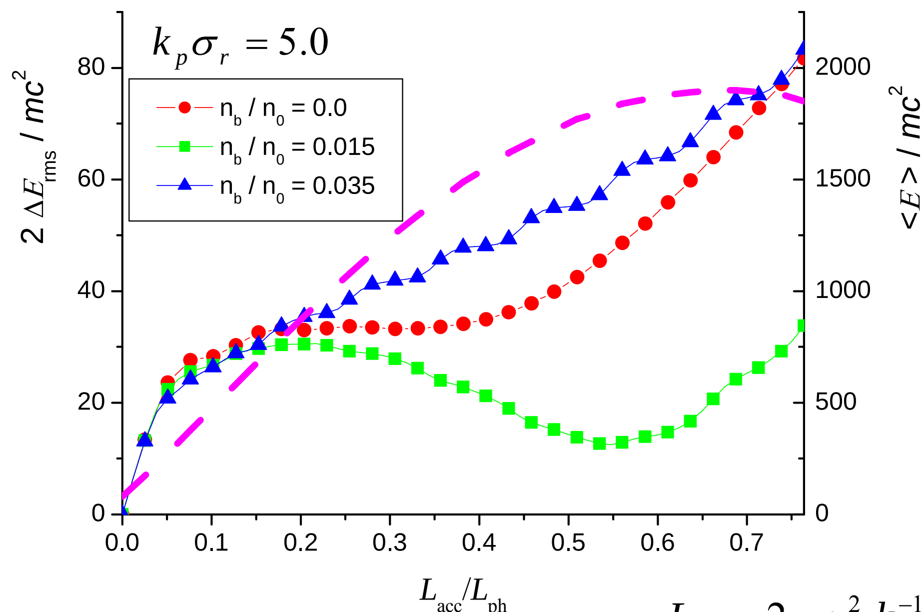
Parameters of laser pulse and electron bunch

$$a_0 = \frac{|e|E_L}{mc\omega} = 1.0 \qquad \gamma_{ph} = \omega/\omega_p = 50 \qquad E_{inj} = 80\ mc^2 \qquad L_b = 0.1k_p^{-1}$$





$$L_{ph} = 2\pi\,\gamma_{ph}^2 k_p^{-1}$$

$$|\Delta E_{min}| / mc^2 = 1.29$$

in agreement with analytical prediction

$$|a(r_\perp, z, t)| = a(\xi)\exp\left[-\rho^2/\left(k_p r_0\right)^2\right]$$

Solid lines are analytical prediction; markers are results of numerical modeling for different bunch densities:

$n_b / n_0 =$ **0 – circles; 0.3 – triangles; 0.121 – squares.**

$$\tau_L = 1.1\omega_p^{-1} \qquad r_0 = 14.14\ k_p^{-1}$$

New scheme of the Electron Bunch Injection in Front of the Laser Pulse

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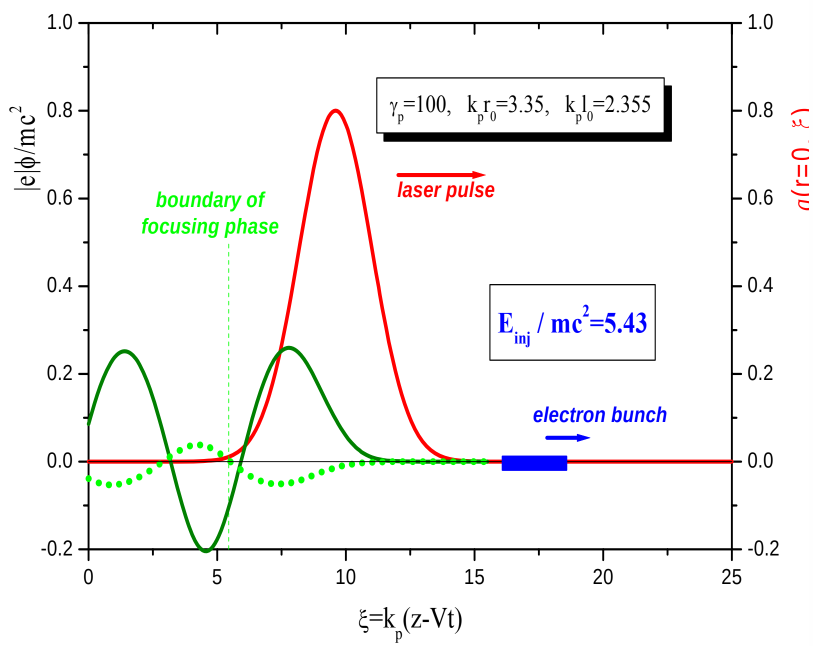

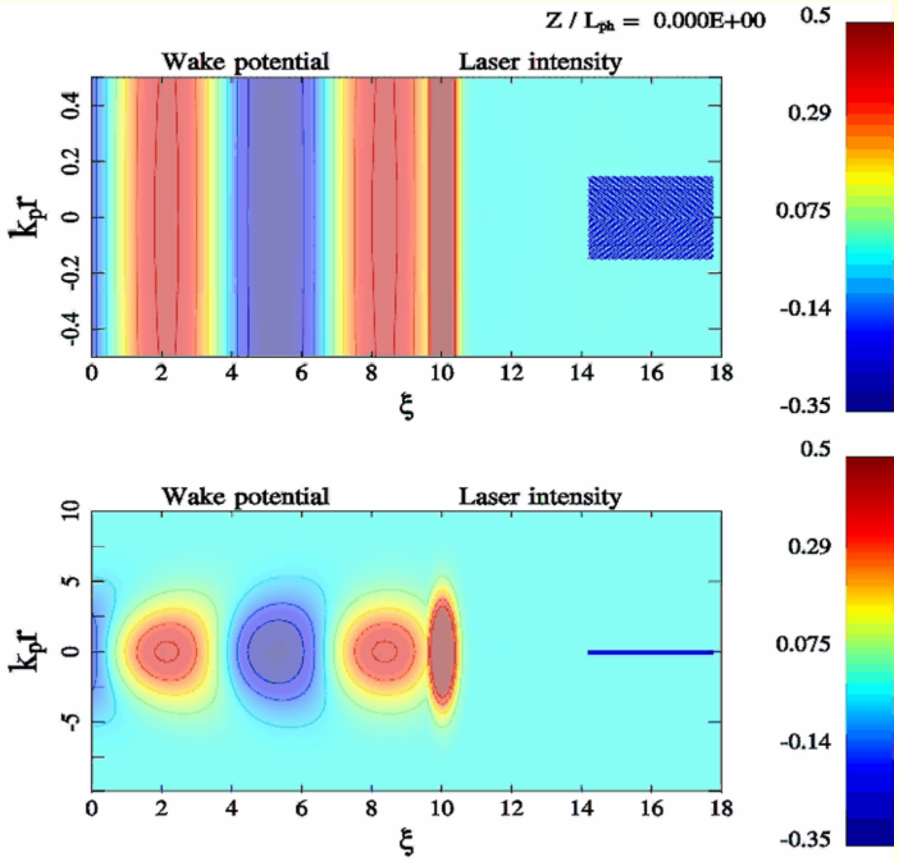

For the velocity of injected electrons

$$u_{inj} = c\sqrt{1-m^2c^4/E_{inj}^2} < v_{ph} \quad :$$

$$-\varphi(\xi_{tr}) = E_{inj}/mc^2 - \left[\left(1-\gamma_{ph}^{-2}\right)\left(E_{inj}^2/m^2c^4-1\right)\right]^{1/2} - 1/\gamma_{ph}$$

$$\frac{L_b}{L_{b0}} = \frac{1-\beta}{\beta - u_{inj}/c} \qquad \beta = v_{ph}/c$$

Long low-energy electron bunch will be trapped and compressed in the wakefield

N.E Andreev., S.V. Kuznezov. **Electron Bunch Compression in Laser Wakefield Acceleration.//** http://www.nuclear.jp/~icfa/GIENS/workshop_papers/giens01.pdf: ***Giens Workshop Proceedings, June 24 - 29, 2001***.

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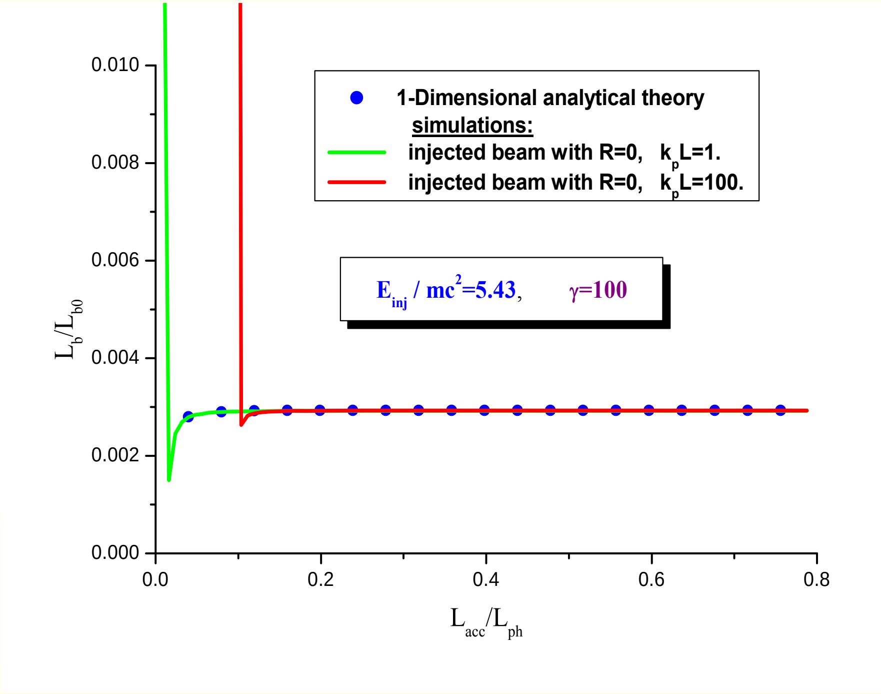
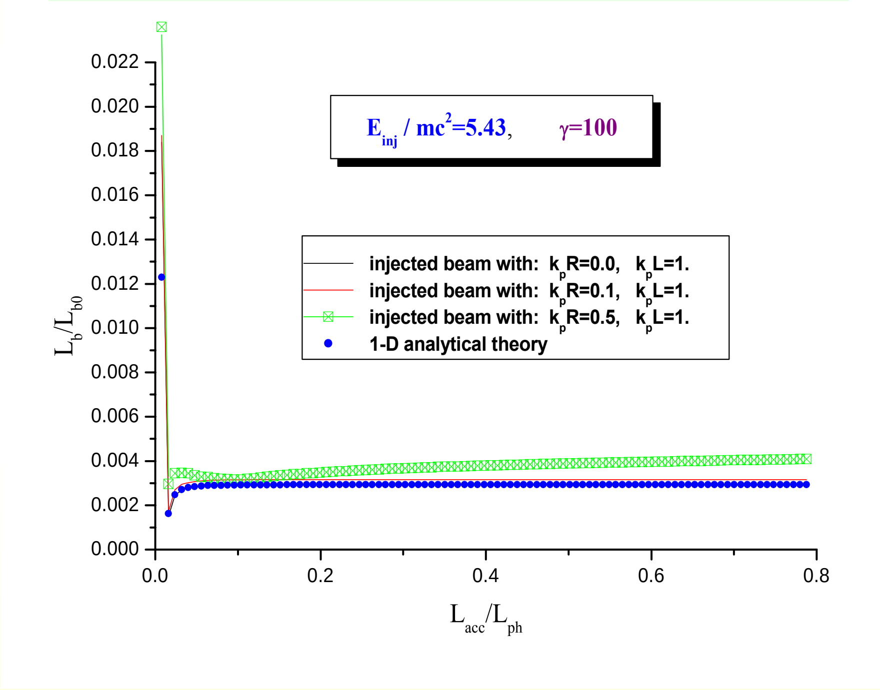
The bunch length decreases substantially in the process of trapping

1-D simulations confirm simple analytical predictions



3-D simulations are in a good agreement with 1-D theory *for injected electron bunches of small radius*

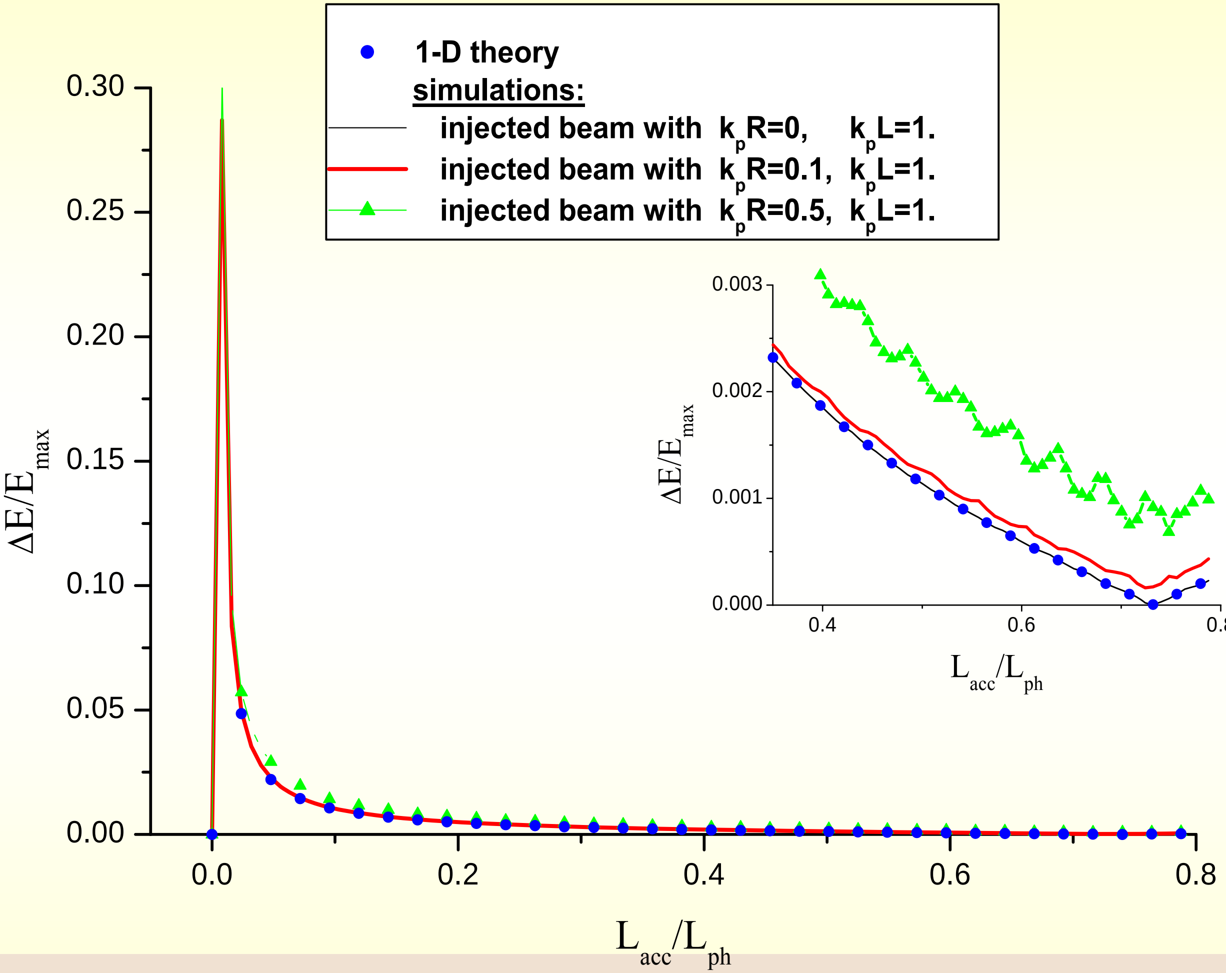


For the velocity of injected electrons

$$u_{inj} = c\sqrt{1 - m^2c^4 / E_{inj}^2} < v_{ph}$$

$$\frac{L_b}{L_{b0}} = \frac{1-\beta}{\beta - u_{inj}/c} \qquad \beta = v_{ph}/c$$

Energy spread decreases substantially at the end of accelerating phase



in accordance with the theory *for injected electron bunches of small radius*

$$\frac{\left|\Delta E(\xi)\right|}{E_{\max}(\xi)} = \frac{1}{1-\beta c/u(\xi)} \cdot \frac{d\phi}{d\xi} k_p L_b(\xi) \cdot E_{\max}^{-1}(\xi)$$

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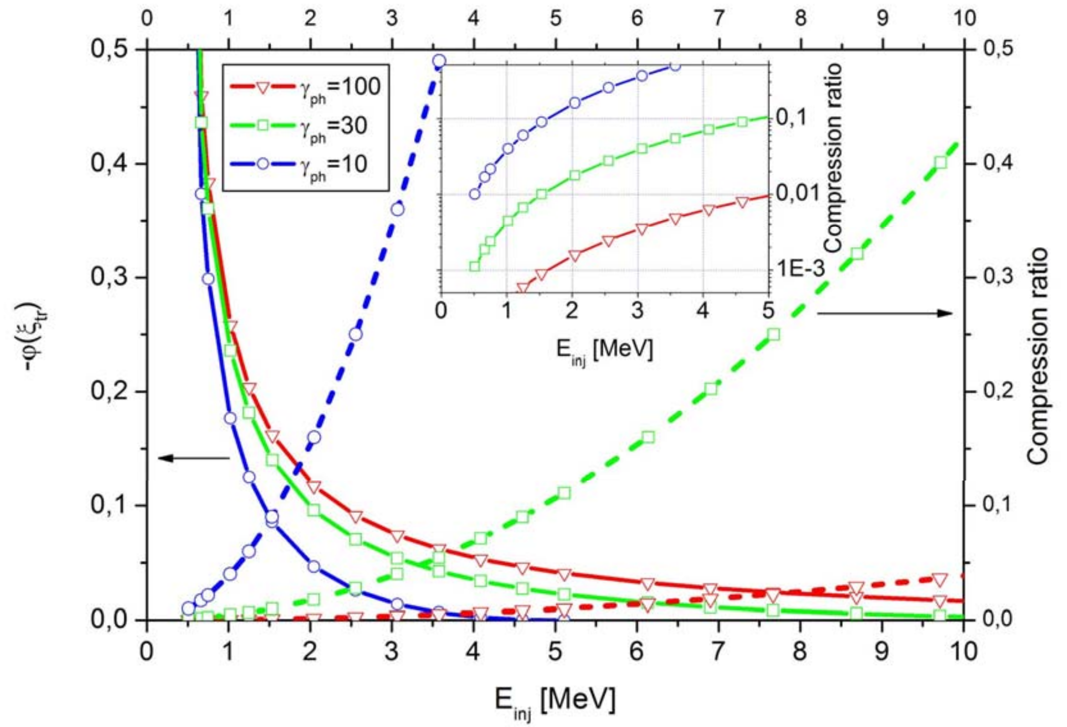
Electron Bunch Injection in Front of the Laser Pulse

trapping and compression

bunch injected in front of the laser pulse can be trapped and compressed in the wake field, if the condition

$$-\varphi(\xi_{tr}) = E_{inj}/mc^2 - \left[\left(1-\gamma_{ph}^{-2}\right)\left(E_{inj}^2/m^2c^4 - 1\right)\right]^{1/2} - 1/\gamma_{ph}$$

is fulfilled in the focusing phase of the wakefield



$$\frac{L_{b,rms}}{L_{b0}} \cong \frac{c - V_{ph}}{V_{ph} - u_{inj}} \approx \frac{E_{inj}^2}{\gamma_{ph}^2 m^2 c^4}$$

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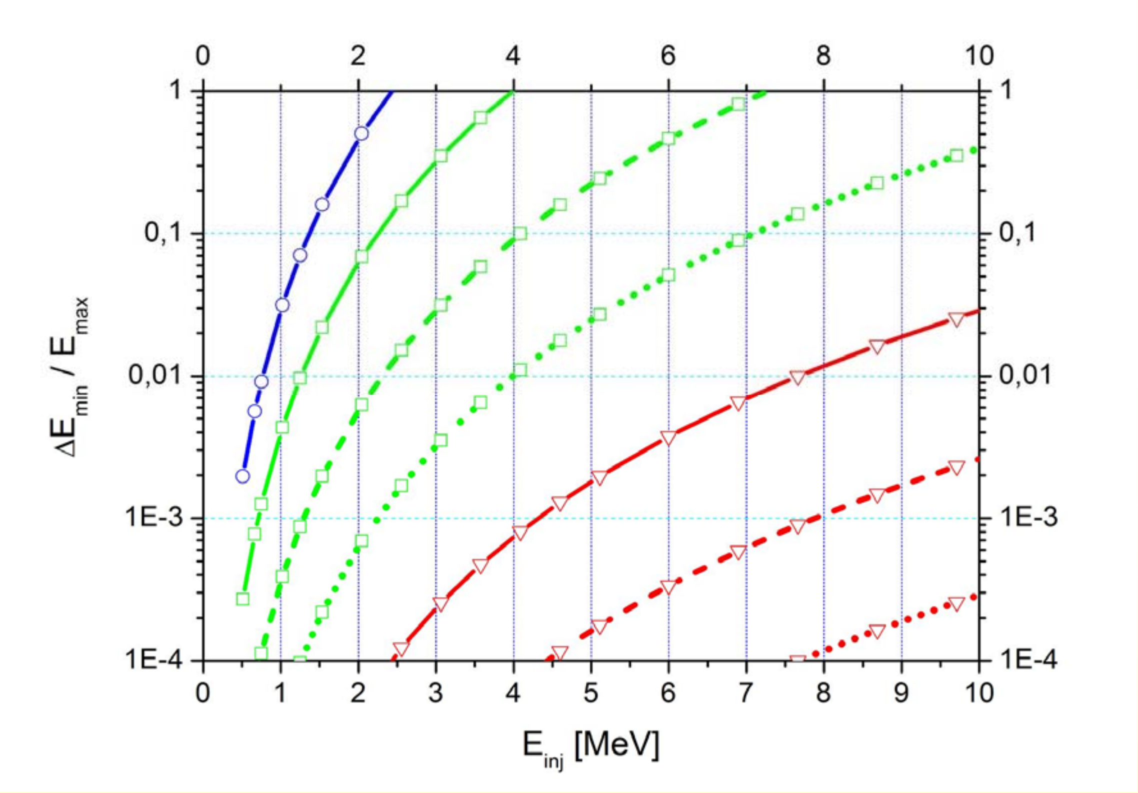
Electron Bunch Injection in Front of the Laser Pulse

energy spread at the end of acceleration

$$E_{\max} \cong 2\gamma_{ph}^2 mc^2 \varphi_{\max}$$

$$\frac{\Delta E_{\min}}{E_{\max}} \cong \frac{1}{2}(k_p L_{b,rms})^2 \cong 2\pi^2 \gamma_{ph}^{-6} \left(\frac{E_{inj}}{mc^2}\right)^4 \left(L_{b0}/\lambda_0\right)^2$$

$$\frac{\Delta E_{\min}}{mc^2} = \gamma_{ph}^2 (k_p L_{b,rms})^2 \left|\frac{d^2\varphi}{d\xi_{\max}^2}\right|$$



$\gamma_{ph} = 100$, 30, and 10 marked by triangles, squares and circles respectively, and for three initial bunch lengths $L_{b0} = 100$, 30, and 10 μm (solid, dashed and dotted lines respectively) for the laser wave length $\lambda_0 = 1$ μm

Computer simulation by the code LAPLAC

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full scale modeling including
laser pulse dynamics, gas ionization and bunch loading

$$\left\{2ik_0\frac{\partial}{\partial z}+2\frac{\partial^2}{\partial z\,\partial\xi}+\Delta_\perp\right\}a=k_0^2\left(\frac{n}{n_c\gamma}a-iG^{(ion)}\right)$$

$$\frac{n}{\gamma}=n_0\frac{1+k_p^{-2}\Delta_\perp\Phi}{\Phi+\delta\Phi_S}$$

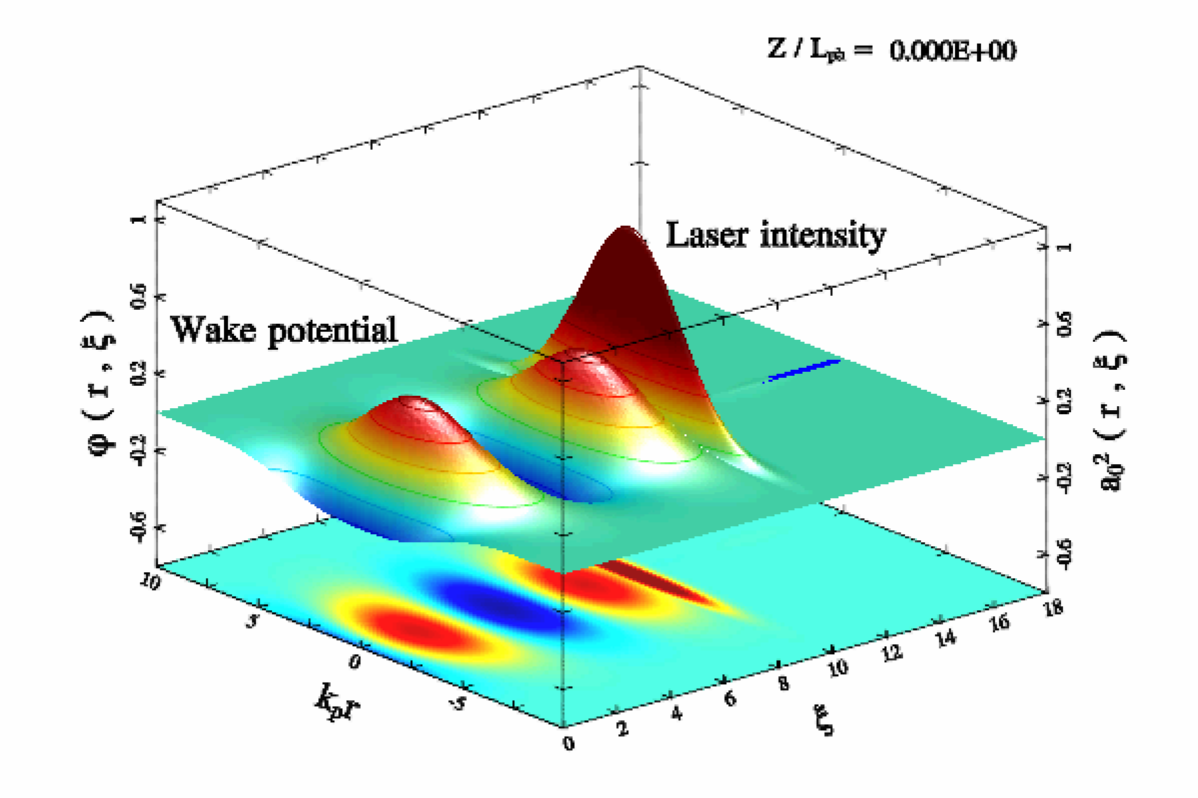
$$\mathbf{G}^{(ion)}=\frac{4\pi e}{m\omega_{p0}^2c}\mathbf{J}^{(ion)}=\frac{k_{p0}}{k_0}\left[\frac{2\boldsymbol{a}}{|\boldsymbol{a}|^2}\sum_{k=0}^{Z_n-1}S_0^{(k)}\frac{U_k}{mc^2}-\frac{\boldsymbol{a}^*}{4}S_2\right]$$

$$S_0=\frac{\Gamma_0}{\mathrm{N}_0\omega_{p0}}=\frac{n_a}{\mathrm{N}_0\omega_{p0}}\sum_{k=0}^{Z_n-1}\overline{W}_kD_k\equiv\sum_{k=0}^{Z_n-1}S_0^{(k)},\qquad S_2=\frac{\Gamma_2}{\mathrm{N}_0\omega_{p0}}\cong 2\mu S_0$$

Computer simulation by the code LAPLAC

Results of full scale modeling including laser pulse dynamics, gas ionization and bunch loading

For symmetrical laser pulse propagation accelerating fields have regular structure over long distances (during acceleration phase)

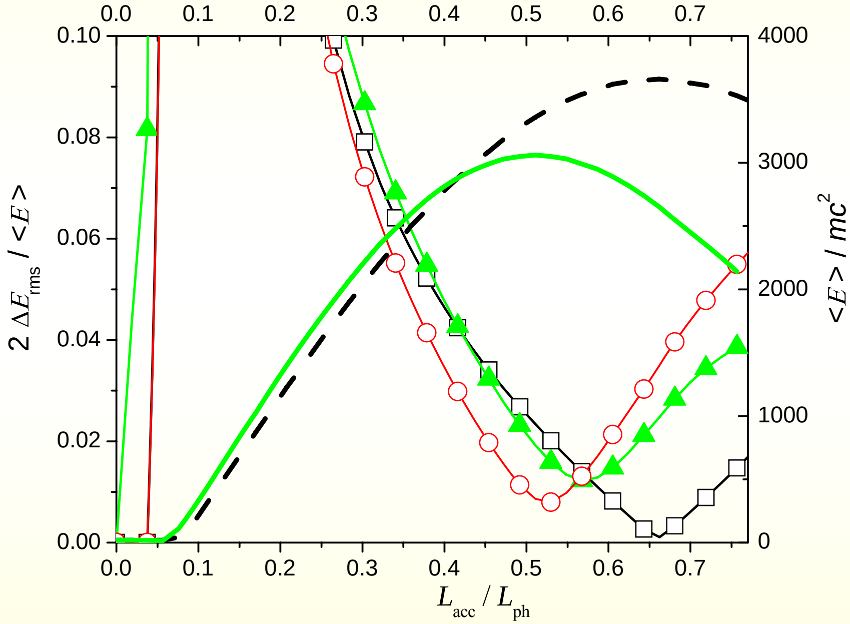


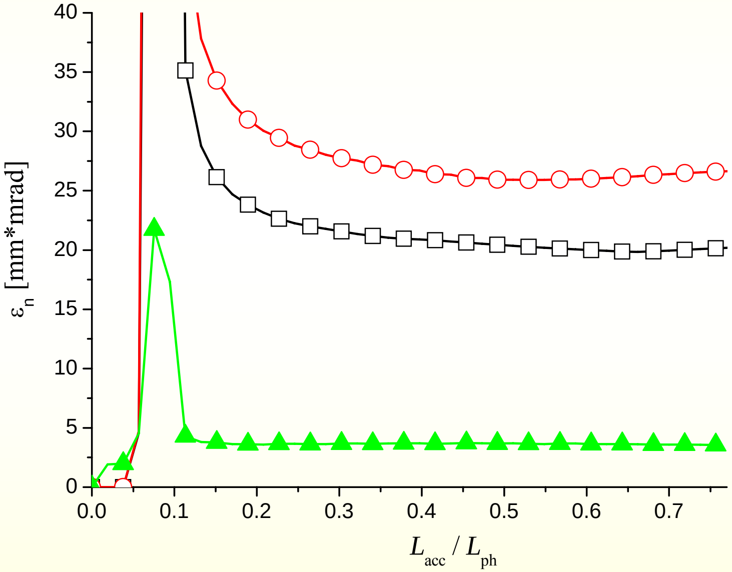
Parameters of the laser pulse and electron bunch

$$a_0 = \frac{|e|E_L}{mc\omega} = 1, \quad \gamma_{ph} = \omega/\omega_p = 100, \quad E_{inj} = 10\ \mathrm{MeV}, \quad L_b = 1.26\,k_p^{-1}, \quad R_b = 0.15\,k_p^{-1}$$

Computer simulation by the code LAPLAC

Results of full scale modeling including laser pulse dynamics, gas ionization and bunch loading





Parameters of the laser pulse and electron bunch

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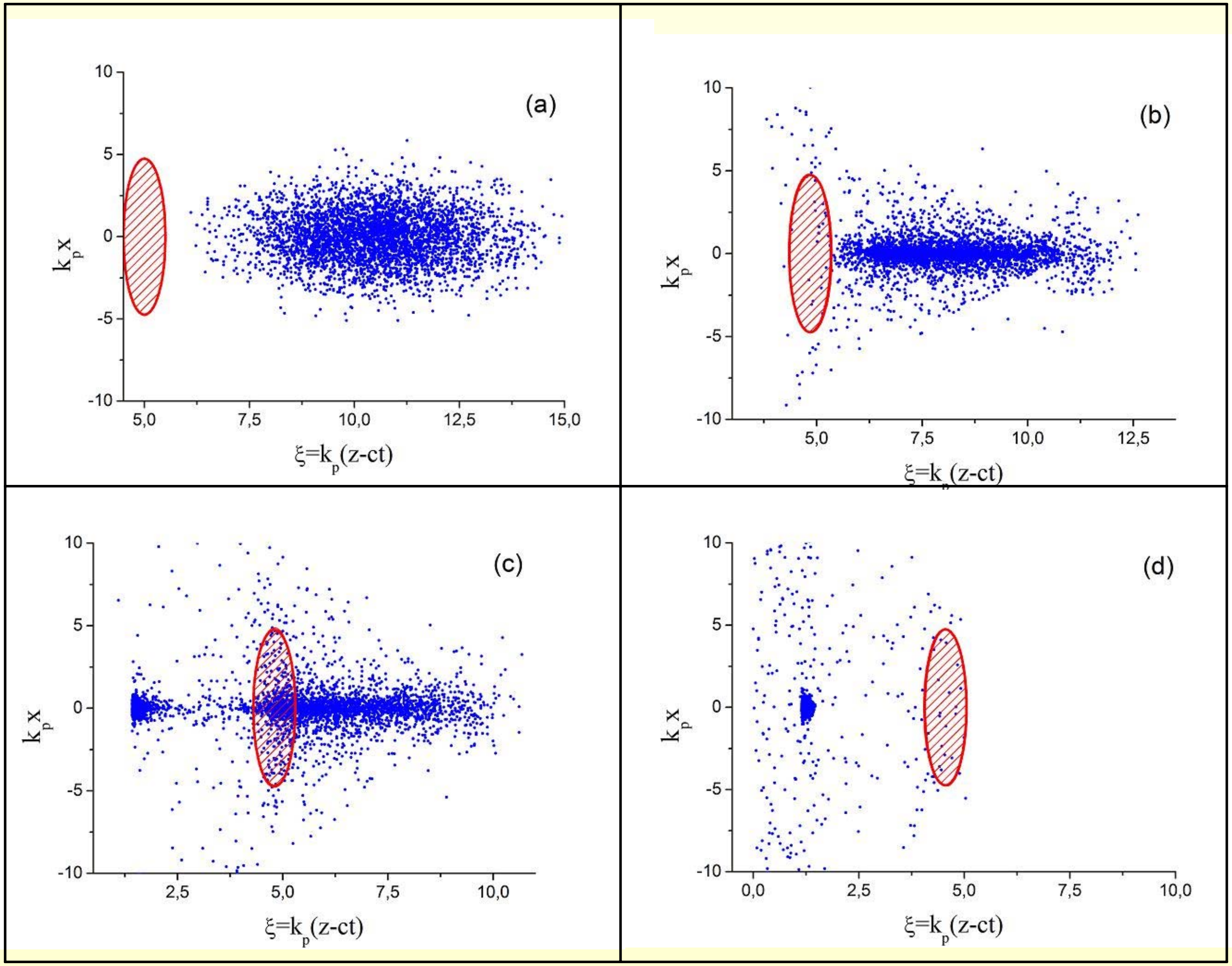
N.E. Andreev and S.V. Kuznetsov, IEEE Trans. on Plasma Sci., vol. 36, No.4. pp. 1765-1772, 2008

Computer simulation by the code LAPLAC

trapping and compression



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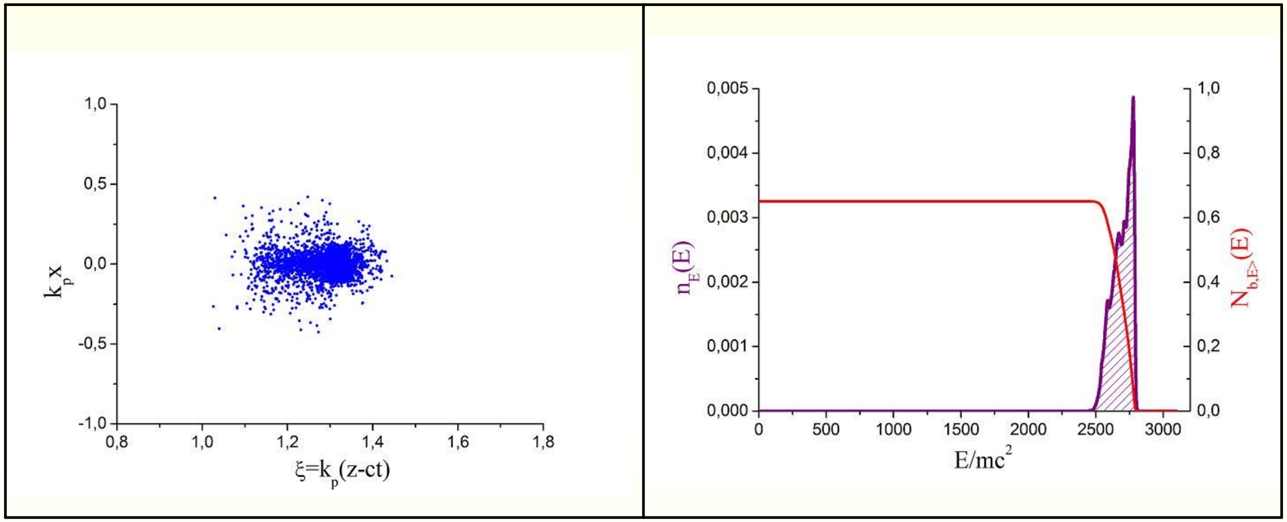
Computer simulation by the code LAPLAC

accelerated electron bunch

the bunch has acquired an energy of 1.4 GeV with a narrow energy spectrum and low emittance 4.8 mm mrad



The total trapped and accelerated number of particles in the bunch is about 65% of the injected electrons

$E_{inj} = 10$ MeV $L_{b0} = 2\sigma_z = 50\,\mu\text{m}$ $r_0 = 80\,\mu\text{m}$ $I_L = 1.2\times10^{18}\ \text{W/cm}^2$ $P_L / P_{cr} = 0.72$

Q_b=5 pC $\sigma_\perp = r_{rms}/\sqrt{2} = 25\,\mu\text{m}$ $\tau_{\text{FWHM}} = 33$ fs laser energy 4.3 J $n_0(0) = 10^{17}\ \text{cm}^{-3}$

$\varepsilon_{N,r} = 4 r_{rms}\sigma_{P_r/mc} = 1\,\text{mm mrad}$

Laser plasma electron acceleration experiments - 2009

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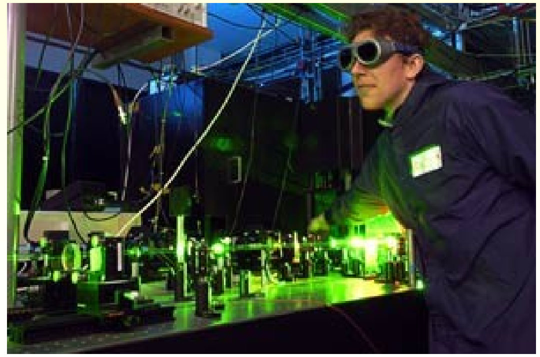
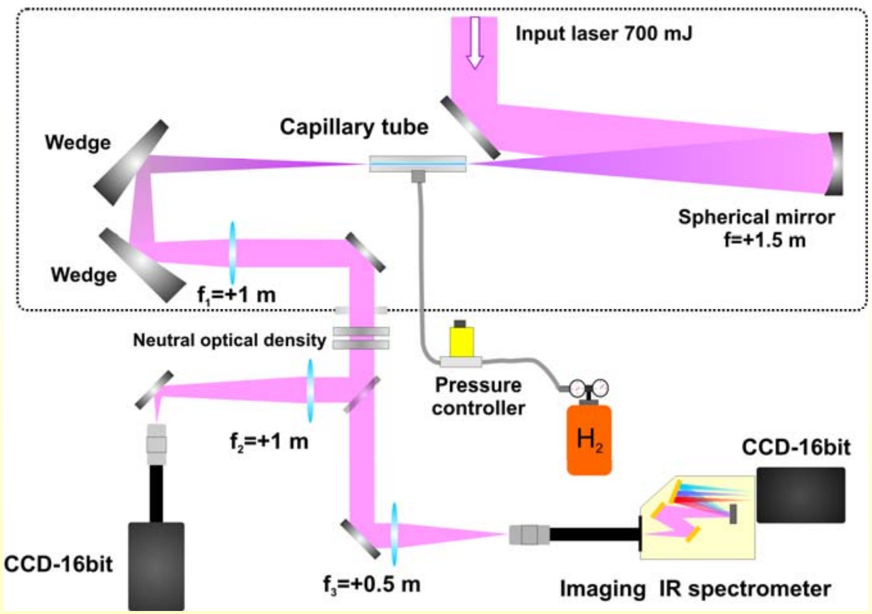
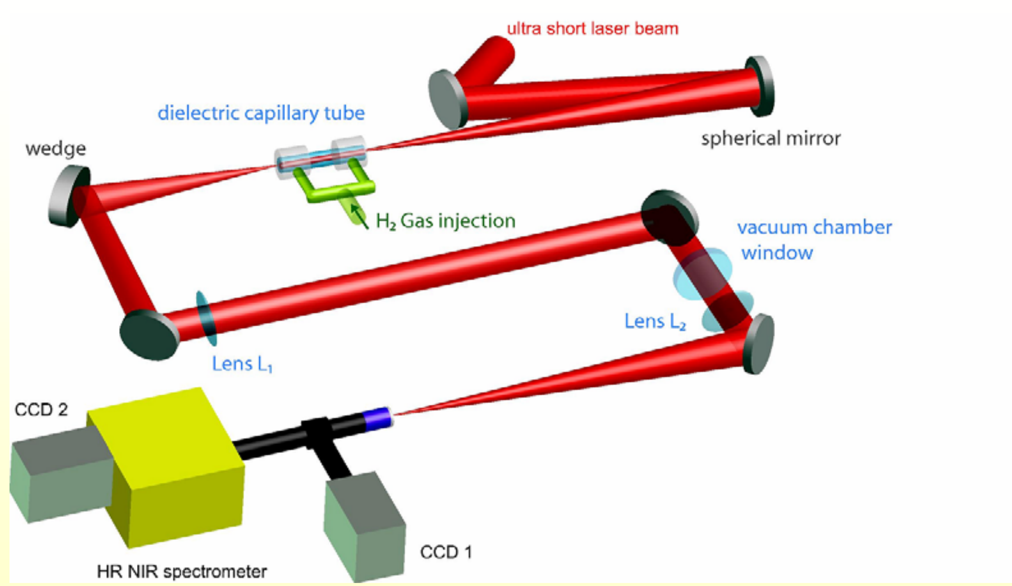
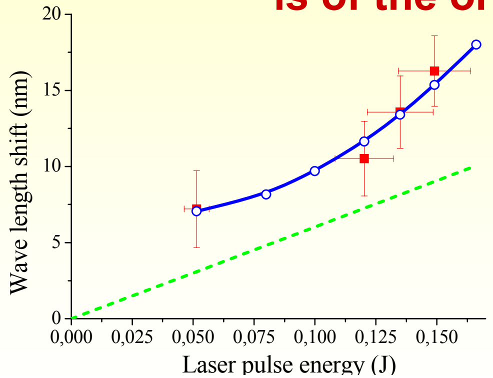


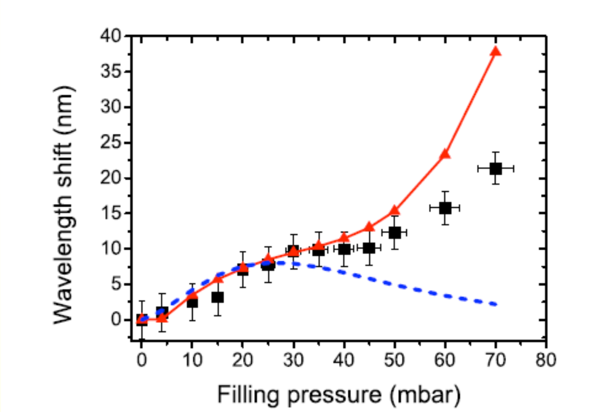


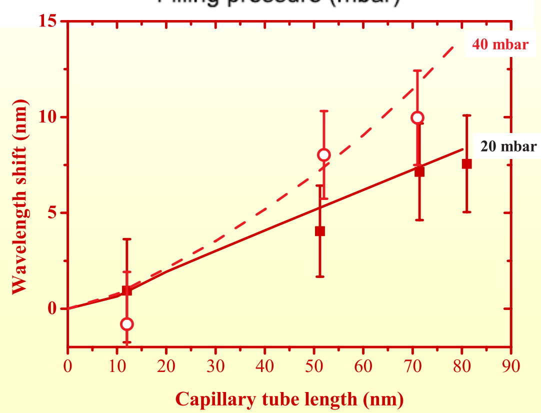


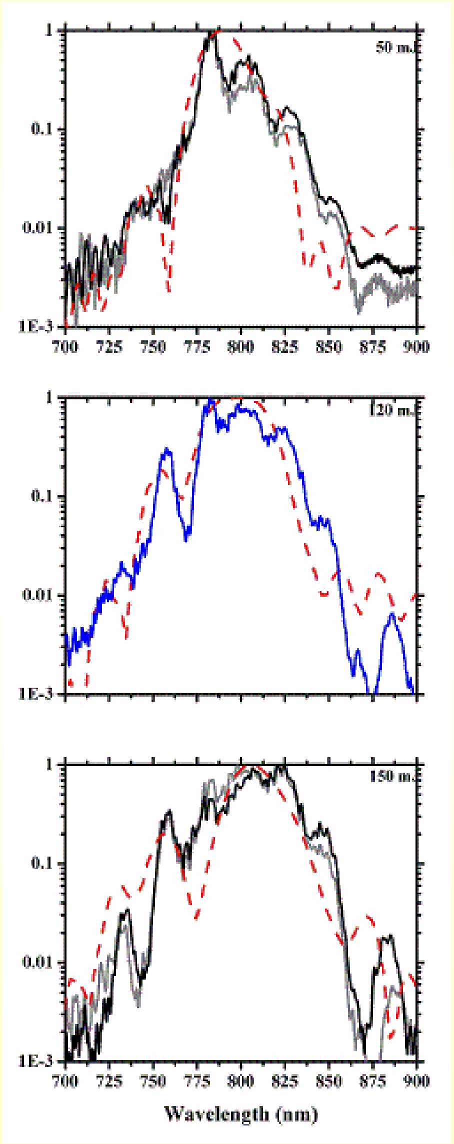
Spectral diagnostics of the laser wake fields in capillary tubes

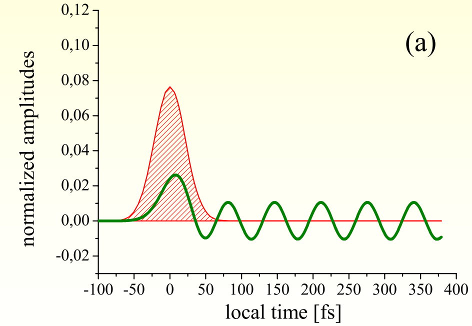
The average product of gradient and length achieved in this experiment
is of the order of 0.4 GV at a pressure of 50 mbar

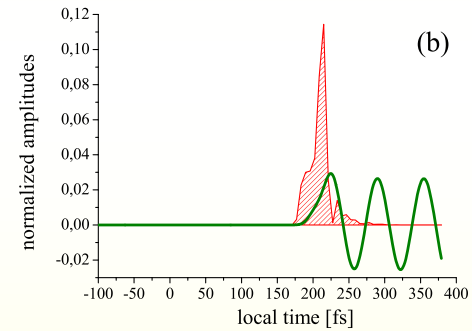


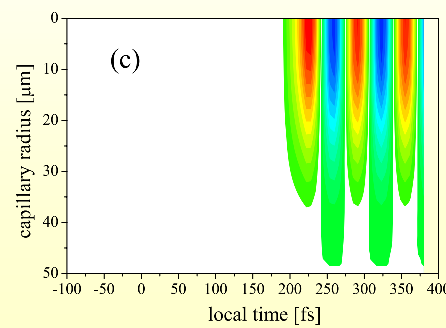












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Conclusions

- The control of the wakefield phase velocity is necessary for an effective electron bunch compression

- The transverse focusing of the bunch, while it propagates in plasma before the laser pulse overtakes the bunch, is important for the decrease of the final bunch emittance

- The effective longitudinal bunch compression in this scheme of injection leads to a small relative energy spread (of order 1%) at the end of the acceleration stage

- Loading effect can be controlled and used to optimize electron bunch parameters for low energy spread

  (but it limits bunch charge!)

With thanks for collaboration to

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Thank
You for attention!