Two-dimensional simulations of strongly radiating plasmas with the RALEF-2D code

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Equations of hydrodynamics

One fluid, one temperature, two spatial dimensions (either x,y, or r,z):

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0, \]
\[ \frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla p = 0, \]
\[ \frac{\partial (\rho E)}{\partial t} + \nabla \cdot [(\rho E + p)\vec{u}] = \nabla \cdot (\kappa \nabla T) + Q_r + Q_{dep}, \]
\[ E = e + \frac{u^2}{2}, \quad e = e(\rho, T) \]

\[ \nabla \cdot (\kappa \nabla T) \] – energy deposition by thermal conduction (local), \[ Q_r \] – energy deposition by radiation (non-local), \[ Q_{dep} \] – eventual external heat sources.
Radiation transport

Transfer equation for radiation intensity $I$ in the quasi-static approximation:

$$\frac{1}{c} \frac{\partial I}{\partial t} + \bar{\Omega} \cdot \nabla I = k_\nu (B_\nu - I), \quad I = I(t, \vec{x}, \nu, \bar{\Omega}), \quad B_\nu = B_\nu(\nu, T)$$

**Quasi-static approximation:** radiation transports energy infinitely fast (compared to the fluid motion) $\Rightarrow$ the energy residing in radiation field at any given time is infinitely small!

Coupling with the fluid energy equation:

$$Q_r = -\nabla \cdot \left( \int d\nu \int d\bar{\Omega} I d\bar{\Omega} \right) = \int d\bar{\Omega} \int_{\frac{4\pi}{\nu}} k_\nu (I - B_\nu) d\nu$$

Radiation transport adds 3 extra dimensions (two angles and the photon frequency) $\Rightarrow$ the 2D hydrodynamics becomes a 5D radiation hydrodynamics!
Radiation transport in the diffusion approximation

In many cases the term “radiation hydrodynamics” is applied to hydrodynamic equations augmented with the multi-frequency diffusion equation

\[
\frac{1}{c} \frac{\partial \mathcal{A}_v}{\partial t} + \nabla \cdot \left( \mathcal{A}_v \frac{\mathbf{u}}{c} \right) + \frac{1}{3} \mathcal{A}_v \nabla \cdot \left( \frac{\mathbf{u}}{c} \right) = \nabla \cdot \left( \frac{1}{3 k_v} \nabla \mathcal{A}_v \right) + k_v \left( \frac{B_v}{c} - \mathcal{A}_v \right)
\]

for the spectral radiation energy density \( \mathcal{A}_v \); the radiation diffusion equation is coupled to the fluid energy equation via the energy exchange term

\[
Q_r = \int \left( k_v \left( c \mathcal{A}_v - B_v \right) \right) dv .
\]

In this treatment, information about the angular dependence of the radiation field is lost (e.g. the CHIC code, P.-H. Maire, J. Breil), and the 2D hydrodynamics becomes a 3D radiation hydrodynamics.

Whereas finite-difference treatment of the multi-frequency diffusion is similar to that of thermal conduction, solution of the transport equation requires a completely different numerical scheme.
To solve numerically the equations of hydrodynamics in two dimensions, we employ the CAVEAT-2D (LANL, 1990) hydrodynamics package. The CAVEAT numerical scheme has the following properties:

- it uses cell-centered principal variables on a multi-block structured quadrilateral mesh (either in the x-y or r-z geometry);
- is fully conservative and belongs to the class of second-order Godunov schemes;
- the mesh is adapted to the hydrodynamic flow by applying the ALE (arbitrary Lagrangian-Eulerian) technique;
- the numerical method is based on a fast non-iterative Riemann solver (J.K.Dukowicz, 1985), easily applied to arbitrary equation of state.
Numerical scheme for thermal conduction

The key ingredient to the RALEF-2D code is the SSI (symmetric semi-implicit) method of E.Livne & A.Glasner (1985), used to incorporate thermal conduction and radiation transport into the 2D Godunov method.

The numerical scheme for thermal conduction (M.Basko, J.Maruhn & A.Tauschwitz, J.Com.Phys., 228, 2175, 2009) has the following features:

- it uses cell-centered temperatures from the FVD (finite volume discretization) hydrodynamics on distorted quadrilateral grids,
- is fully conservative (based on intercellular fluxes with an SSI energy correction for the next time step),
- (almost) unconditionally stable,
- space second-order accurate on all grids for smooth $\kappa$,
- symmetric on a local 9-point stencil,
- computationally efficient.
Numerical scheme for radiation transport

Our numerical scheme for radiation transport (not published yet) has the following basic properties:

- radiation coupling to the fluid is combined with thermal conduction within the unified SSI approach;
- the angular dependence is treated by applying the classical $S_n$ method with $n(n+2)$ fixed photon propagation directions over the $4\pi$ solid angle;
- the scheme is non-conservative in the sense that energy deposition by radiation $Q_r$ is calculated not via elementary fluxes,
- the algorithm has the important property of correct transition to the diffusion limit when mesh cells become optically thick.

Presently, we do not see any practical alternative to the SSI method to cope with numerical instabilities in cases where radiation transport has a strong dynamic impact on hydrodynamics.
The method of short characteristics

For each angular direction $\tilde{\Omega}_L$ and frequency $\nu$, the radiation field $I$ is found by solving the transfer equation

$$\tilde{\Omega}_L \cdot \nabla I = k_\nu (B_\nu - I), \quad I = I(t, \bar{x}, \nu, \tilde{\Omega}_L),$$

with the method of short characteristics (A. Dedner, P. Vollmöller, JCP, 178, 263, 2002). Mesh nodes are chosen as collocation points for the radiation intensity $I$.

**Advantages:**
- even on strongly distorted meshes, it is guaranteed that light rays pass through each mesh cell;
- the algorithm is generally computationally more efficient than that of long characteristics.

**Disadvantages:**
- a significant amount of numerical diffusion in space.

**Difference with R. Ramis:** in the MULTI-2D code $I$ is assigned to cell edges.

Only Cartesian geometry with straight characteristics has been treated so far.
The method of short characteristics produces a significant amount of numerical diffusion for light beams with sharp edges.

However, for thermal radiation a certain amount of numerical diffusion may be more an advantage than a shortcoming.
Vital importance of the ALE technique

*The problem:* a thin foil is irradiated by an intense laser beam.
Problem 1: A strongly radiating central Z-pinch in tungsten multi-wire arrays
Multi-wire Z-pinches (Sandia, Angara-5)

40-mm diameter array of 240, 7.5-\(\mu\)m-diam. wires.

Z-machine at Sandia (USA):

- 11.5 MJ stored energy
- 19 MA peak load current
- 40 TW electrical power to load
- 100-250 TW x-ray power
- 1-1.8 MJ x-ray energy
Problem statement for the Angara-5 parameters

Cylindrical implosion of an initially cold tungsten plasma cloud

Initial shell parameters:

- radial thickness 2 mm;
- implosion velocity $v_0 = 400$ km/s;
- uniform temperature $T_0 = 20$ eV;
- mass per unit length 0.286 mg/cm;
- initial kinetic energy 23 kJ/cm;
- far from the axis, mass is uniformly distributed over the radius;
- possible influence of magnetic field is neglected.

In its present formulation, the problem is one-dimensional.
Pseudocolor
Var: T

Max: 0.4783
Min: 0.05678

user: Mikhail Basko
Sat Nov 07 12:13:34 2009
$t = 3 \text{ ns}$
Shock structure

Here we deal with a radiation-dominated shock front, which has a supercritical amplitude (see Zel’dovich and Raizer, chapter VII). The shocked material radiates away about 90% of its initial kinetic energy.
Plasma opacities

Plasma opacities for the above simulation have been provided by the group of V.G. Novikov from the Keldysh institute (KIAM) with the THERMOS code.

**Diagram:**

- **W:** $T=0.25$ keV, $\rho=0.01$ g/cc

Legend:
- Original THERMOS data
- Group-averaged (8 groups)
- Group-averaged (32 groups)
- Group-averaged (200 groups)
The imploding tungsten plasma radiates away about 90% of its initial kinetic energy.

The nominal power of implosion is

\[ W_{\text{nom}} = \frac{1}{2} MU_{\text{in}}^2 / t_{\text{pulse}} = 23 \text{ kJ cm}^{-1} / 5 \text{ ns} = 4.6 \text{ TW cm}^{-1} \]
High-resolution emission spectra can be obtained in the post-processor regime even when hydrodynamics is simulated with a relatively small number (~10) of spectral groups.
Optical thickness of the imploding plasma

Over a wide range of frequencies, the shock front lies at an optical depth of $\tau = 0.5$–2.0

⇒ the multi-frequency diffusion is not an adequate approach for such situations!
Calculated X-ray images of the central pinch

Spectral X-ray images provide information on the internal structure of the imploding pinch.

$W: t=3 \text{ ns}, 32 \nu\text{-groups, S}_{14}, \text{mesh } 40\times200$

- $h\nu = 0.172 \text{ keV}$
- $h\nu = 2.00 \text{ keV}$

$x_s (\text{mm})$

$\frac{\Delta \nu}{\nu}$

$x\times1000$
Problem 2: Hole boring in a copper foil by an intense nanosecond laser pulse
Copper foil under PHELIX (GSI) irradiation

**Cu foil:** \( 4 \text{g/cc} \times 2 \mu \text{m} = 0.8 \text{ mg/cm}^2, T_0 = 0.58 \text{ eV} \)

**PHELIX beam (prepulse):** 2-3 ns with \( 2 \times 10^{13} \text{ W/cm}^2 \)

**Temporal pulse profile**

- Laser intensity: \( 2 \times 10^{13} \text{ W/cm}^2 \)
- Time: \( 0 \text{ to } 2.5 \text{ ns} \)

**Spatial pulse profile**

- \[ 1 + \left( \frac{y}{r_{\text{foc}}} \right)^2 \]^{-1}
Simulation setup

Initial conditions: \( \rho_0 = 4.0 \text{ g/cc}, \ \Delta x = 2 \mu\text{m}, \ \rho_0 = 3 \text{ kbar}, \ T_0 = 0.58 \text{ eV} \)

Boundary conditions: \( p_b = 3 \text{ kbar} \)

EOS: Novikov et al.

Opacity: Novikov et al.
9 freq. groups

Laser deposition: transport of one beam with the inverse bremsstrahlung absorption coefficient

Mesh: \( n_x \times n_y = 150 \times 300 \)

Spectral opacity of Cu
Temperature
Temperature

Max: 0.3419
Min: 0.0005477
Temperature

Max: 0.3367
Min: 0.0005312

user: Mikhail Basko
Wed Nov 25 14:52:50 2009
Temperature

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Wed Nov 25 14:53:01 2009
Time dependence of mass column density \( \int \rho \, dx \) along the central axis at \( y = 0 \).

The effect of hole boring: the evacuated mass exceeds the depth of the critical surface by as large a factor as 20–30.
Conclusions

The status of the RALEF-2D code:

- the first results are encouraging,
- the work on further development continues.