RECENT TRENDS IN THE HIGH-DENSITY EOS

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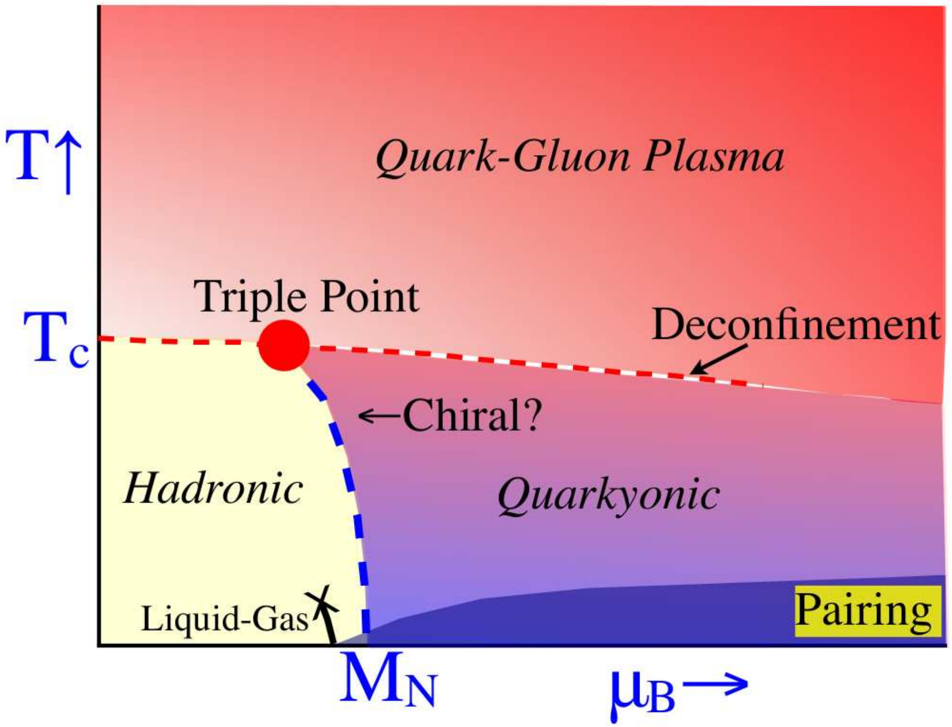
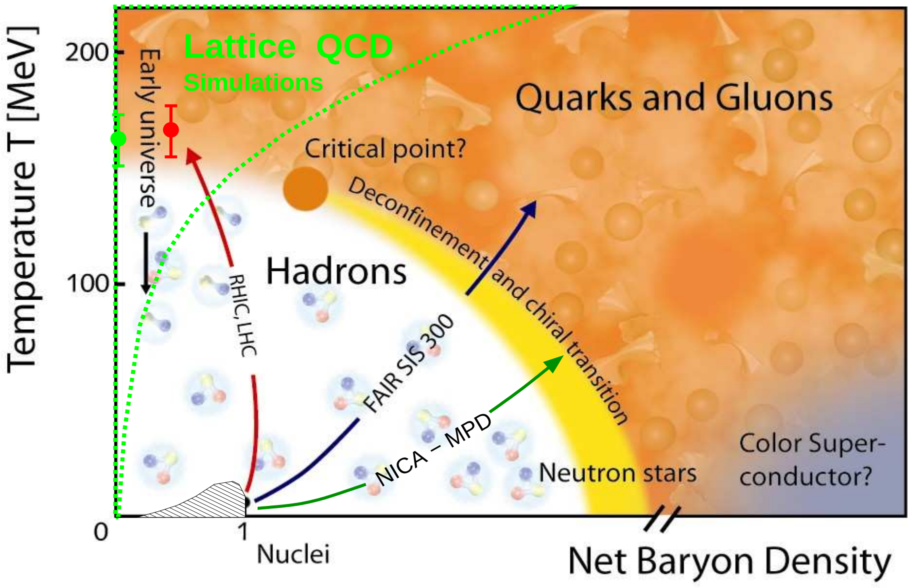
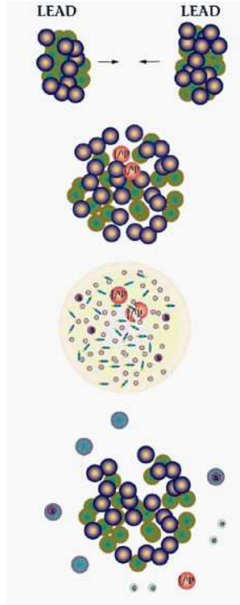


- Current QCD Phase Diagram:
  - Experiment: Chem. Freezeout
  - Theory: Quarkyonic Phase
- (Nonlocal) PNJL Model
  - Beyond MF: Mesons ($\bar{q}q$)
  - Baryons: $q - (qq)$ Loop Expansion
- HIC, Supernovae & Compact Stars:

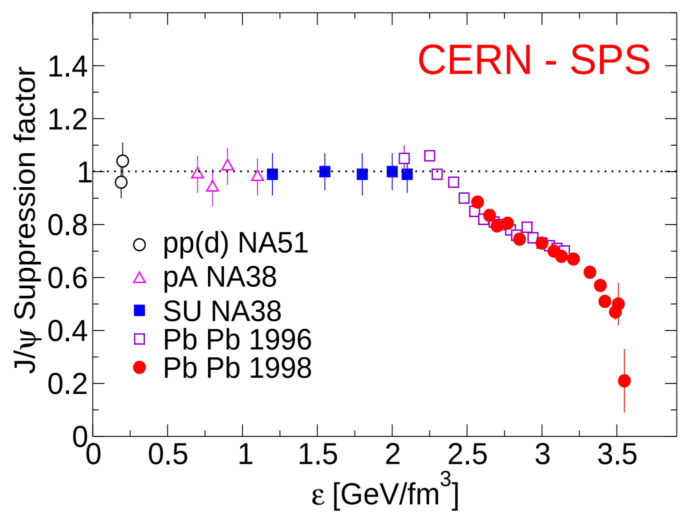
Andronic, D.B., Braun-Munzinger, Cleymans, Fukushima, Oeschler, Pisarski, McLerran, Redlich, Sasaki, Satz, Stachel, **arxiv:0911.4806 [hep-ph]**

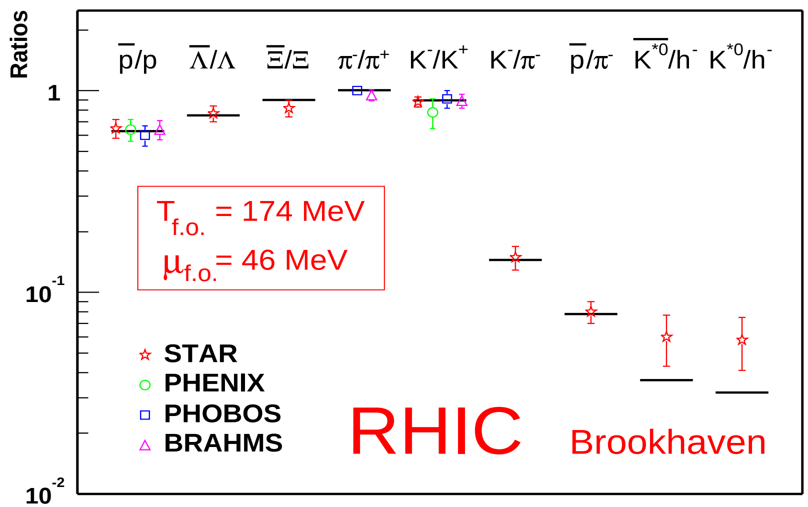
Phasediagram: Freeze-out in Heavy-Ion Collisions





QGP Signal: Anomalous J/ψ suppression





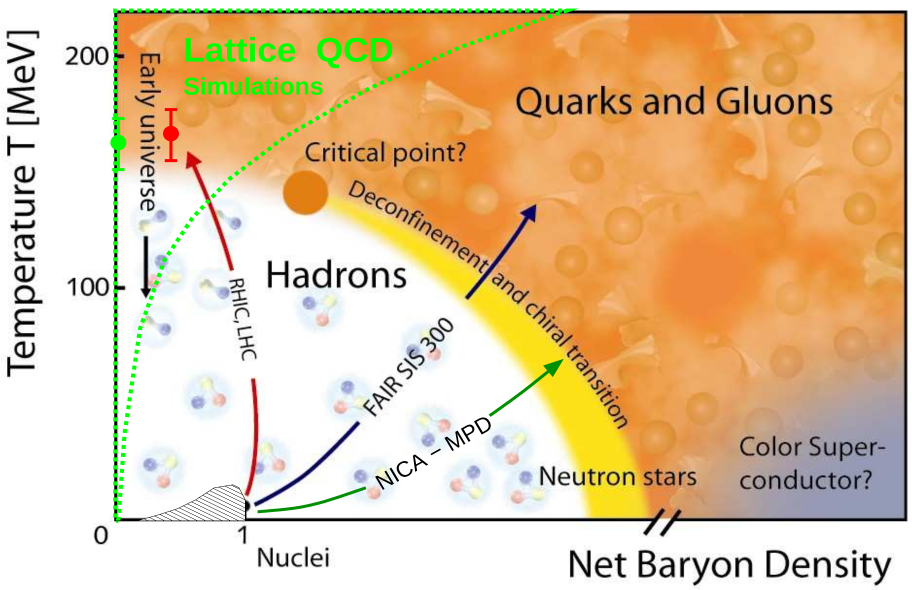
Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

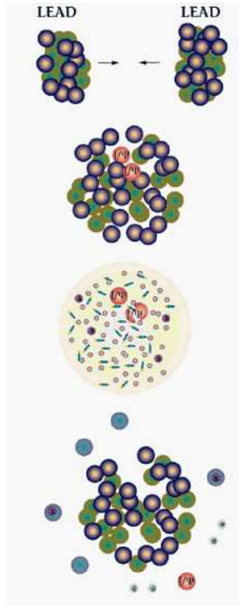
$$\ln Z[T,V,\{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty dp\, p^2 \ln[1 \pm \lambda_i \exp(-\beta\varepsilon_i(p))]$$

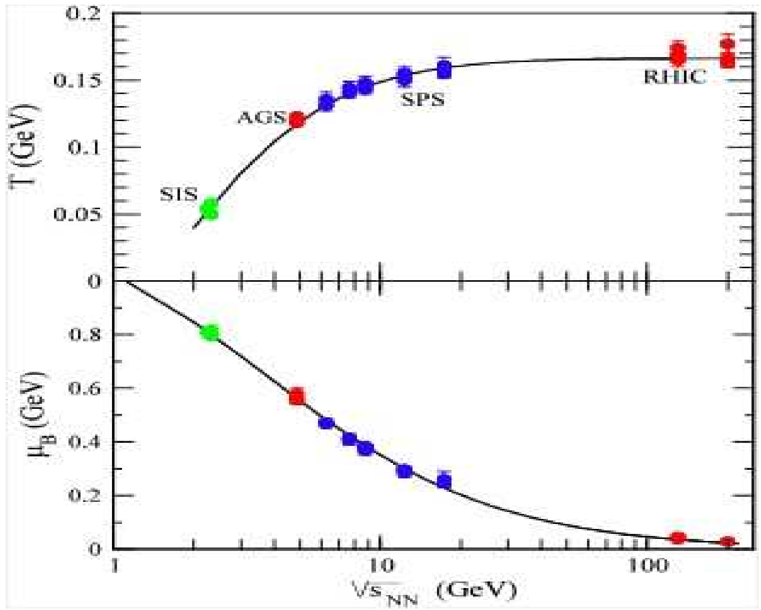
$$\lambda_i(T,\{\mu\}) = \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

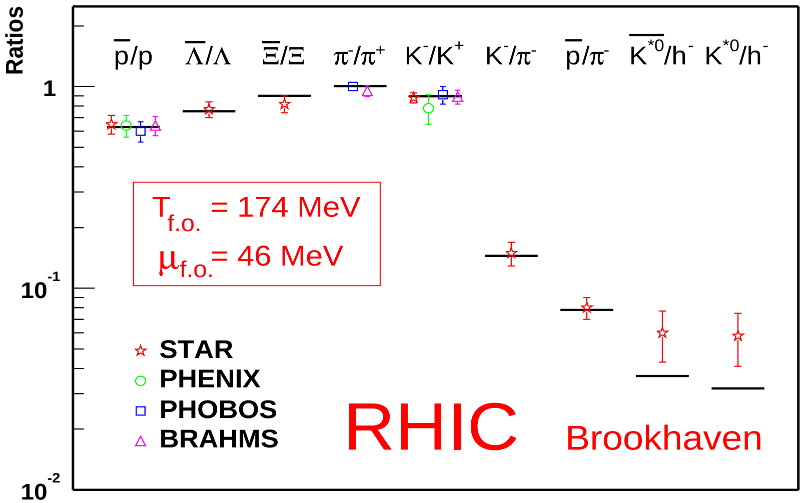
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (II)









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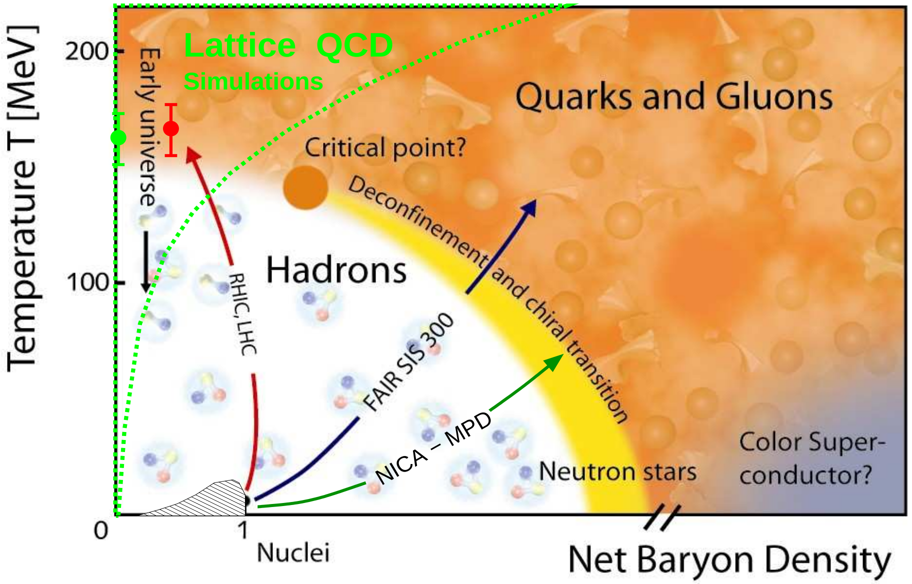
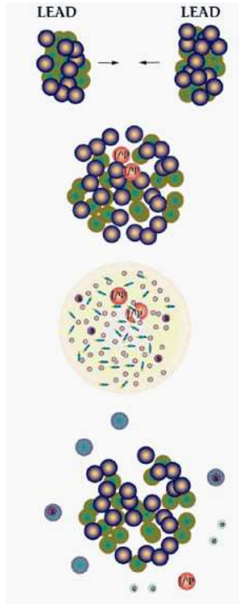
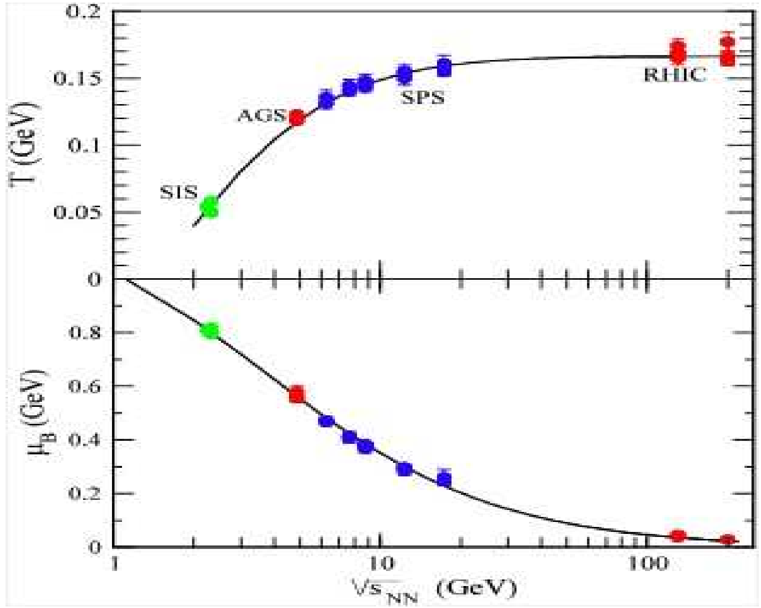
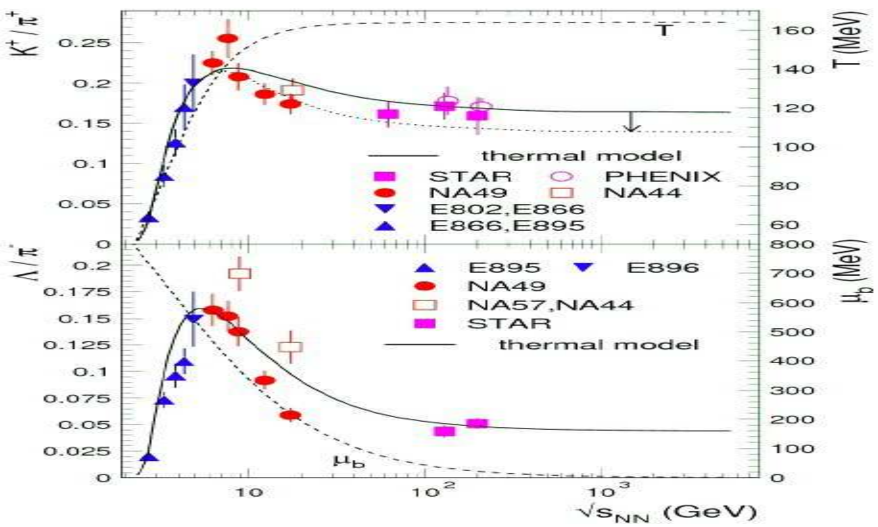
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (III)



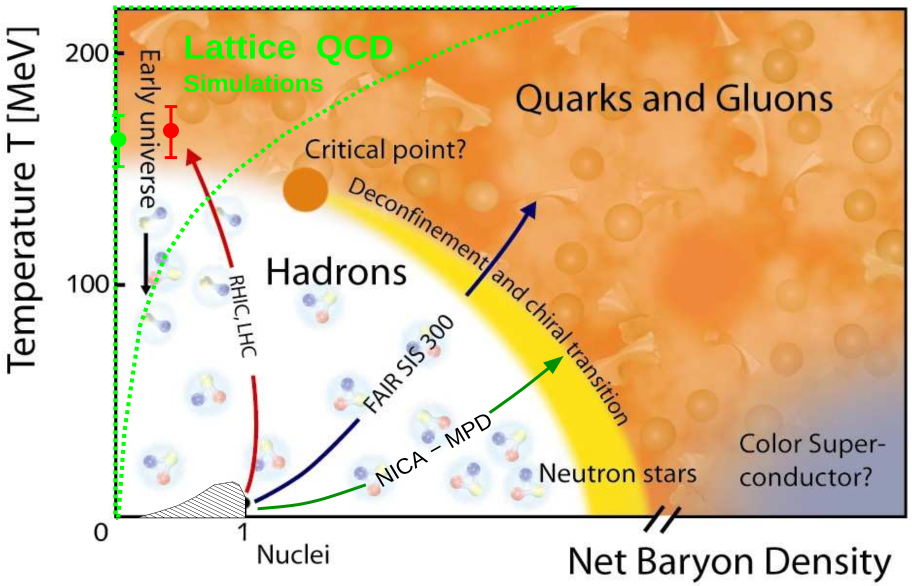


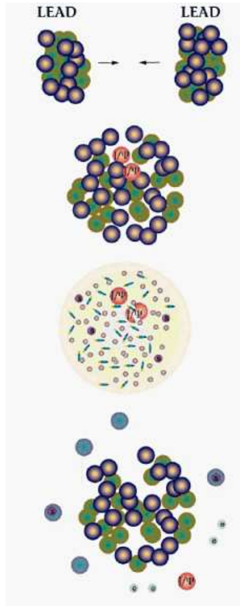


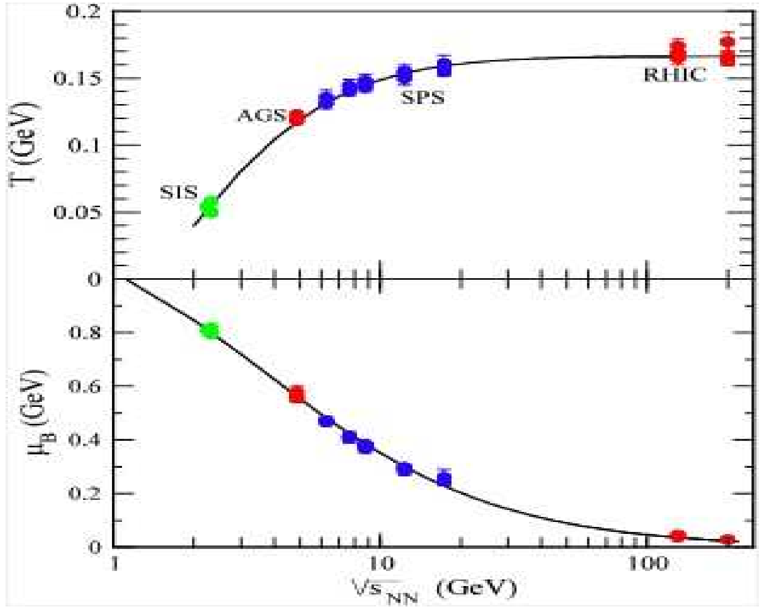


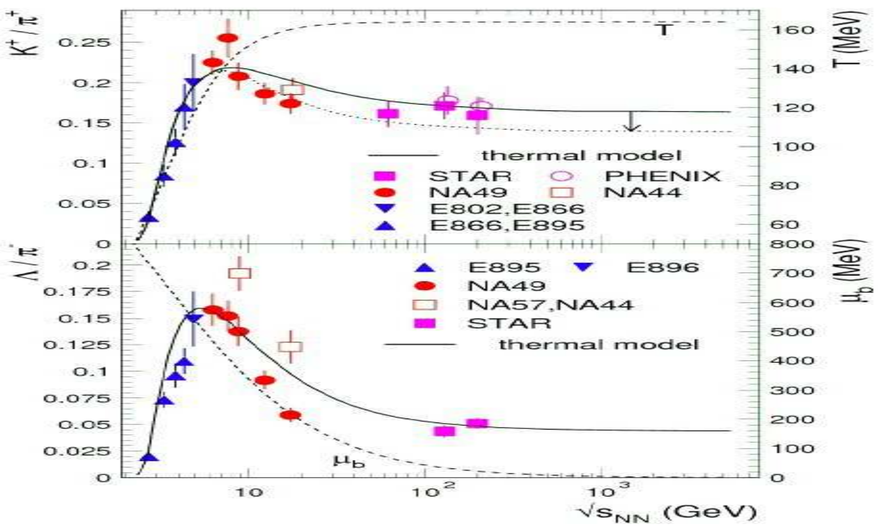
Strange MatterHorn (Pisarski)

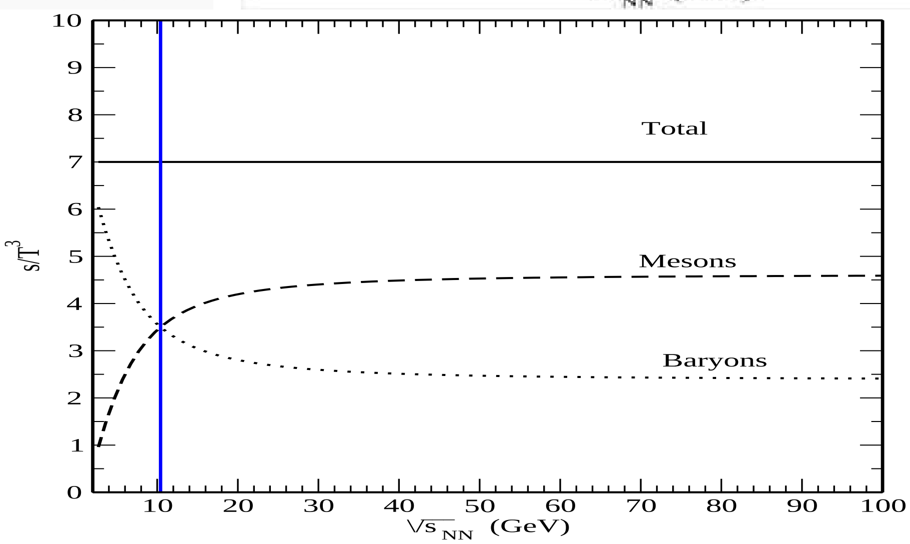
PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (III)





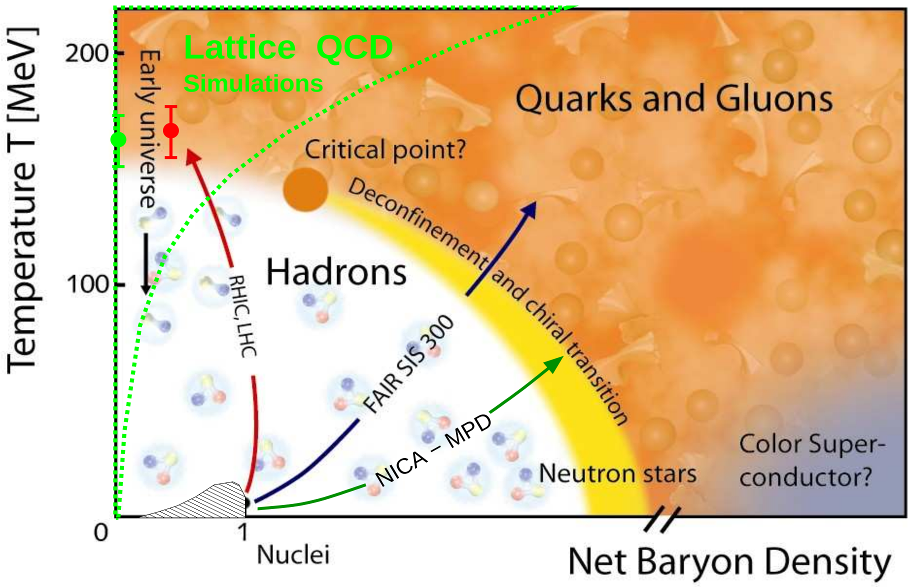


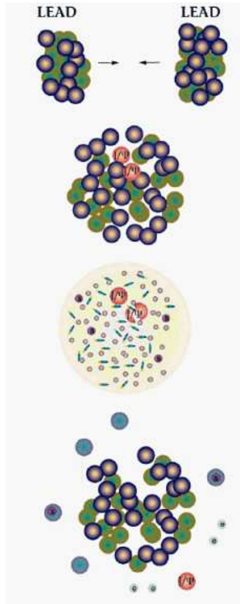


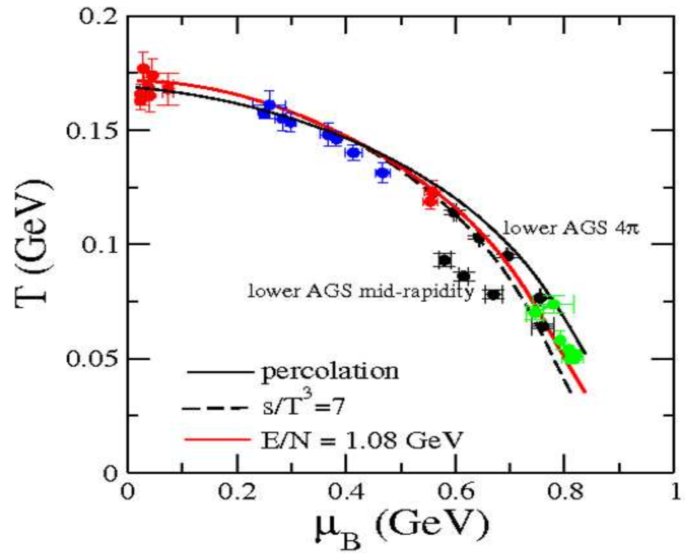


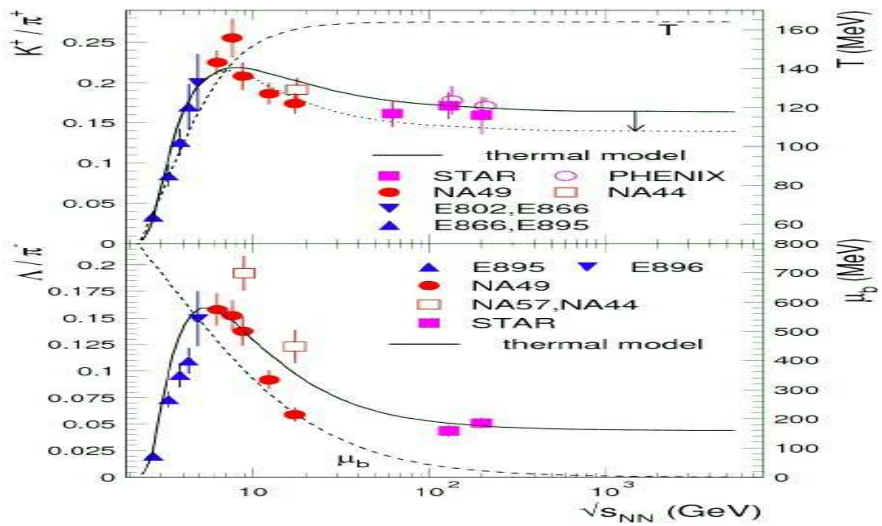
Baryon → Meson Dominance

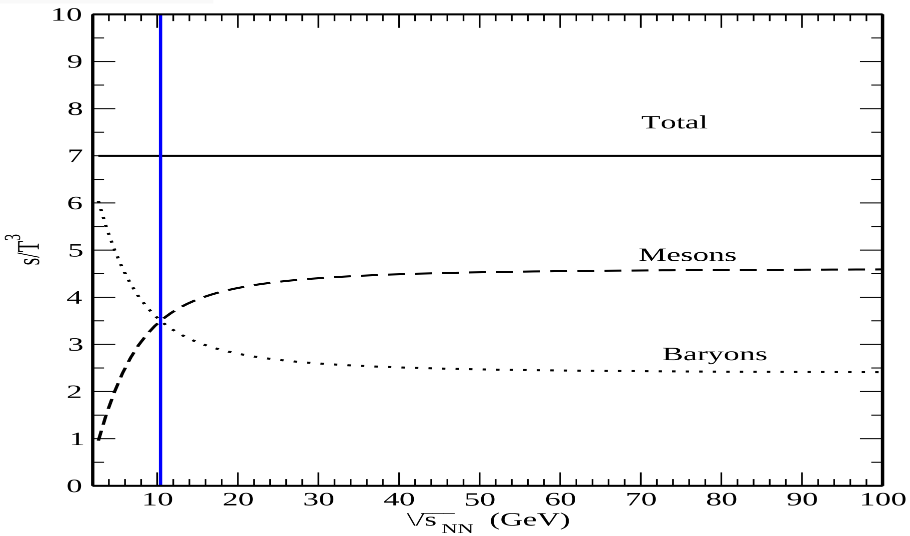
PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (IV)





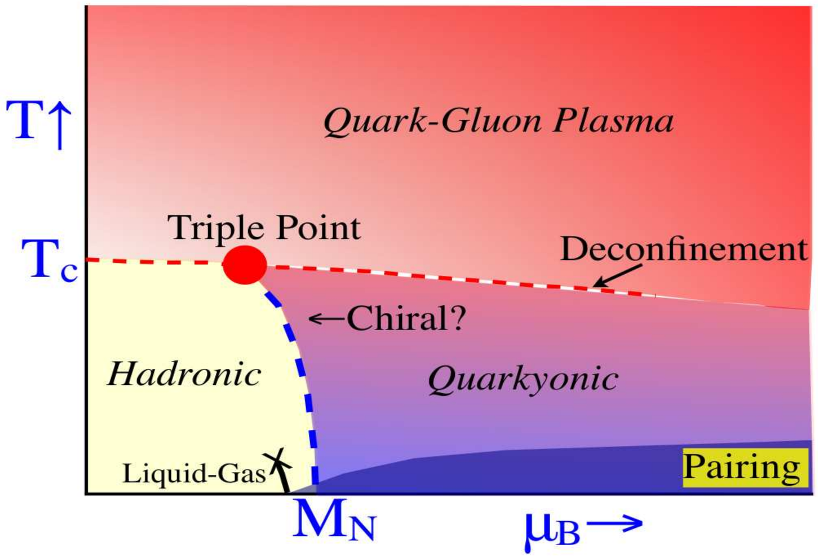


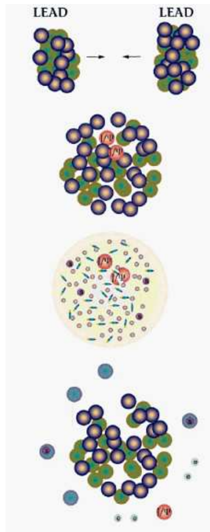


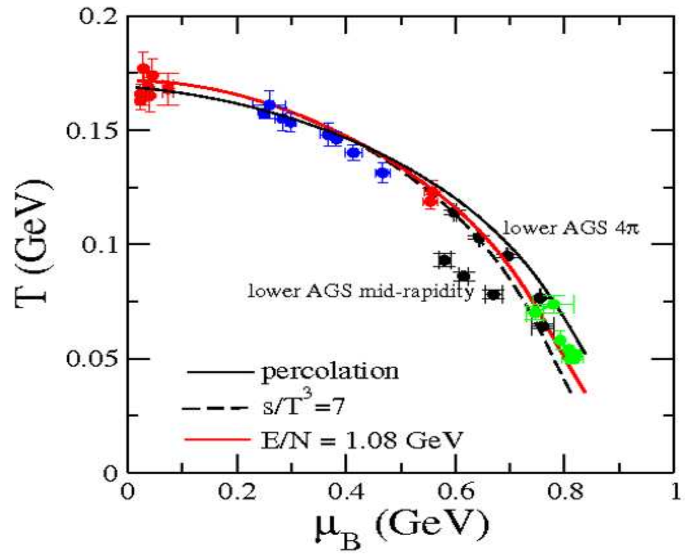


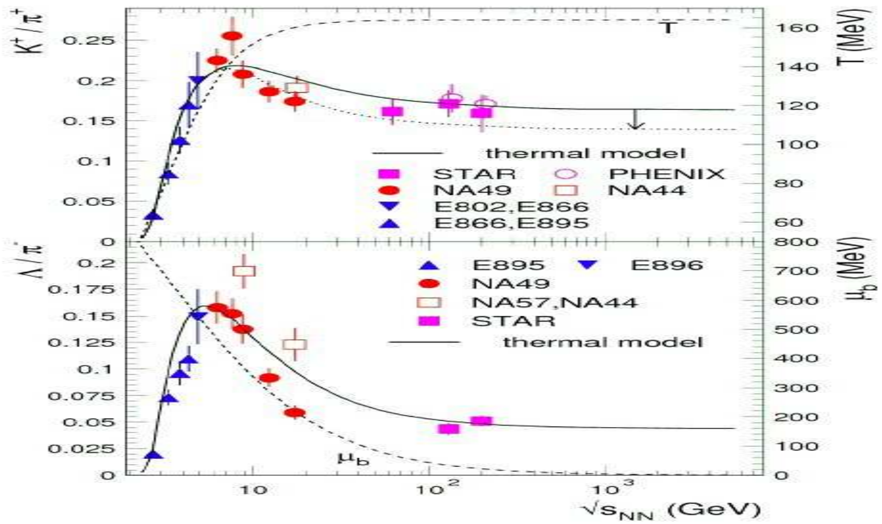
Baryon → Meson Dominance

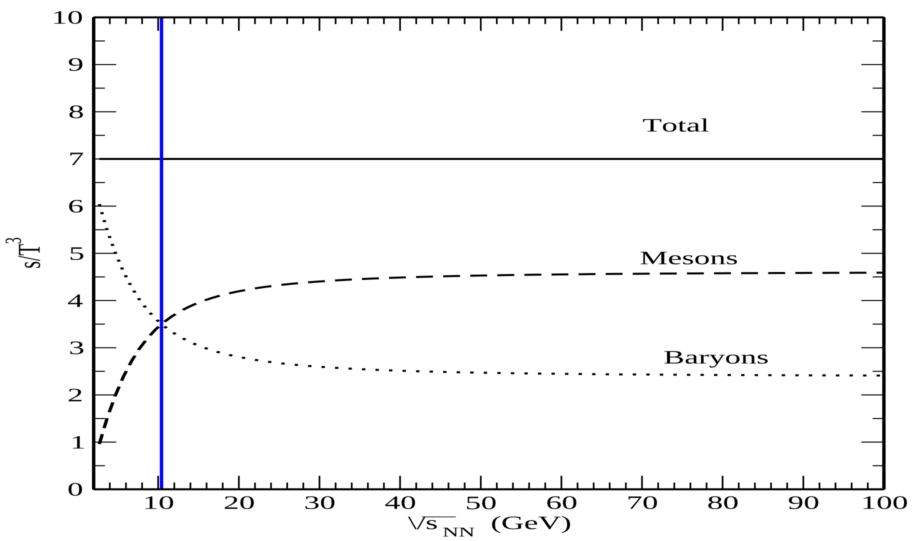
Phasediagram: Freeze-out in Heavy-Ion Collisions (V)





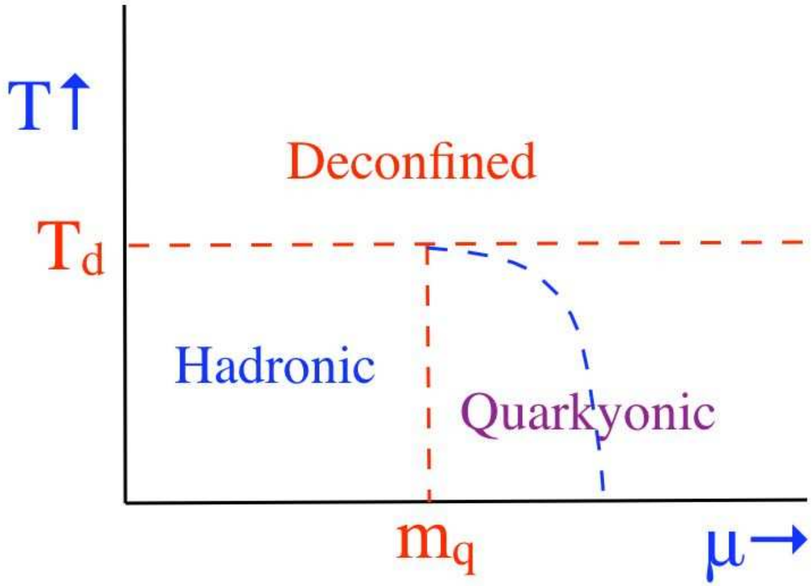




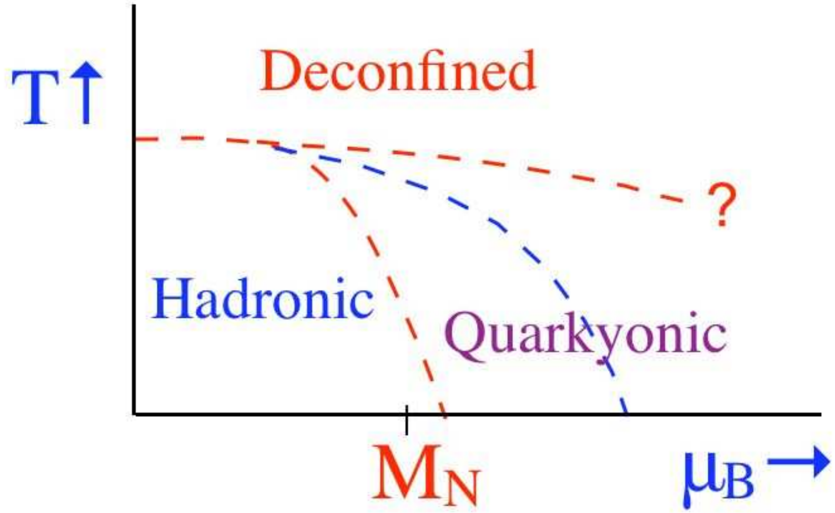


Andronic, D.B., et al., arxiv:0911.4806 [hep-ph]

QUARKYONIC PHASE = CHIRAL SYMMETRY + CONFINEMENT



Phase diagram for $N_c \to \infty$ and finite N_f



Phase diagram for $N_c \to \infty$ and small N_f/N_c

Hidaka, McLerran, Pisarski, Nucl. Phys. A 808 (2008) 117.
McLerran, Pisarski, Nucl. Phys. A 796 (2007) 83.
McLerran, Redlich, Sasaki, arXiv:0812.3585

PNJL BEYOND MF: PION $(q\bar{q})$ AND NUCLEON (qqq) MEDIUM

Idea: melting $\langle\bar{q}q\rangle$ → swelling hadrons → flavor kinetics = quark percolation → freeze-out

$$\langle\bar{q}q\rangle(T,\mu) = \frac{\partial}{\partial m_0}\Omega(T,\mu)\ , \quad \Omega(T,\mu) = \Omega_{\rm PNJL,MF}(T,\mu) + \Omega_{\rm meson}(T,\mu) + \Omega_{\rm baryon}(T,\mu)$$

$$\Omega_{\rm meson}(T,\mu) = \sum_{M=\pi,\dots} d_M \int\frac{d\omega}{\pi}\int\frac{d^3k}{(2\pi)^3}\left\{\frac{\omega}{2} + T\ \ln\left[1-e^{-\beta\omega}\right]\right\} A_M(\omega,k)\ ,$$

$$\Omega_{\rm baryon}(T,\mu) = -\sum_{B=N,\dots} d_B \int\frac{d\omega}{\pi}\int\frac{d^3k}{(2\pi)^3}\left\{\frac{\omega}{2} + T\ \ln\left[1+e^{-\beta(\omega-\mu_B)}\right] + (\mu_B\leftrightarrow-\mu_B)\right\} A_B(\omega,k)\ ,$$

$$A_M(\omega,k) = \pi\delta(\omega - E_M(k)) + \text{continuum}\ , \quad A_B(\omega,k)\dots\text{analoguous}$$

Remove vacuum terms; neglect continuum (for the freeze-out);
use GMOR: $M_\pi^2\ f_\pi^2 = -m_0\langle\bar{q}q\rangle$ and $\sigma_N = m_0(\partial m_N/\partial m_0) = 45$ MeV,
Enforce $M_\pi(T,\mu) = \text{const}$ by setting $f_\pi^2(T,\mu) = -m_0\langle\bar{q}q\rangle(T,\mu)/M_\pi^2$,

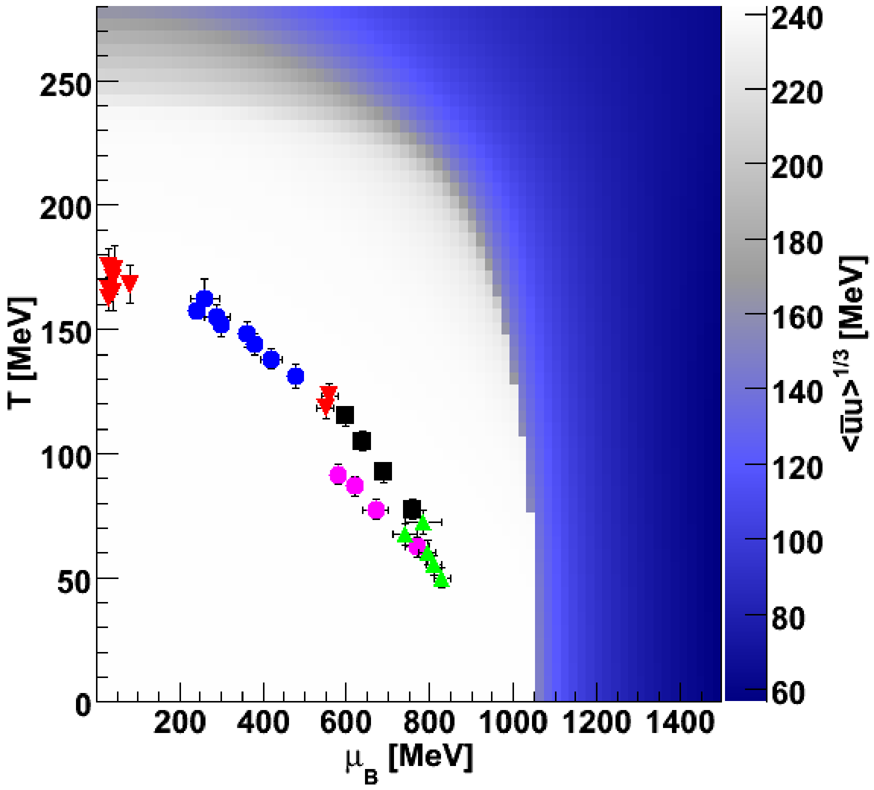
$$-\langle\bar{q}q\rangle(T,\mu) = -\langle\bar{q}q\rangle_{\rm PNJL,MF}(T,\mu) + \frac{M_\pi^2 T^2}{8m_0} + \frac{\sigma_N}{m_0}n_{s,N}(T,\mu)$$

with the scalar nucleon density $n_{s,N}(T,\mu) = \frac{2}{\pi^2}\int_0^\infty dp\,p^2\left\{f_N(T,\mu)+f_N(T,-\mu)\right\}$

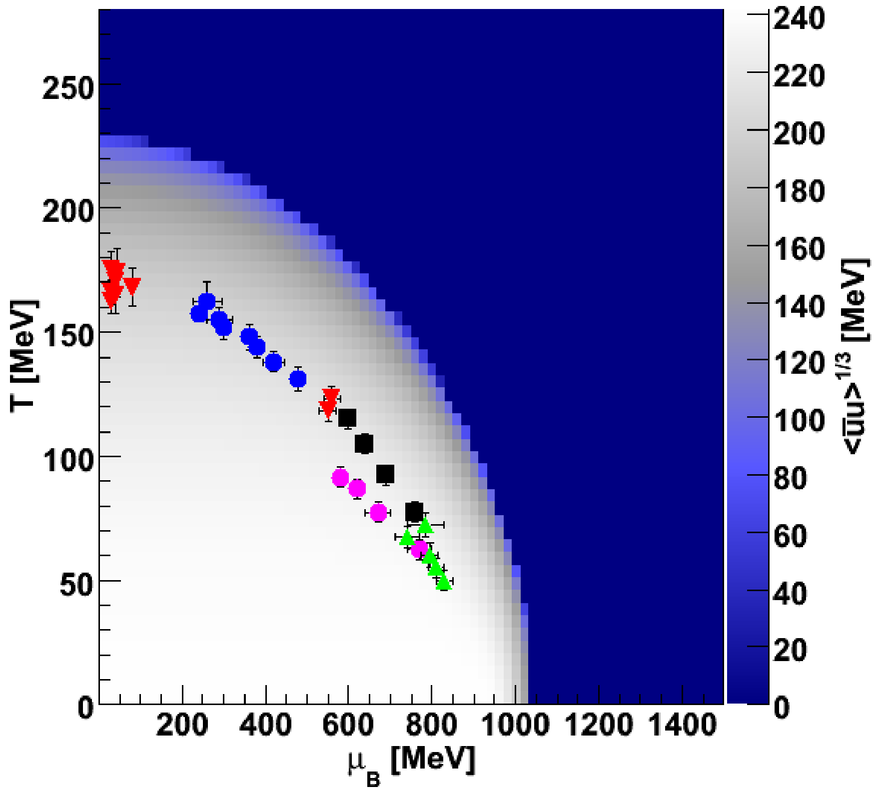
J. Berdermann, D.B., J. Cleymans, K. Redlich, in progress (2009)

PNJL Model beyond MF - results

$$-\langle\bar{q}q\rangle = -\langle\bar{q}q\rangle_{\mathrm{PNJL,MF}}$$



$$-\langle\bar{q}q\rangle = -\langle\bar{q}q\rangle_{\mathrm{PNJL,MF}} + \frac{M_\pi^2 T^2}{8m_0} + \frac{\sigma_N}{m_0} n_{s,N}(T,\mu) + \ldots$$

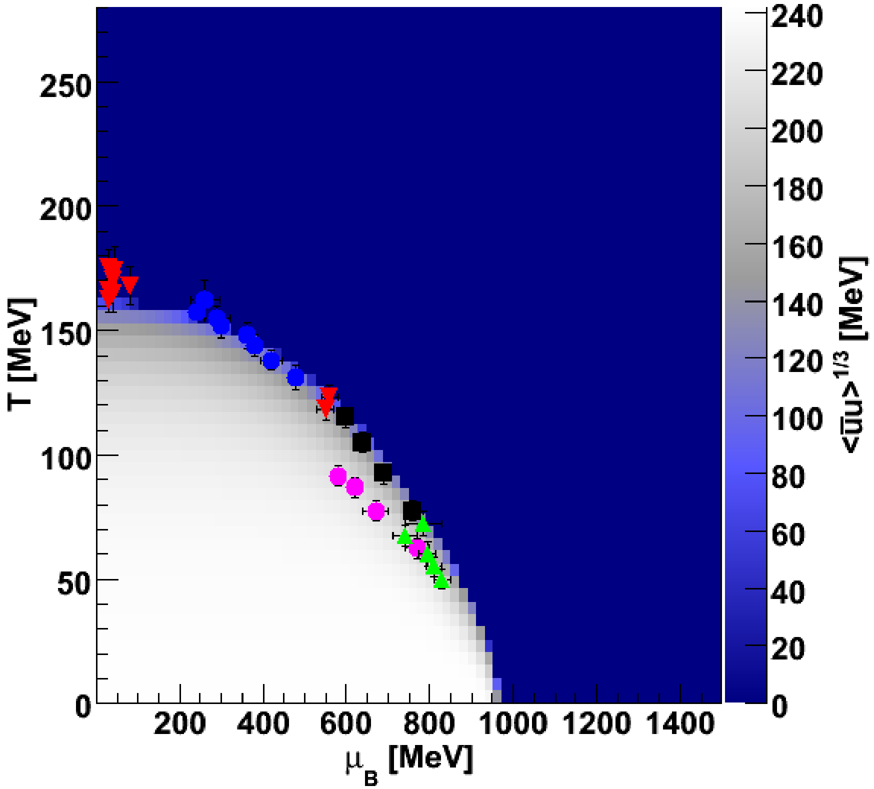


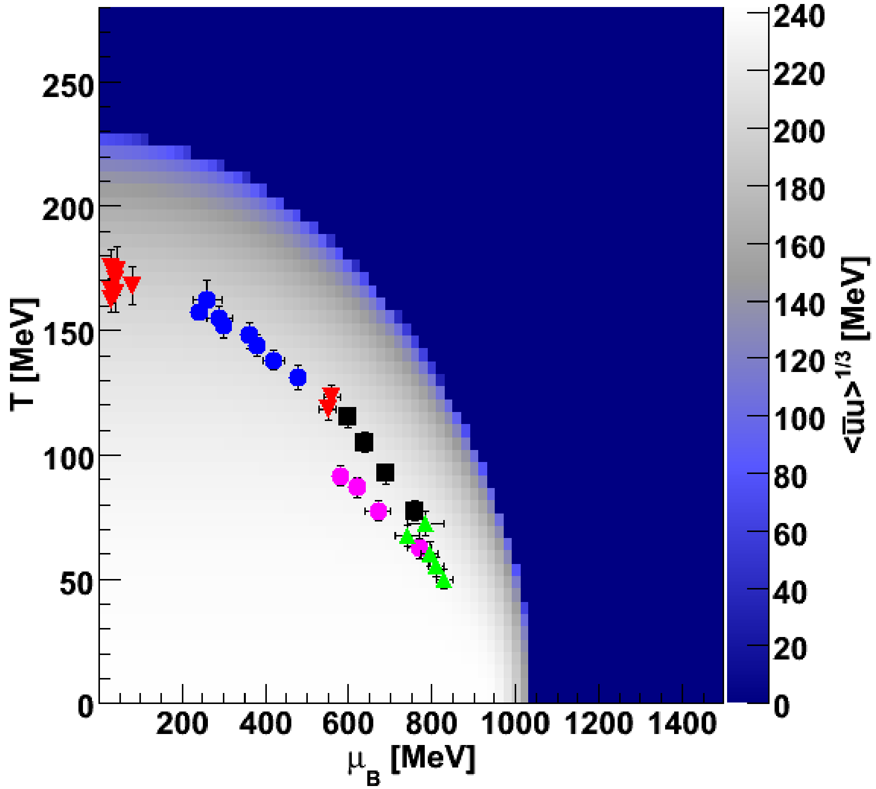
J. Berdermann, D.B., J. Cleymans, K. Redlich, in progress (2009)

PNJL MODEL BEYOND MF - RESULTS

$$-\langle\bar{q}q\rangle = -\langle\bar{q}q\rangle_{\mathrm{PNJL,MF}} + \kappa_M \frac{M_\pi^2 T^2}{8m_0} + \kappa_B \frac{\sigma_N}{m_0} n_{s,N}(T,\mu)$$

$$-\langle\bar{q}q\rangle = -\langle\bar{q}q\rangle_{\mathrm{PNJL,MF}} + \frac{M_\pi^2 T^2}{8m_0} + \frac{\sigma_N}{m_0} n_{s,N}(T,\mu) + \ldots$$





J. Berdermann, D.B., J. Cleymans, K. Redlich, in progress (2009)

CHIRAL MODEL FIELD THEORY FOR QUARK MATTER

- Partition function as a Path Integral (imaginary time $\tau = i\ t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}[i\gamma^\mu\partial_\mu - m - \gamma^0(\mu + \lambda_8\mu_8 + i\lambda_3\phi_3]\psi - \mathcal{L}_{\rm int} + U(\Phi)]\right\}$$

Polyakov loop: $\Phi = N_c^{-1}{\rm Tr}_c[\exp(i\beta\lambda_3\phi_3)]$

- Current-current interaction (4-Fermion coupling)

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

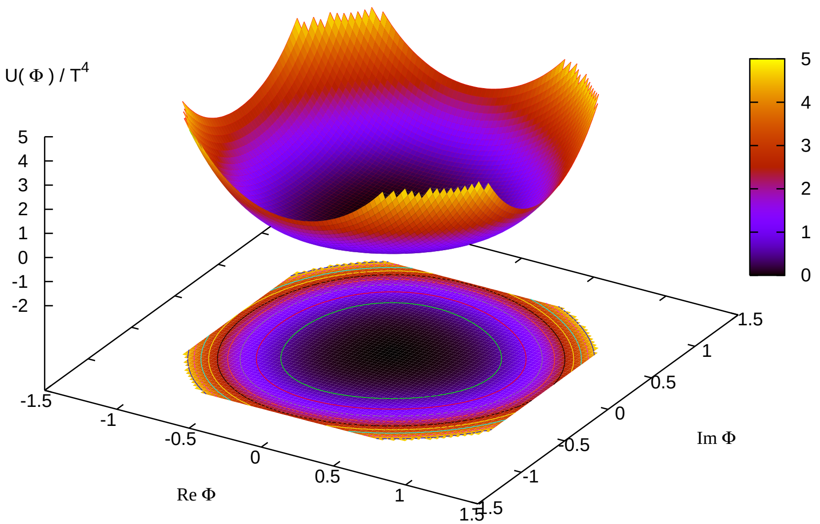
- Bosonization (Hubbard-Stratonovich Transformation)

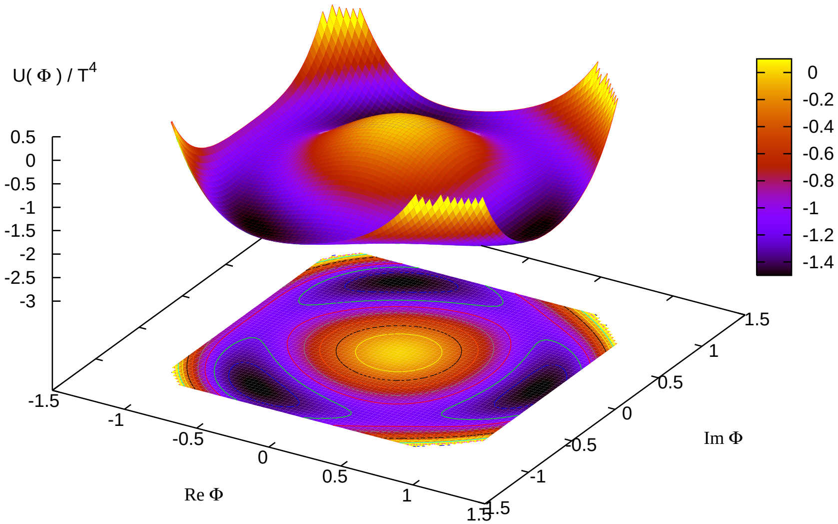
$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_{M,D}\frac{M_M^2}{4G_M} - \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\ \ln S^{-1}[\{M_M\},\{\Delta_D\},\Phi] + U(\Phi)\right\}$$

- Collective quark fields: Mesons (M_M) and Diquarks (Δ_D); Gluon mean field: Φ
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (II)

Temperature dependence of the Polyakov-loop potential $U(\Phi, \bar{\Phi}; T)$



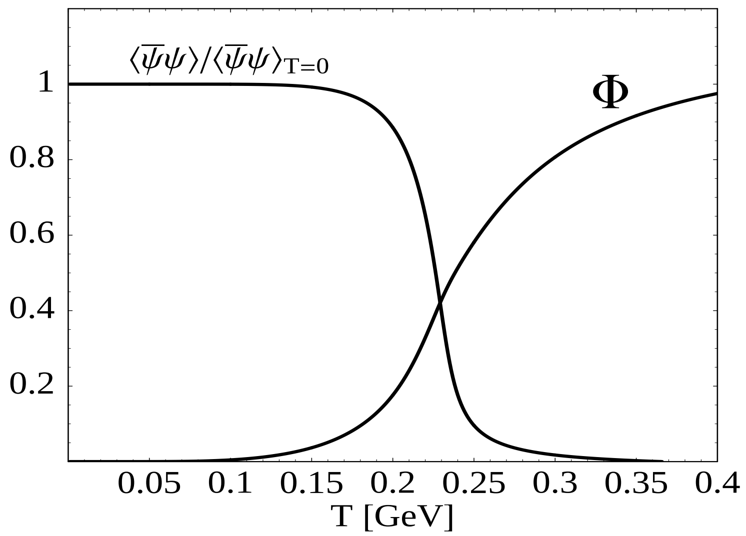


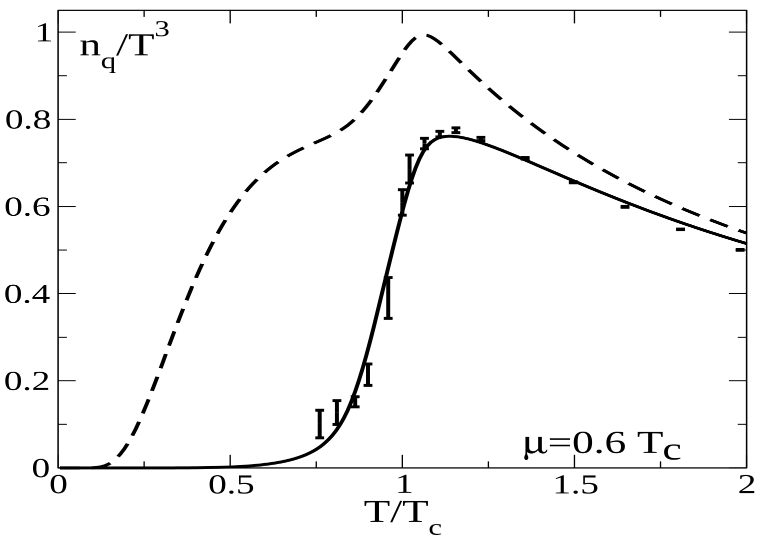
$T = 0.26$ GeV$< T_0$
"Color confinement"

$T = 1.0$ GeV$> T_0$
"Color deconfinement"

Critical temperature for pure gauge $SU_c(3)$ lattice simulations: $T_0 = 270$ MeV.

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (IV)





Grand canonical thermodynamical potential

$$\begin{aligned}\Omega(T,\mu;\Phi,m) &= \frac{\sigma^2}{2G} - 6N_f \int \frac{\mathrm{d}^3p}{(2\pi)^3} E\,\theta\left(\Lambda^2 - \vec{p}^{\,2}\right) \\ &- 2N_f\,T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \Big\{ \mathrm{Tr}_c \ln \left[1 + L\,\mathrm{e}^{-(E-\mu)/T}\right] \\ &+ \mathrm{Tr}_c \ln \left[1 + L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right] \Big\} + \mathcal{U}\left(\Phi,\bar{\Phi},T\right)\end{aligned}$$

Appearance of quarks below T_c largely suppressed:

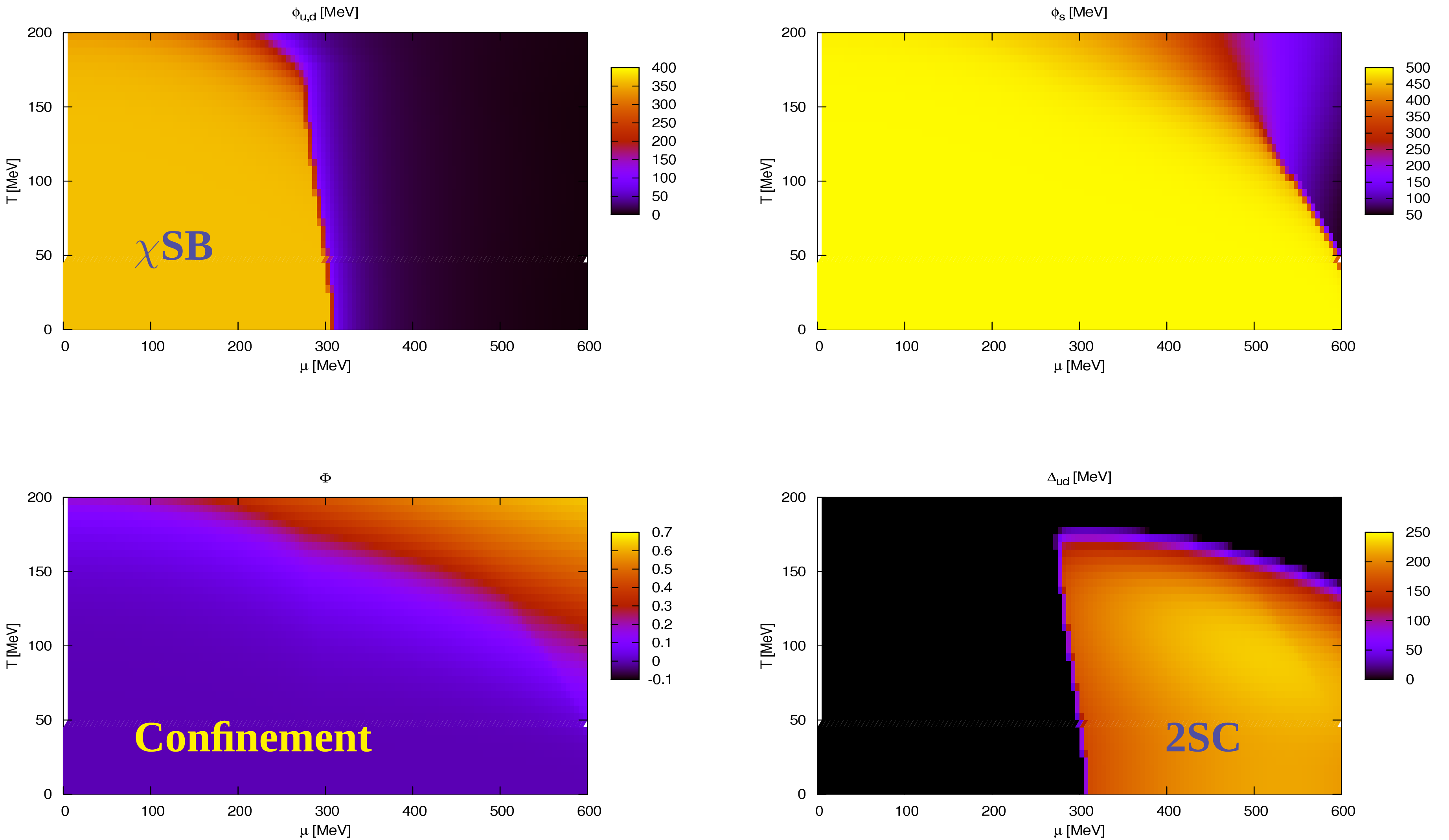
$$\begin{aligned}&\ln\det\left[1 + L\,\mathrm{e}^{-(E-\mu)/T}\right] + \ln\det\left[1 + L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right] \\ &= \ln\left[1 + 3\left(\Phi + \bar{\Phi}\mathrm{e}^{-(E-\mu)/T}\right)\mathrm{e}^{-(E-\mu)/T} + \mathrm{e}^{-3(E-\mu)/T}\right] \\ &+ \ln\left[1 + 3\left(\bar{\Phi} + \Phi\mathrm{e}^{-(E+\mu)/T}\right)\mathrm{e}^{-(E+\mu)/T} + \mathrm{e}^{-3(E+\mu)/T}\right].\end{aligned}$$

Accordance with QCD lattice susceptibilities! Example:

$$\frac{n_q(T,\mu)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega(T,\mu)}{\partial\mu},$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

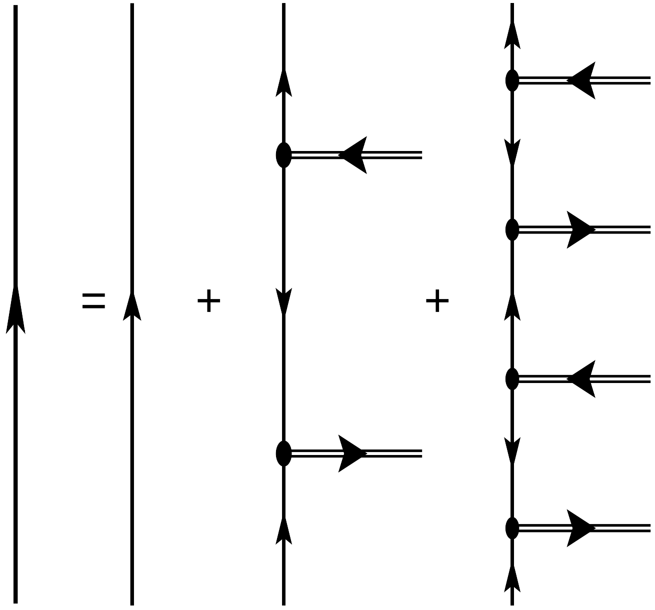
Phases of QCD @ Extremes: no Color Neutrality

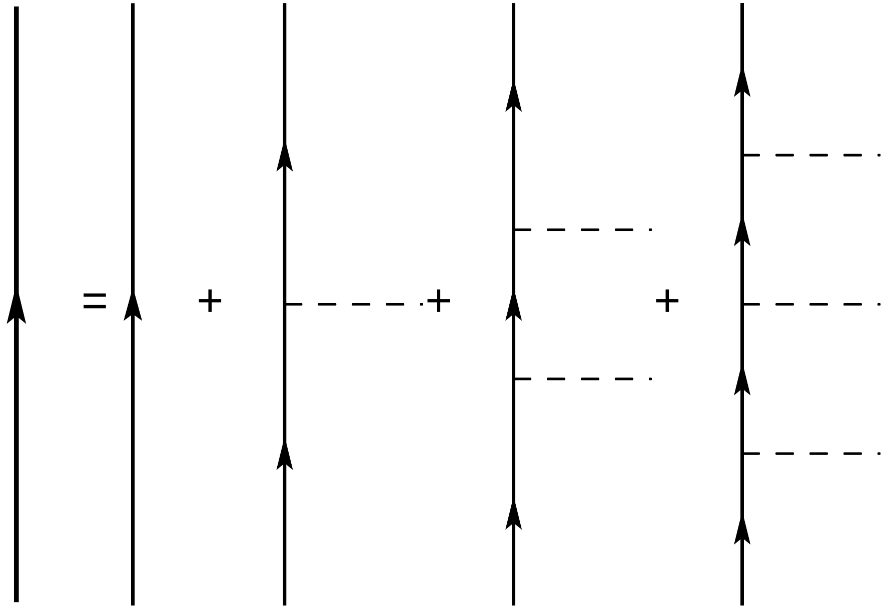


Expansion in mesonic and diquark fluctuations

$$Z_{\rm fluct} = \int D\Delta^\dagger D\Delta D\phi \exp\{-\frac{|\Delta|^2}{4G_D} - \frac{\phi^2}{4G} - Tr\ lnS^{-1}[\Delta, \Delta^\dagger, \phi]\}$$

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29

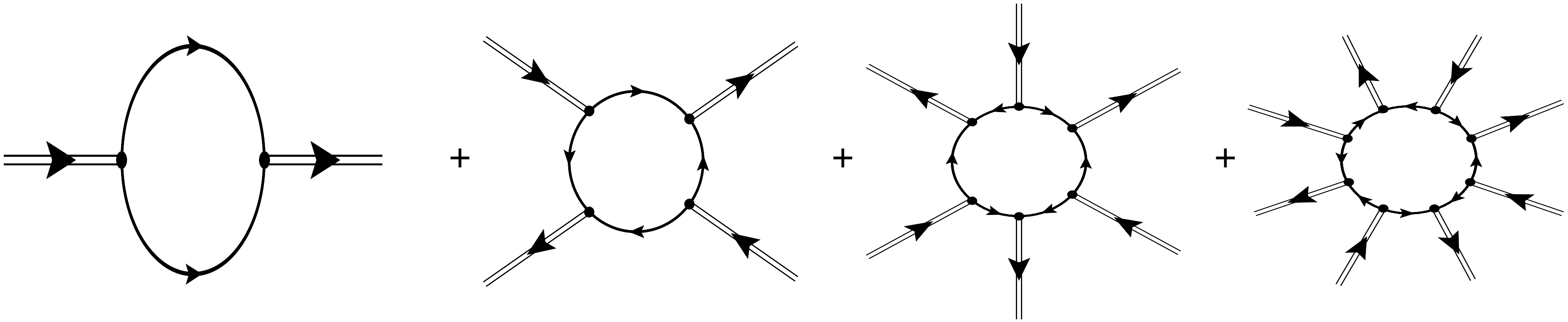


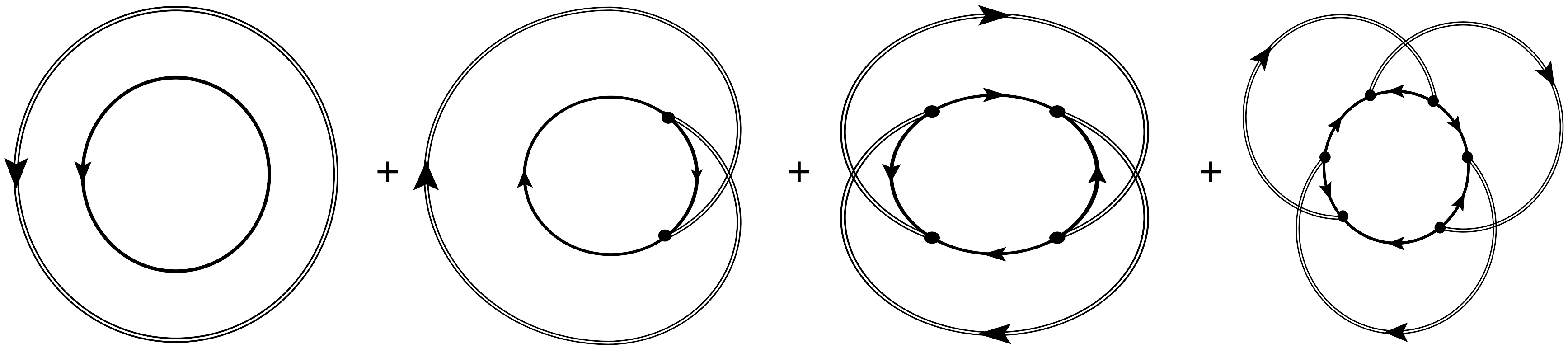


How to perform the path integral over diquark and meson fields?

TRACE OVER QUARK, INTEGRATION OVER DIQUARK FIELDS

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29





Very nice! But where is the nucleon?

Baryon as a partial diagram resummation

$$Z_{\text{fluct}} = \int D\Delta^\dagger D\Delta D\phi \exp\{-\frac{|\Delta|^2}{4G_D} - \frac{\phi^2}{4G} - Tr\; lnS^{-1}[\Delta, \Delta^\dagger, \phi]\}$$

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
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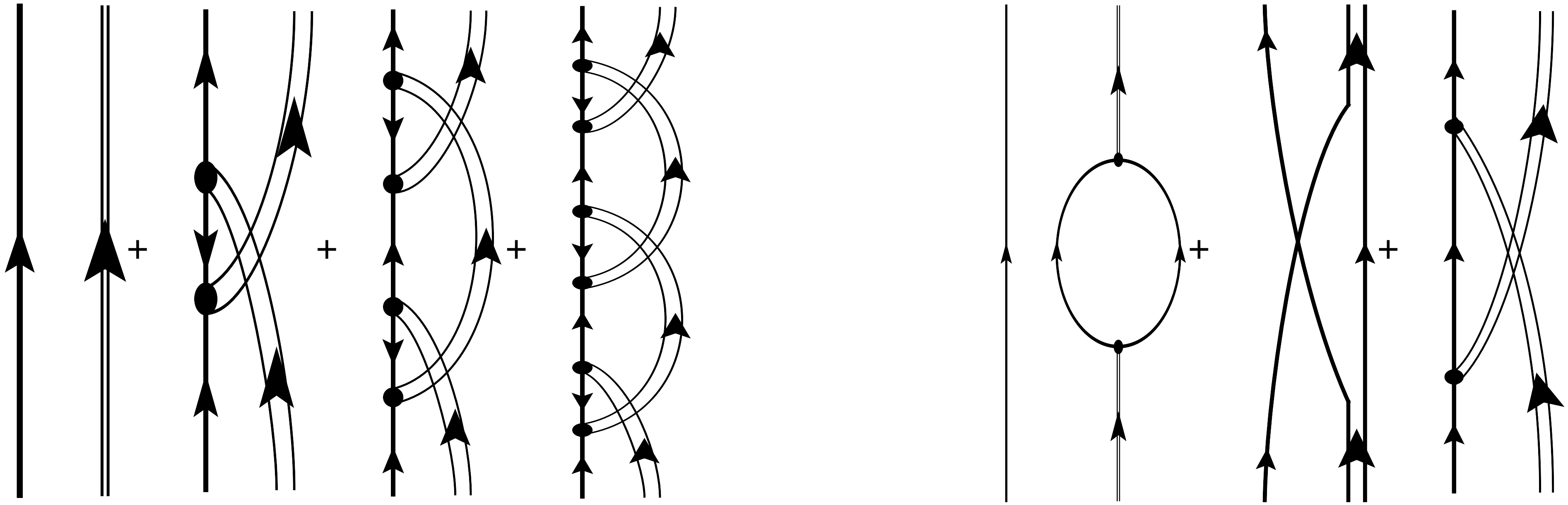
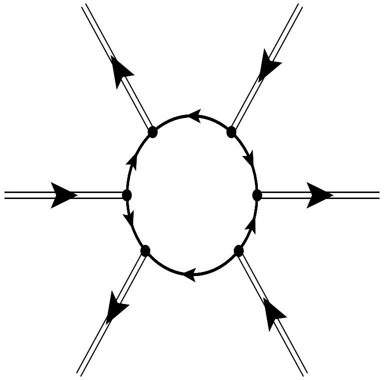
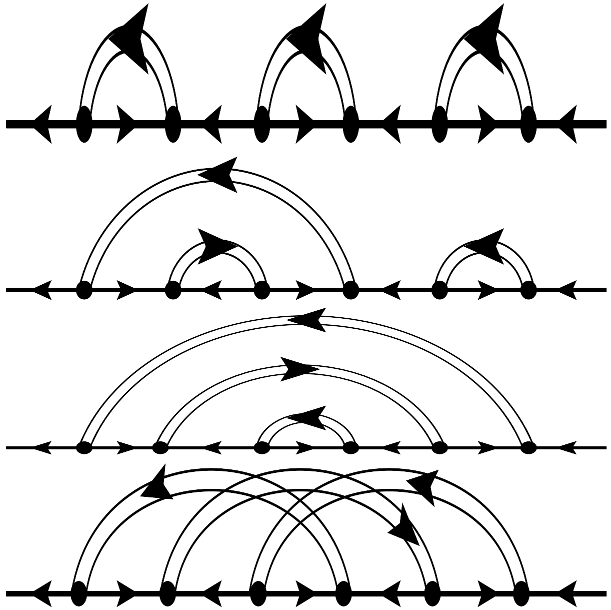
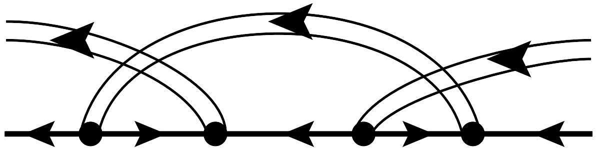


Faddeev equation for quark-diquark states, bound by quark exchange

WHICH DIAGRAMS HAVE BEEN FORGOTTEN ?

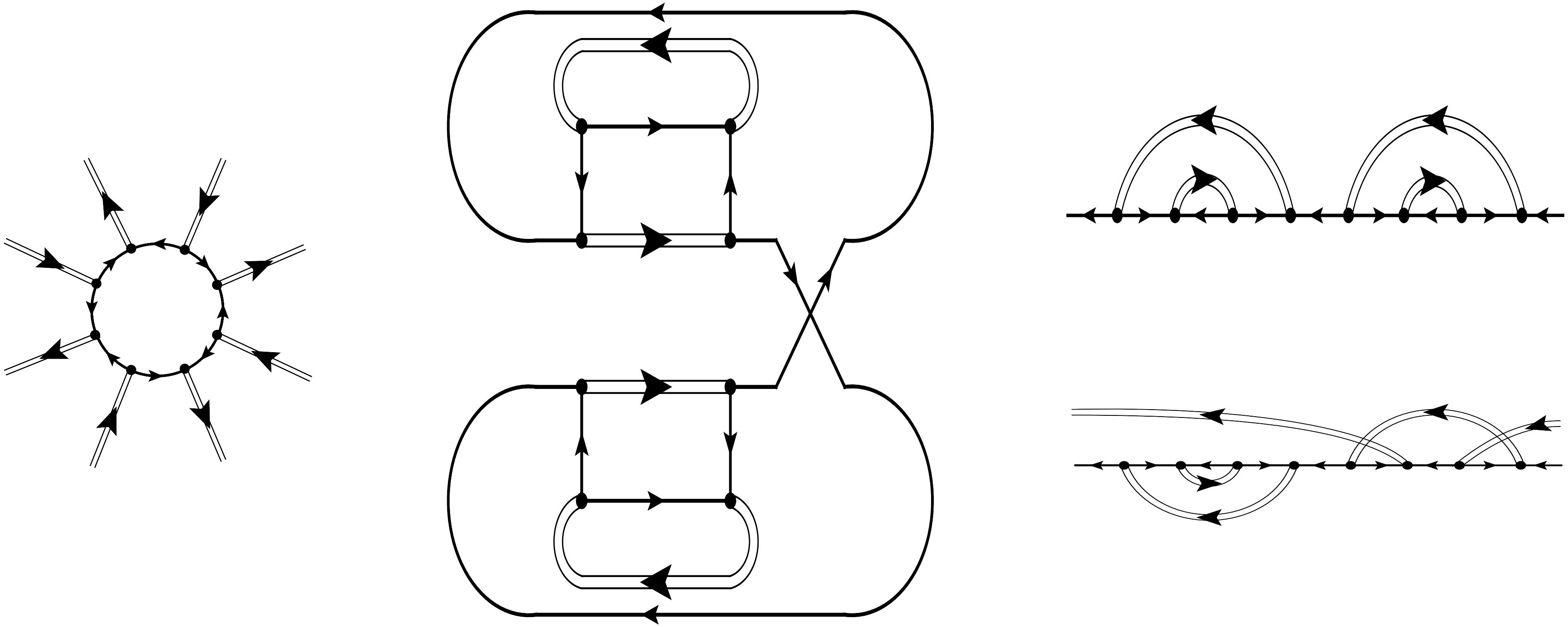






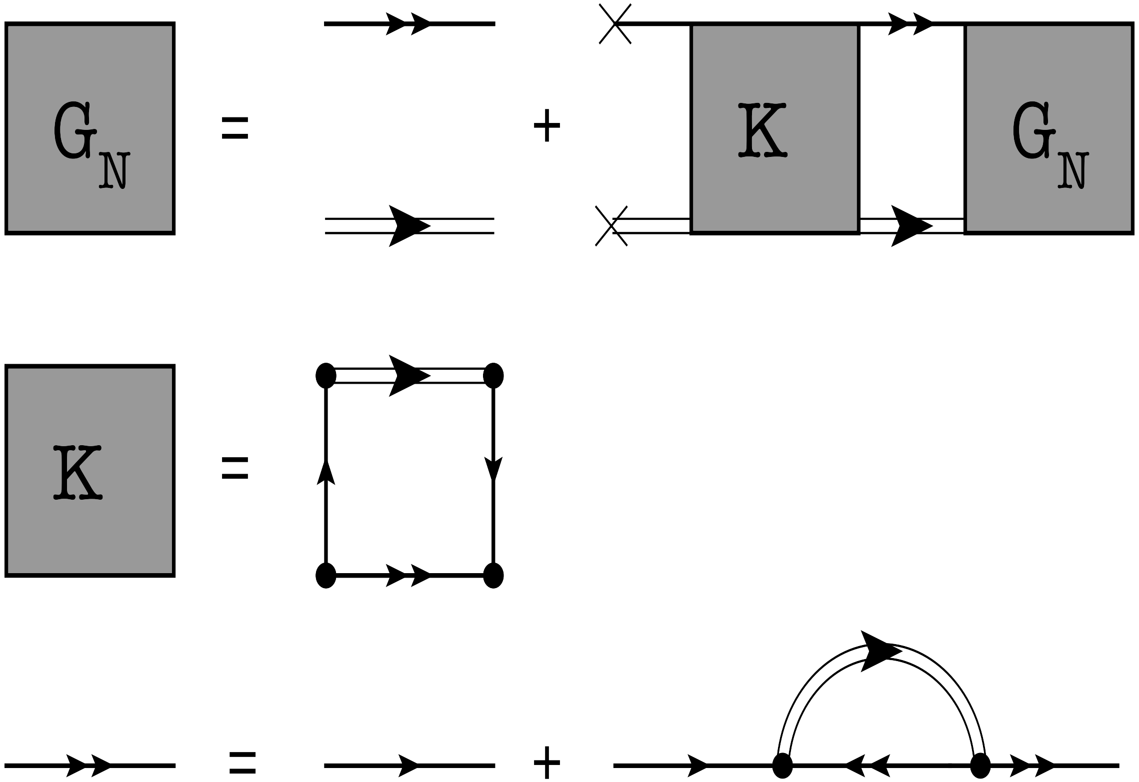
Self-energy type diagrams for the quark propagator

Which diagrams have been forgotten ?

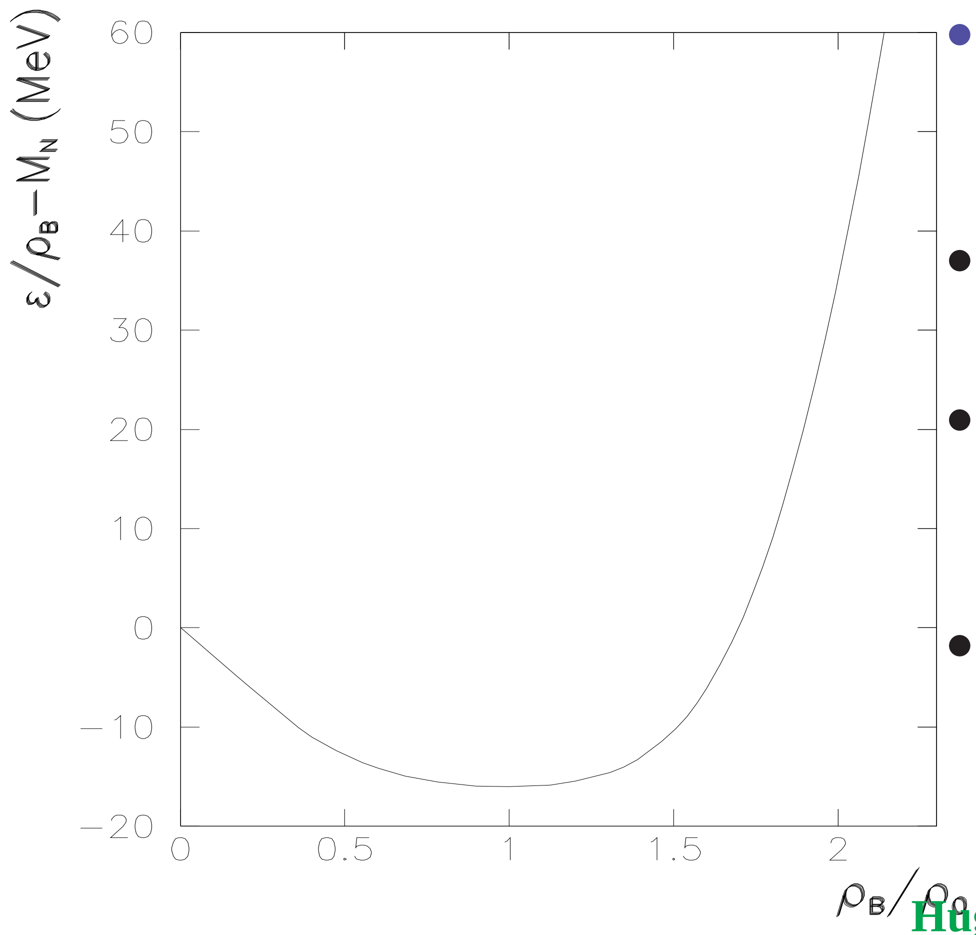


Self-energy diagrams for the quark propagator → quark exchange between nucleons

General. Bethe-Salpeter eq. for quark-diquark states



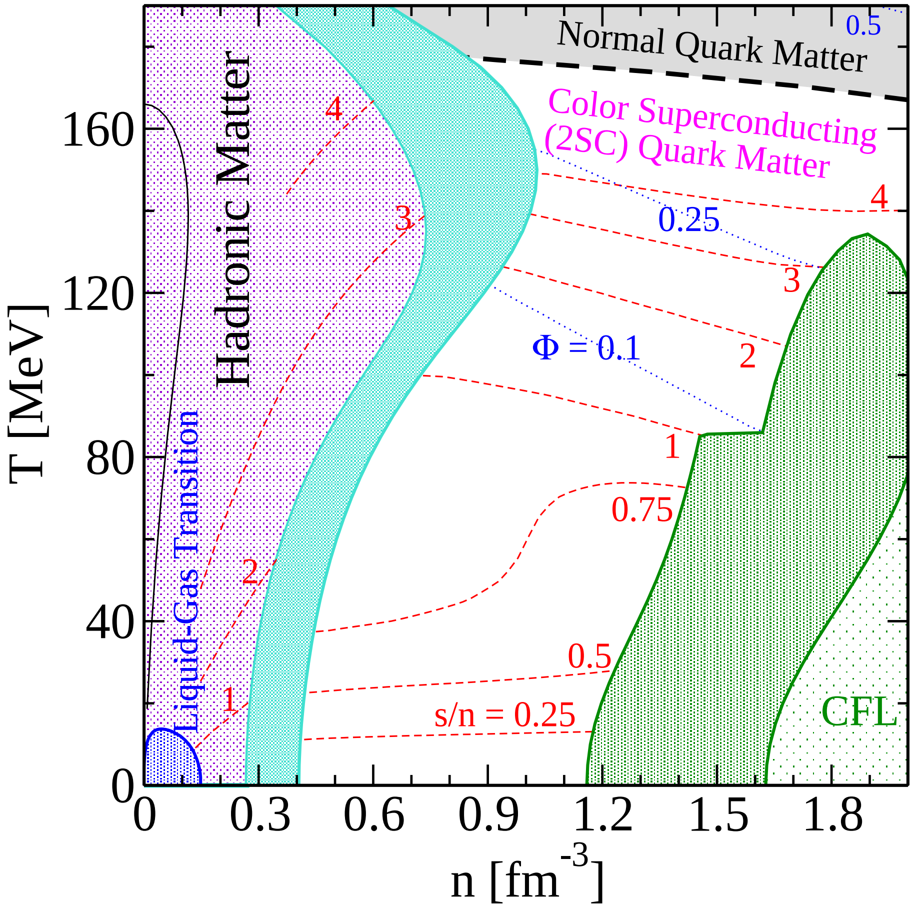
Towards Nuclear Matter from Chiral Quark Models



- RMF with density-dependent meson masses and couplings from NJL* model
- Next steps: solve nucleon EoM at finite μ including chiral transition
- Is Polyakov-loop NJL sufficient for a description of the Quarkyonic Phase?
- Thermodynamics from the QCD-DSE approach ? (Roberts, Klähn, ...)

Huguet, Caillon, Labarsouque, NPA 781 (2007) 448

PHASE DIAGRAM FOR SYMMETRIC MATTER (HIC)

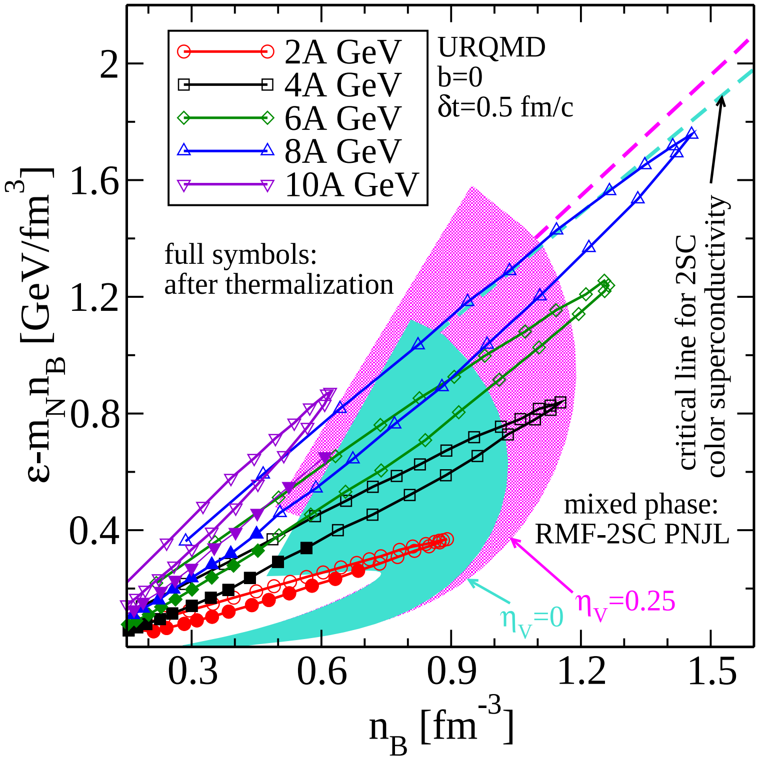


D.B., Sandin, Typel, Klähn, in preparation

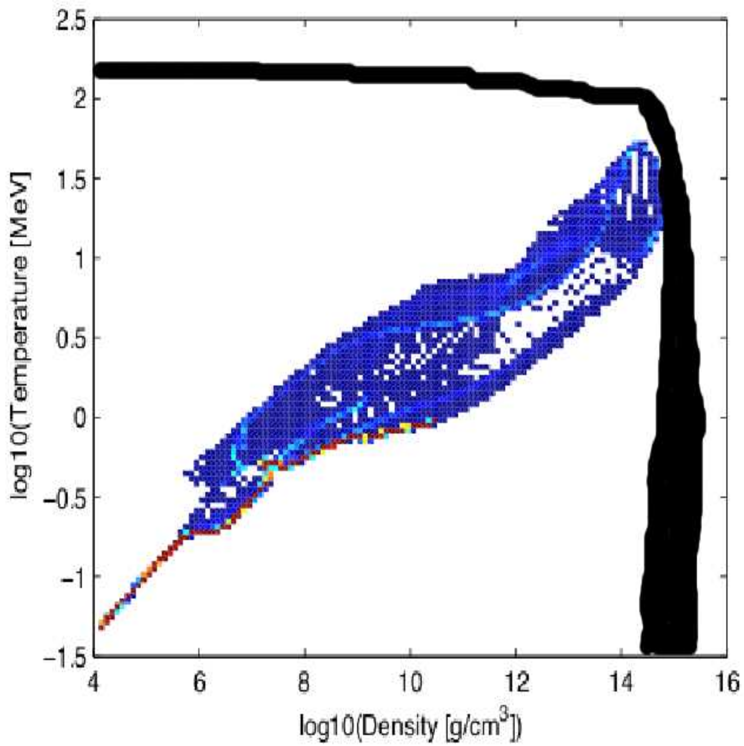
- Critical density for chiral restoration $n_\chi \geq 1.5\, n_0$ increasing (!) with low T
- Almost crossover (masquerade!), i.e. small density jump, small latent heat/ time delay in heavy-ion collision!
- High $T_c \approx 0.9 T_d$ for 2SC phase due to Polyakov loop.
- 2SC - CFL phase transition at $n \geq 6\ n_0$ with density jump and latent heat/ time delay!
  Provided the temperature can be kept low $T \leq 100$ MeV

Exploring the QCD Phase Diagram: Trajectories

Heavy-Ion Collisions:



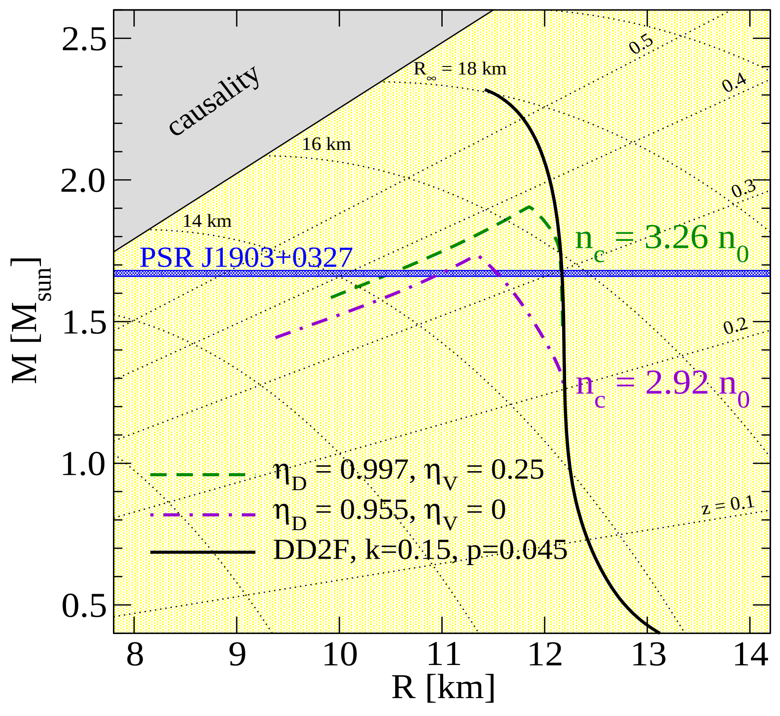
Supernova Explosions (15 $M_{\odot}$):

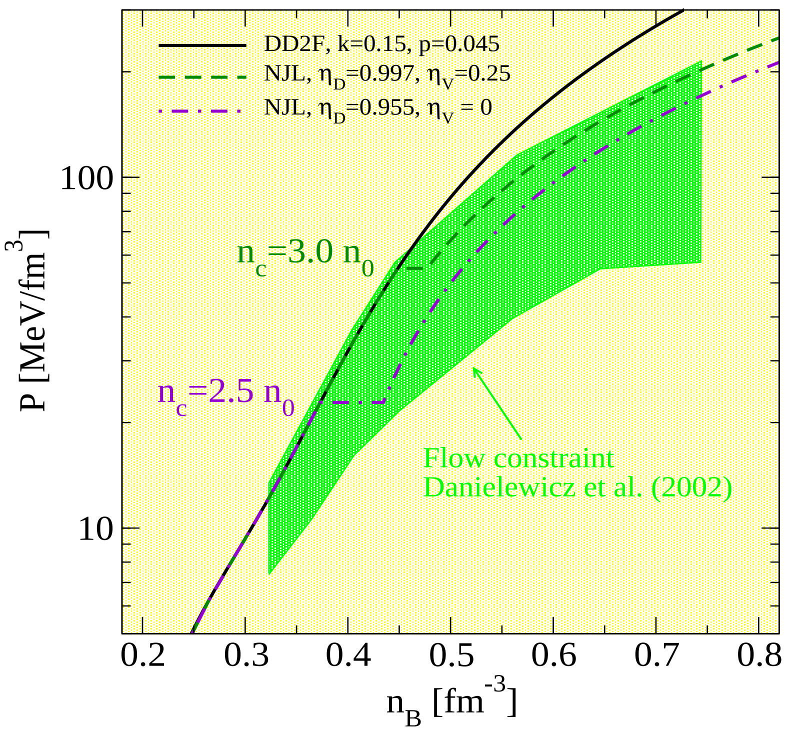


D.B.,Skokov, Sandin, NICA WhitePaper (2009)

Liebendoefer et al. (2005)
Sagert et al., PRL 102 (2009)

Mass-Radius constraint and Flow constraint





- Large Mass ($\sim 2\ M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff EoS;
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !

Sandin et al. (in preparation), See also:
Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Röpke, Trümper, [arxiv:nucl-th/0609067]

SUMMARY

- hadron production in HIC $\rightarrow$ Triple point in QCD phase diagram!
- Compressed nuclear matter: **quarkyonic phase** (QP)! Coexisting chiral symm. + conf.
- Here: PNJL model as microscopic formulation of the QP
- Prospects for HIC (CBM & NICA) and Supernovae: color superconducting (quarkyonic) phases accessible!

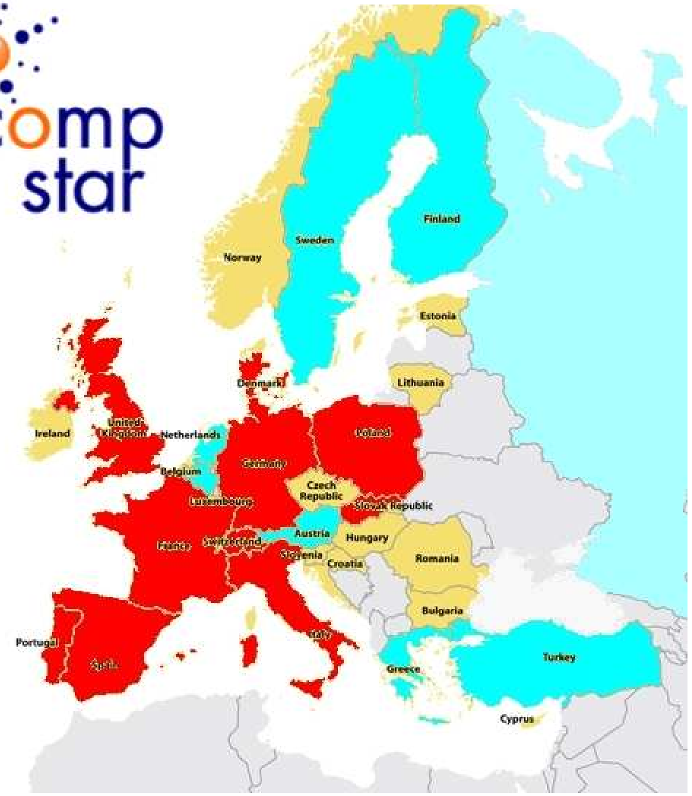
OUTLOOK: NEXT STEPS ...

- Walecka model as limit of PNJL model: chiral transition effects in nuclear EoS
- Beyond meanfield: mesons and baryons in the PNJL, higher clusters: sextetting
- Astrophysics: Maximum mass & cooling of quarkyonic stars; quarkyonic supernovae
- HIC: signals of CSC phase transition (dilepton enhancement?)

Collaborations

Thanks to:

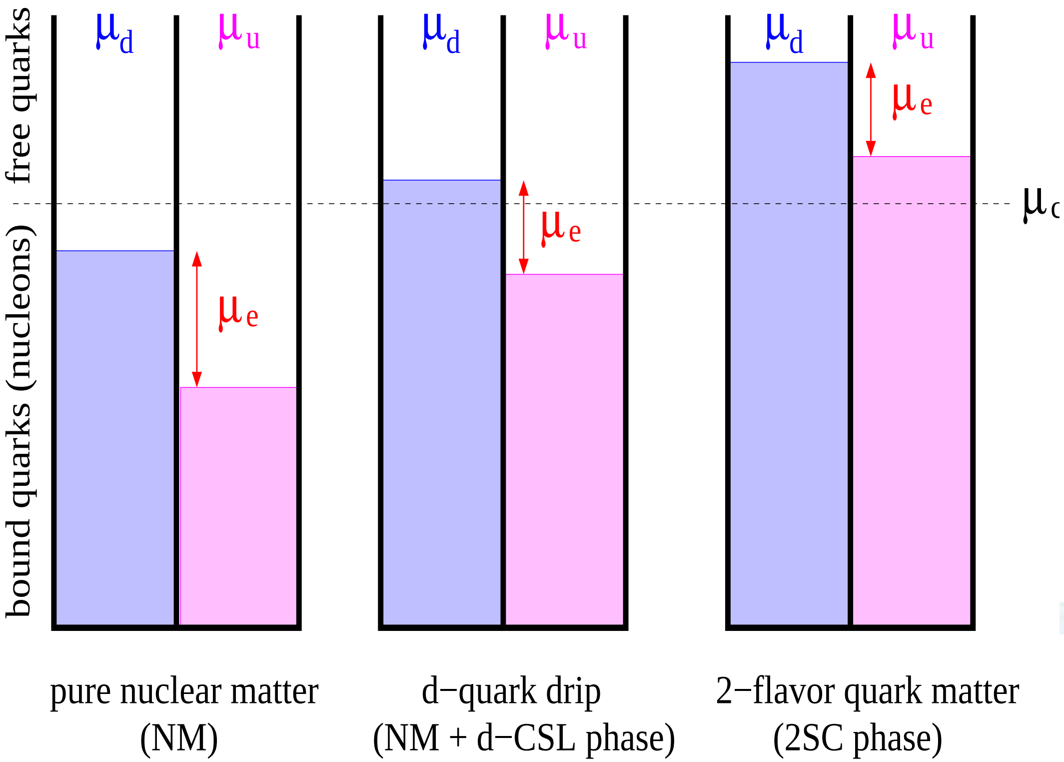
Deborah Aguilera, Jens Berdermann, Michael Buballa, Fiorella Burgio, Christian Fuchs, Daniel Gomez Dumm, Hovik Grigorian, Gabriela Grunfeld, Davor Horvatic, Igor Iosilevsky, Dubravko Klabucar, Thomas Klähn, Gevorg Poghosyan, Sergey Popov, Gerd Röpke, Fredrik Sandin, Norberto Scoccola, Joachim Trümper, Stefan Typel, Dima Voskresensky, a.m.m.

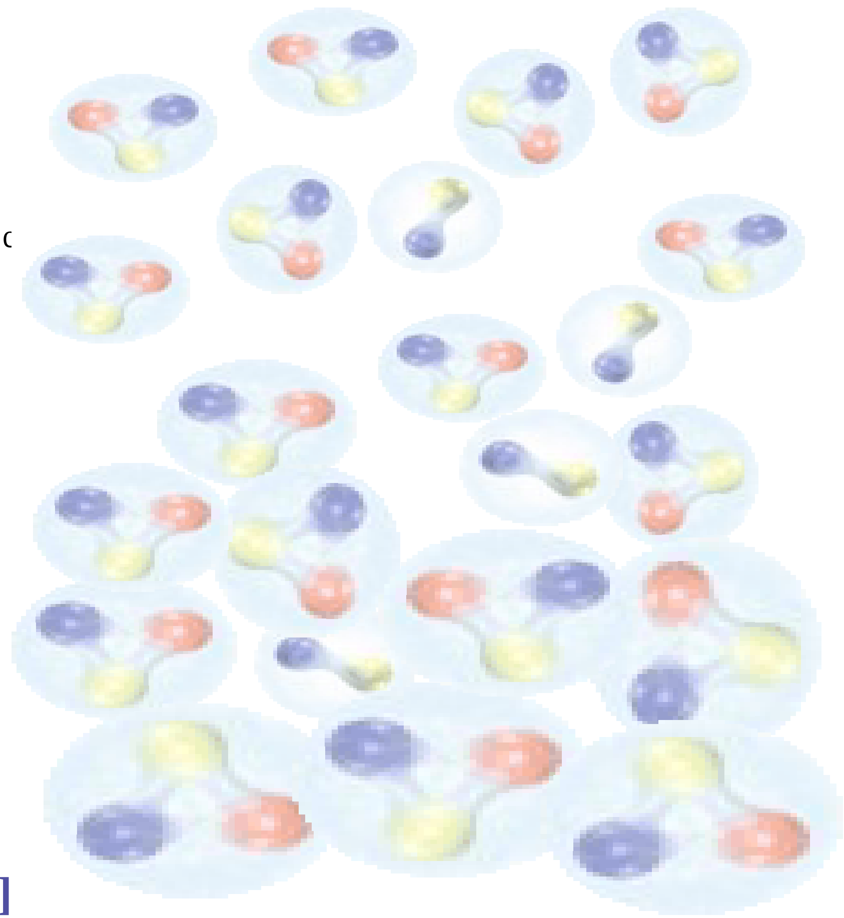


ESF Research Networking Programme
“CompStar”, 2008 - 2013
http://www.compstar-esf.org

D-QUARK 'DRIPLINE' AND SINGLE-FLAVOR (D-CSL) PHASE

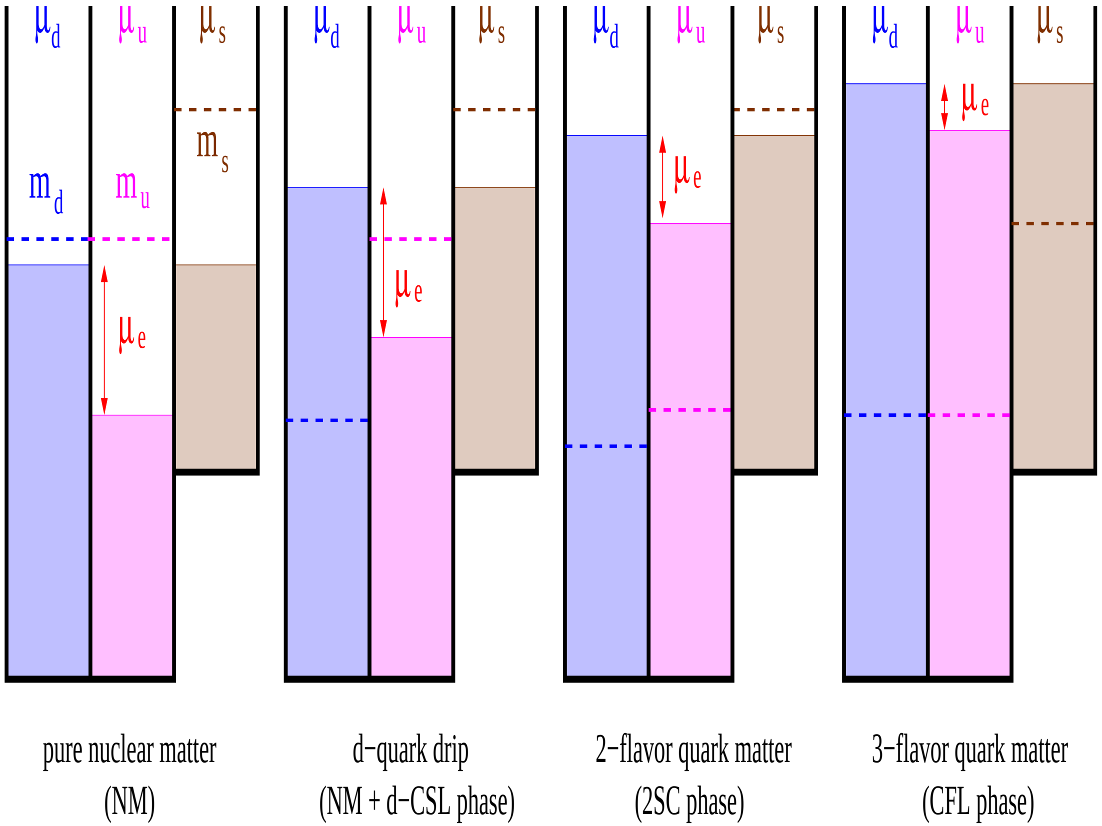
Sequential 'deconfinement' of quark flavors

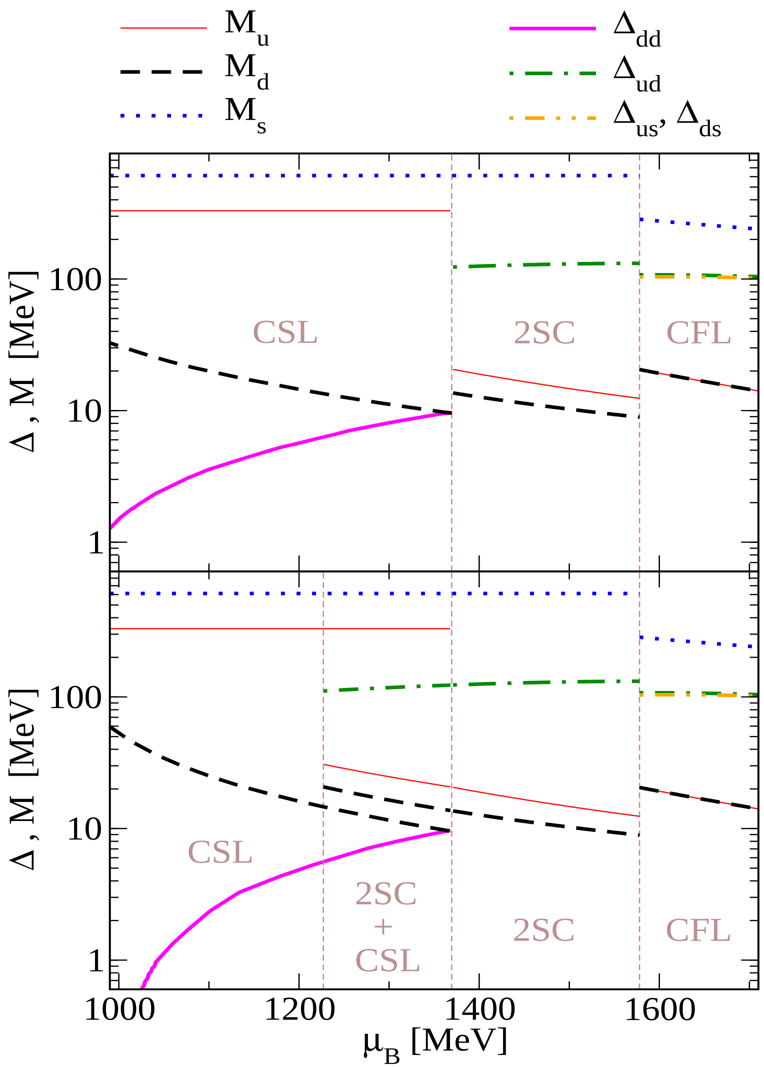




D.B., F. Sandin, T. Klähn, J. Berdermann,
arXiv:0807.0414 [nucl-th]; arXiv:0808.1369 [astro-ph]
arXiv:0808.0181 [nucl-th], J. Phys. G, in press

SEQUENTIAL DECONFINEMENT IN ASYMMETRIC NS MATTER



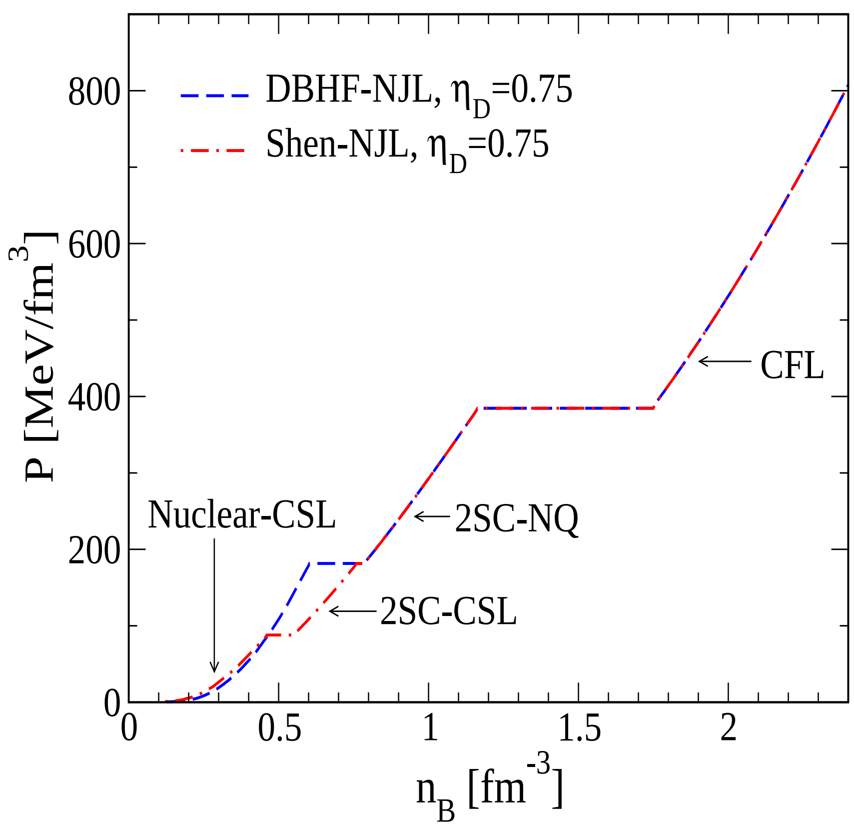


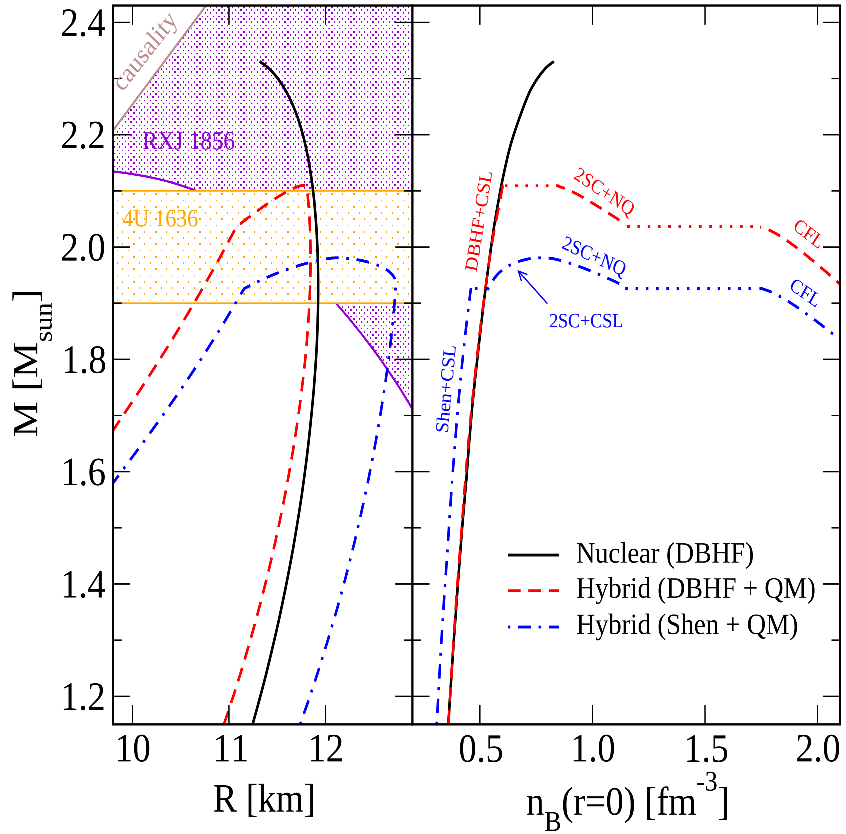
D.B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]; arXiv:0808.1369 [astro-ph]

D-CSL: SINGLE-FLAVOR PHASE IN NEUTRON STARS

Configuration Sequences

Equation of state

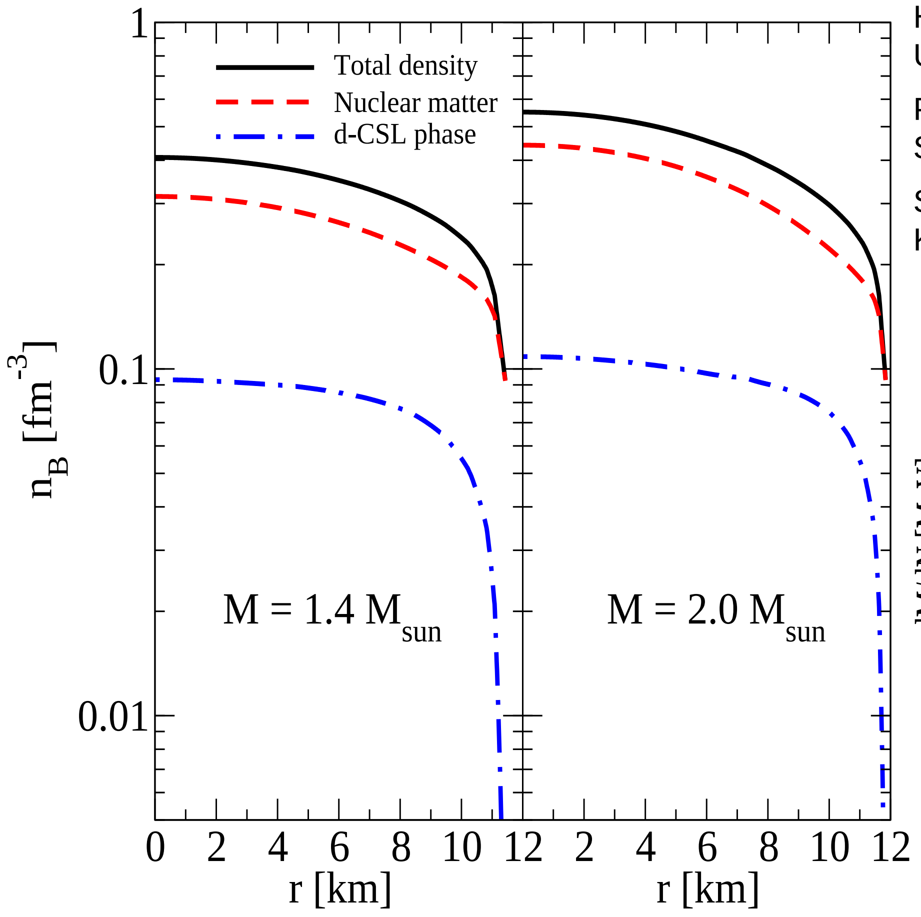




D. B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]; arXiv:0808.1369 [astro-ph]; arXiv:0808.0181 [nucl-th], J. Phys. G 35, 104077 (2008).

d-CSL: SINGLE-FLAVOR PHASE IN NEUTRON STARS

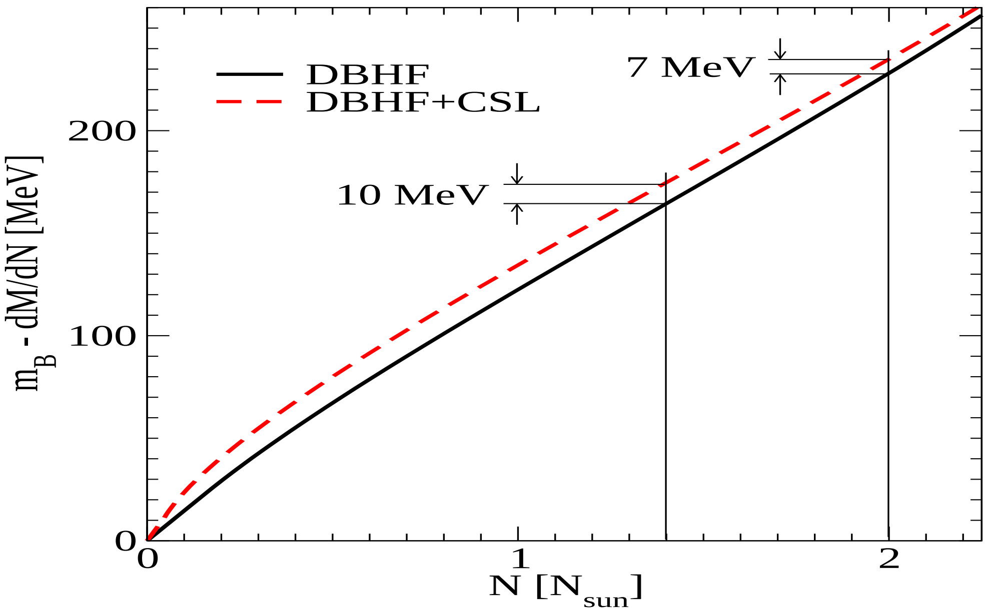
d-quark drip at crust-core boundary: Candidate for "deep crustal heating" (DCH) process?



Haensel and Zdunik,A& A **227**, 431 (1990)
Ushomirsky and Rutledge, MNRAS **325**, 1157 (2001)

Page and Cumming,ApJ **635**, L157 (2005): Superbursts & Strange Stars
Stejner and Madsen,A& A **458**, 523 (2006): SS + Transient Cooling

Shternin, Yakovlev, Haensel and Potekhin, MNRAS **382**, L43 (2007): KS1731



D. B., F. Sandin, T. Klähn, J. Berdermann, arXiv:0807.0414 [nucl-th]

Polyakov-loop Nambu–Jona-Lasinio Model (I)

$SU(N_c)$ pure gauge sector: Polyakov line

$$L(\vec{x}) \equiv \mathcal{P} \exp\left[i \int_0^\beta d\tau\, A_4(\vec{x},\tau)\right] \;;\quad A_4 = iA^0 = \lambda_3\phi_3 + \lambda_8\phi_8$$

Polyakov loop

$$l(\vec{x}) = \frac{1}{N_c}\mathrm{Tr}L(\vec{x})\;,\quad \langle l(\vec{x})\rangle = e^{-\beta\Delta F_Q(\vec{x})}.$$

$\mathbf{Z}_{N_c}$ symmetric phase: $\langle l(\vec{x})\rangle = 0 \quad \Longrightarrow \Delta F_Q \to \infty$: Confinement !
Polyakov loop field:

$$\Phi(\vec{x}) \equiv \langle\langle l(\vec{x})\rangle\rangle = \frac{1}{N_c}\,\mathrm{Tr}_c\,\langle\langle L(\vec{x})\rangle\rangle$$

Potential for the PL-meanfield $\Phi(\vec{x})$ =const., which fits quenched QCD lattice thermodynamics

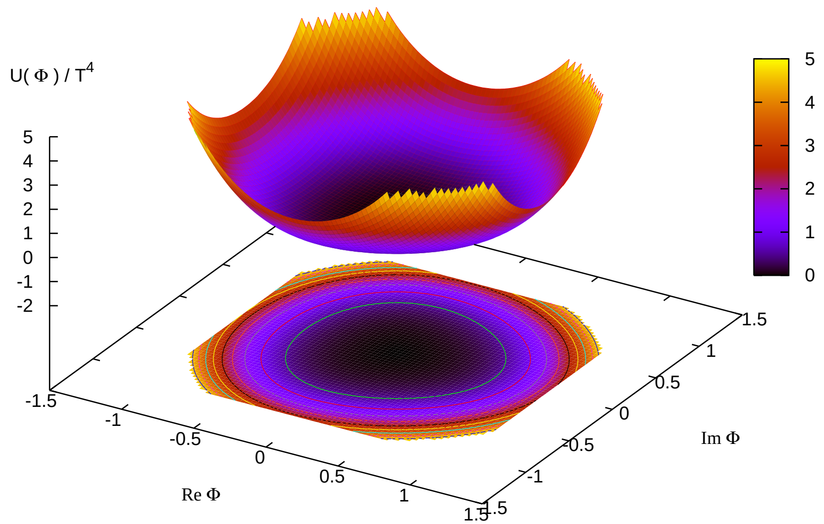
$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \;,$$

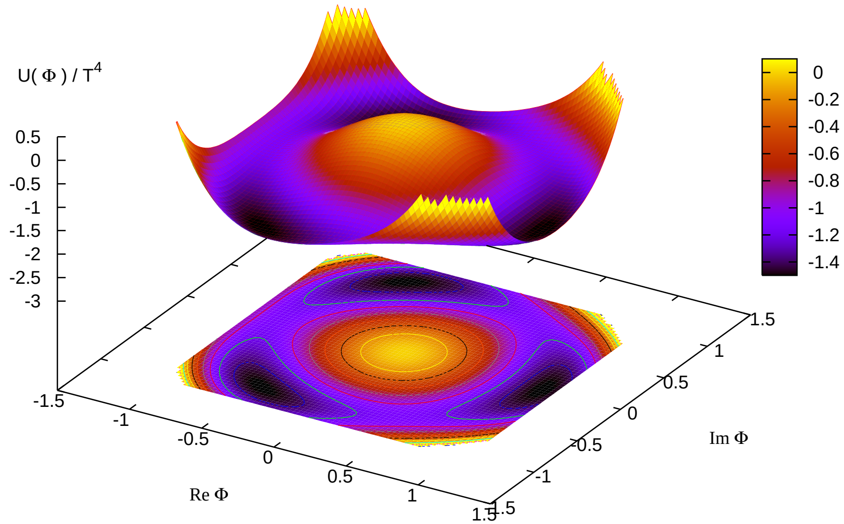
$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

Polyakov-loop Nambu–Jona-Lasinio Model (II)

Temperature dependence of the Polyakov-loop potential $U(\Phi, \bar{\Phi}; T)$





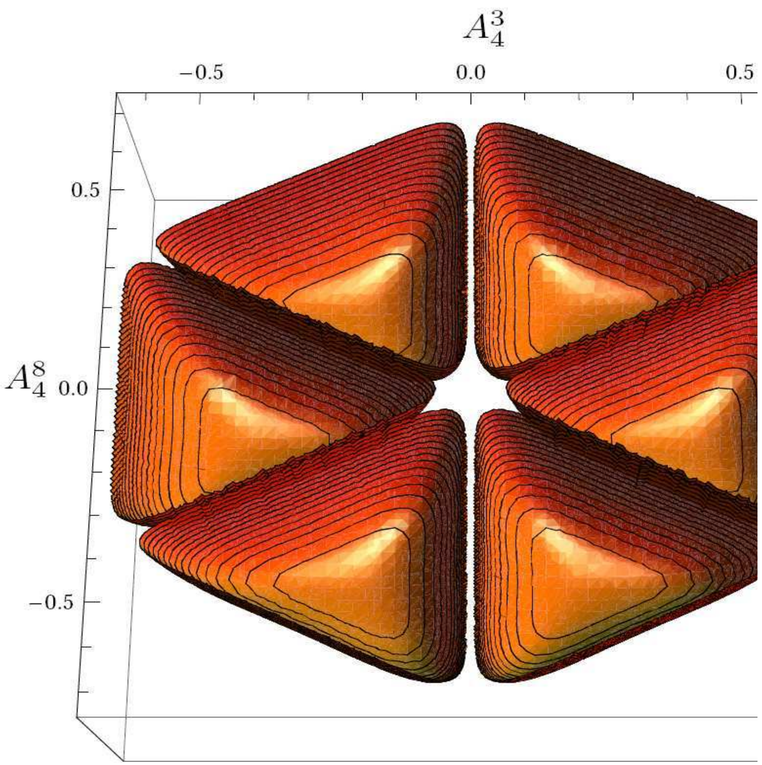
$T = 0.26$ GeV$< T_0$
"Color confinement"

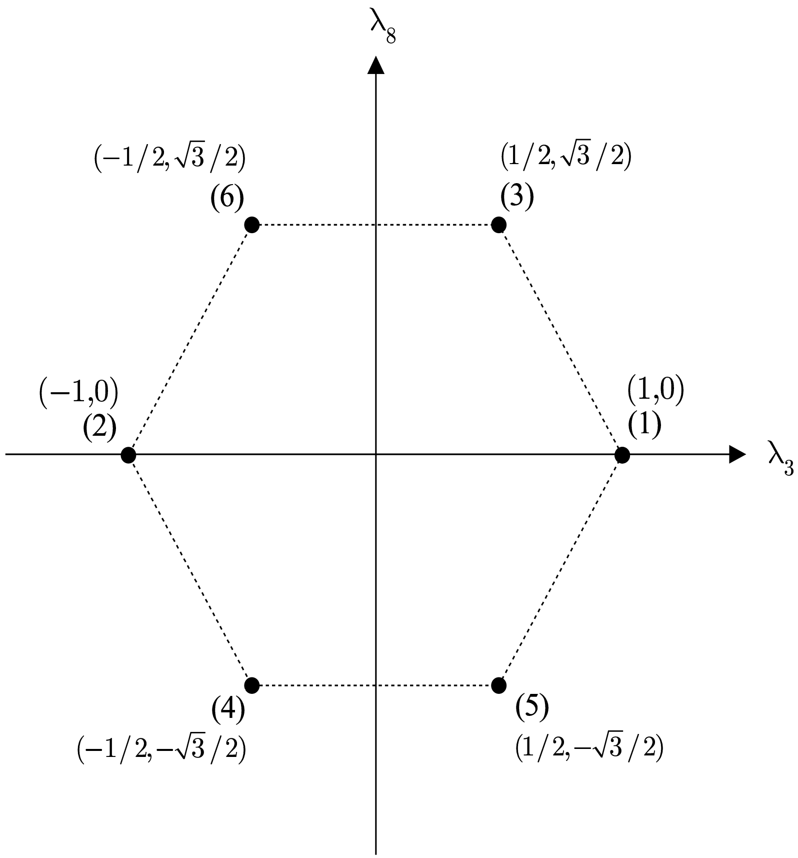
$T = 1.0$ GeV$> T_0$
"Color deconfinement"

Critical temperature for pure gauge $SU_c(3)$ lattice simulations: $T_0 = 270$ MeV.

POLYAKOV-LOOP VARIABLE Φ

Degeneracy in $\Phi = Tr_c\{\exp[i\beta A_4]\}/N_c$; $A_4 = \lambda_3\phi_3 + \lambda_8\phi_8$; Internal Z(3) Symmetry





Hell et al., 0810.1099 [hep-ph]

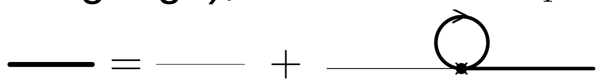
Abuki et al., 0811.1512 [hep-ph]

Polyakov-loop Nambu–Jona-Lasinio Model (III)

Lagrangian for $N_f = 2$, $N_c = 3$ quark matter, coupled to the gauge sector

$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma^\mu D_\mu - \hat{m} + \gamma_0\mu)q + G_1\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right),$$

$D^\mu = \partial^\mu - iA^\mu$; $A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge), with $A^0 = -iA_4$

Diagrammatic Hartree equation: 

$$S_0(p) = \text{———} = -(\not{p} - m_0 + \gamma^0(\mu - iA_4))^{-1}\ ; \quad S(p) = \text{———} = -(\not{p} - m + \gamma^0(\mu - iA_4))^{-1}$$

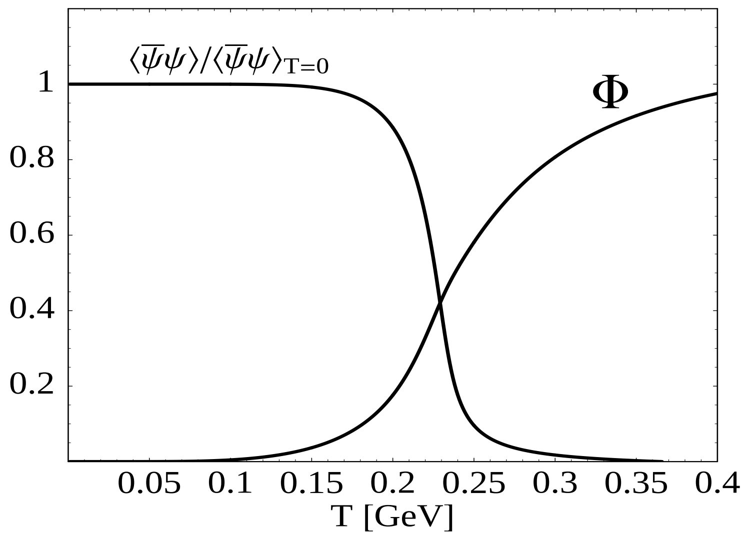
Dynamical chiral symmetry breaking $\sigma = m - m_0 \neq 0$? Solve Gap Equation! ($E = \sqrt{p^2 + m^2}$)

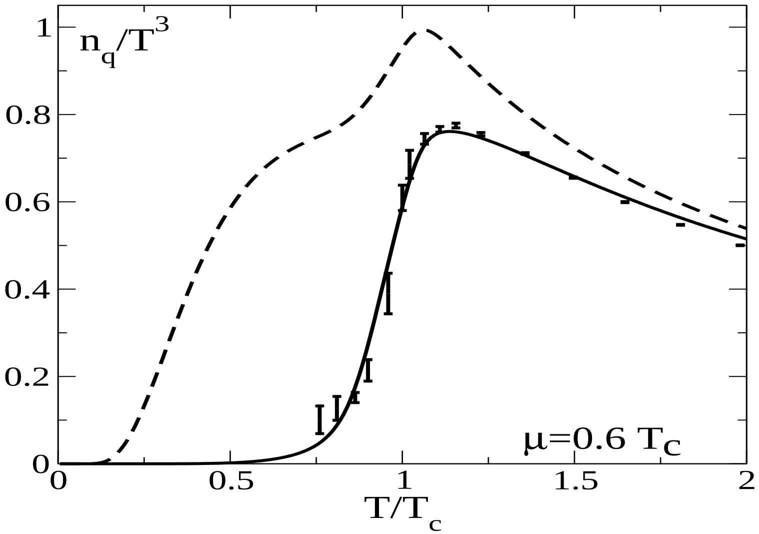
$$\begin{aligned} m - m_0 &= 2G_1 T\ \mathrm{Tr} \sum_{n=-\infty}^{+\infty} \int_\Lambda \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{-1}{\not{p} - m + \gamma^0(\mu - iA_4)} \\ &= 2G_1 N_f N_c \int_\Lambda \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{2m}{E}[1 - f^+_\Phi(E) - f^-_\Phi(E)] \end{aligned}$$

Modified quark distribution functions ($\Phi = \bar{\Phi} = 0$: "poor man's nucleon": $E_N = 3E$, $\mu_N = 3\mu$)

$$f^\pm_\Phi(E) = \frac{\left(\Phi + 2\bar{\Phi}e^{-\beta\left(E_p \mp \mu\right)}\right)e^{-\beta\left(E_p\mp\mu\right)} + e^{-3\beta\left(E_p\mp\mu\right)}}{1 + 3\left(\Phi + \bar{\Phi}e^{-\beta\left(E_p\mp\mu\right)}\right)e^{-\beta\left(E_p\mp\mu\right)} + e^{-3\beta\left(E_p\mp\mu\right)}} \longrightarrow f^\pm_0(E) = \frac{1}{1 + e^{\beta(E_N \mp \mu_N)}}$$

Polyakov-loop Nambu–Jona-Lasinio Model (IV)





Grand canonical thermodynamical potential

$$\begin{aligned}\Omega(T,\mu;\Phi,m) &= \frac{\sigma^2}{2G} - 6N_f \int \frac{\mathrm{d}^3p}{(2\pi)^3} E\,\theta\left(\Lambda^2 - \vec{p}^{\,2}\right) \\ &- 2N_f\,T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \Big\{ \mathrm{Tr}_c \ln\left[1 + L\,\mathrm{e}^{-(E-\mu)/T}\right] \\ &+ \mathrm{Tr}_c \ln\left[1 + L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right] \Big\} + \mathcal{U}\left(\Phi,\bar\Phi,T\right)\end{aligned}$$

Appearance of quarks below T_c largely suppressed:

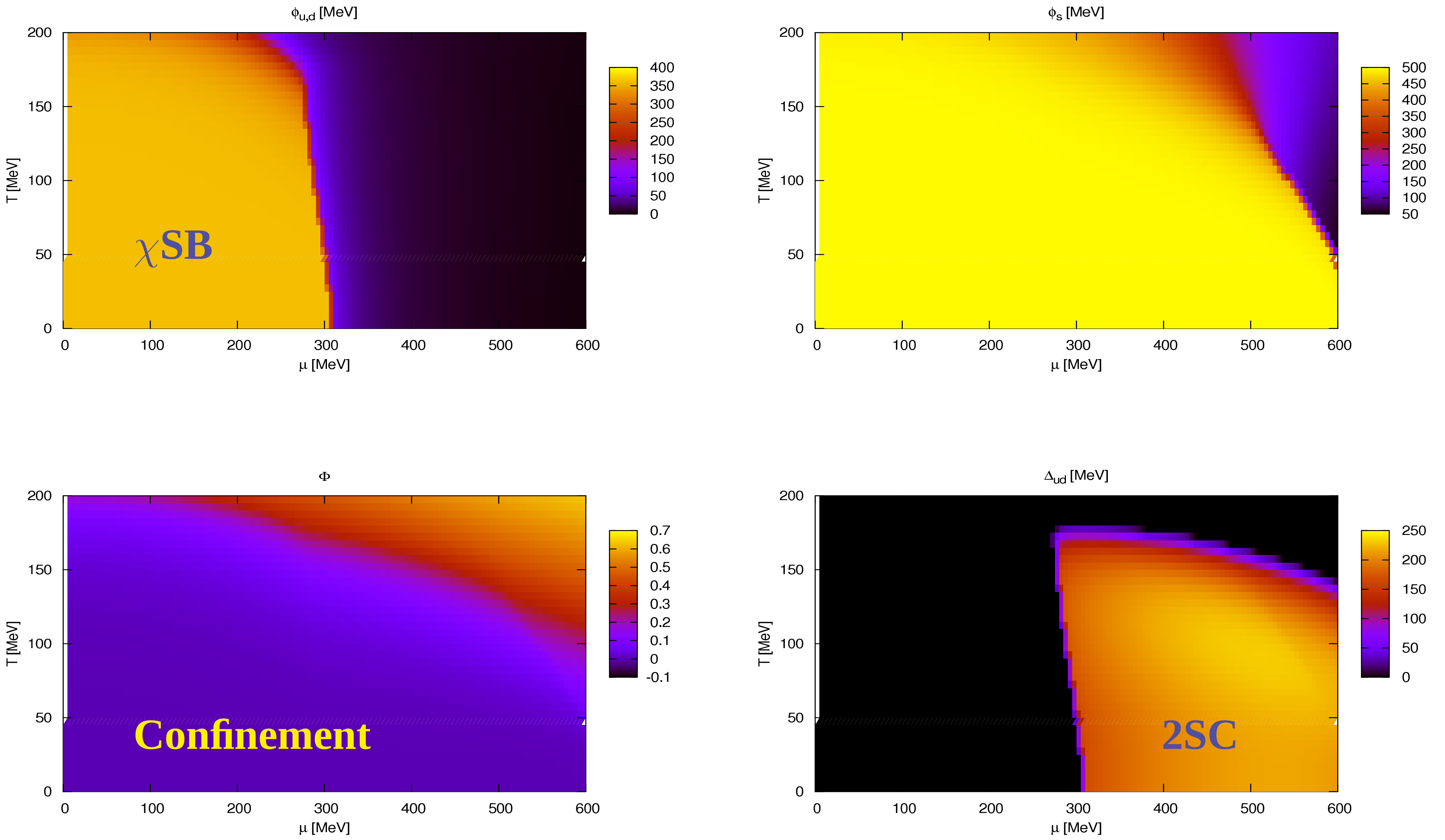
$$\begin{aligned}&\ln\det\left[1 + L\,\mathrm{e}^{-(E-\mu)/T}\right] + \ln\det\left[1 + L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right] \\ = &\ln\left[1 + 3\left(\Phi + \bar\Phi \mathrm{e}^{-(E-\mu)/T}\right)\mathrm{e}^{-(E-\mu)/T} + \mathrm{e}^{-3(E-\mu)/T}\right] \\ + &\ln\left[1 + 3\left(\bar\Phi + \Phi \mathrm{e}^{-(E+\mu)/T}\right)\mathrm{e}^{-(E+\mu)/T} + \mathrm{e}^{-3(E+\mu)/T}\right].\end{aligned}$$

Accordance with QCD lattice susceptibilities! Example:

$$\frac{n_q(T,\mu)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega(T,\mu)}{\partial\mu},$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

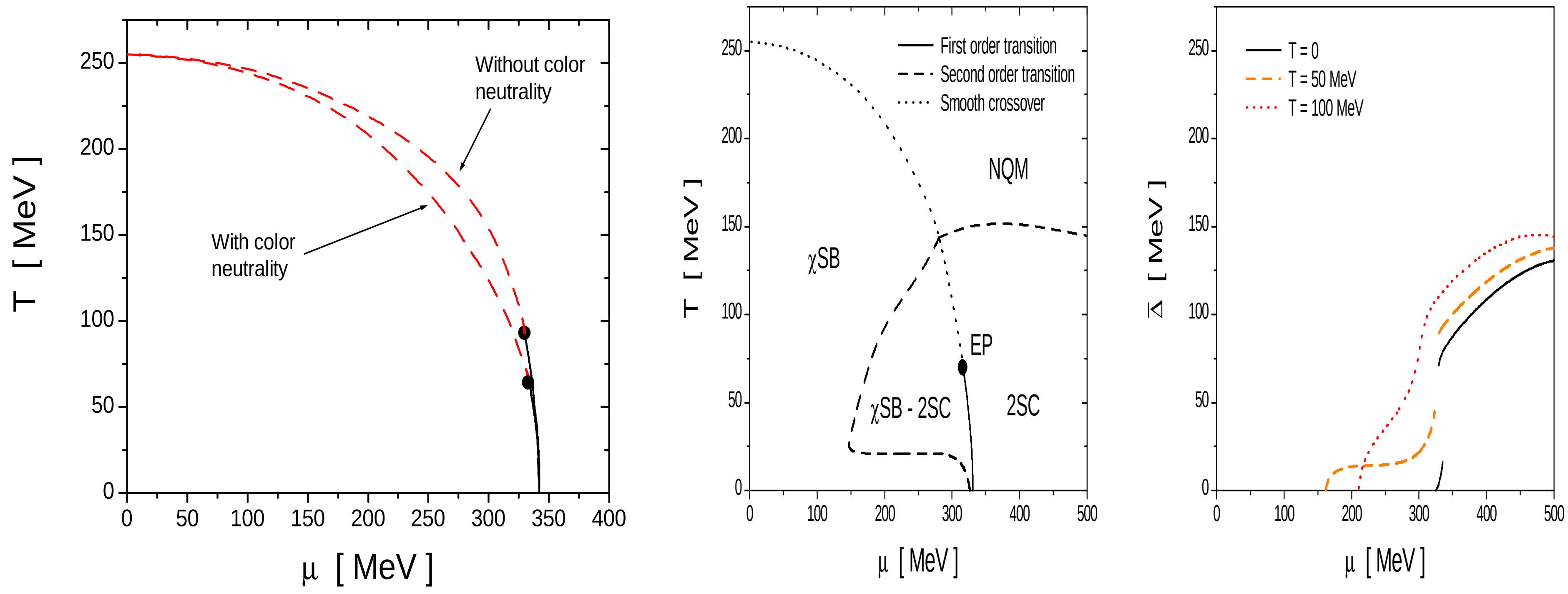
Phases of QCD @ Extremes: no Color Neutrality



Color Neutrality in the PNJL Phase Diagram

Color neutrality constraint: $\tilde{\mu} = \mu \mathbf{1} + \mu_8 \lambda_8 + i\phi_3 \lambda_3$; $\partial\Omega_{MF}/\partial\mu_8 = n_8 = n_r + n_g - 2n_b = 0$

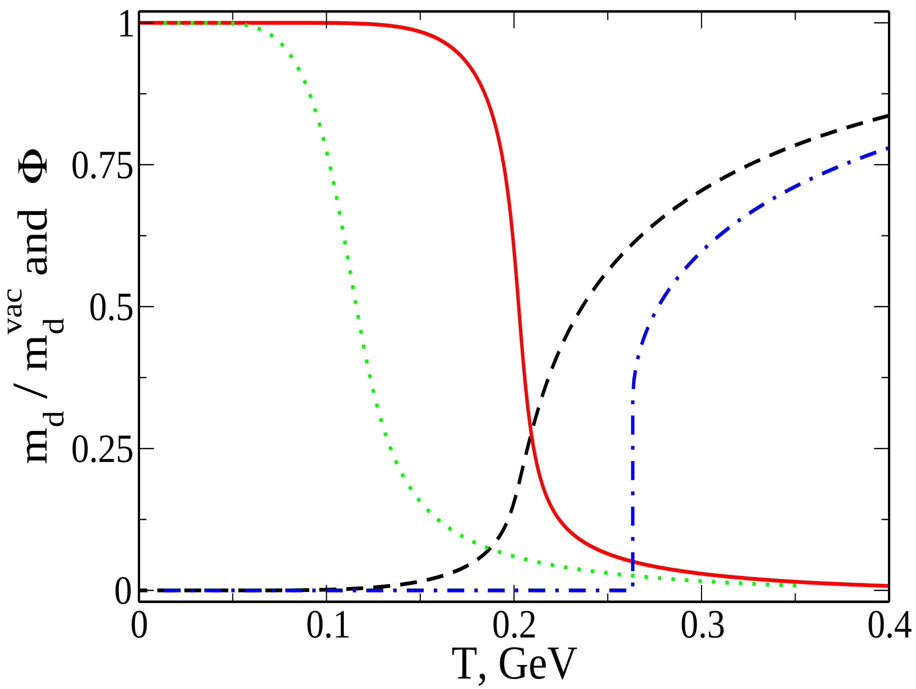
Gap equations: $\partial\Omega_{MF}/(\partial\sigma, \partial\Delta, \partial\phi_3) = 0$



Gomez-Dumm, D.B., Grunfeld, Scoccola, PRD 78, 114021 (2008) [arXiv:0807.1660]

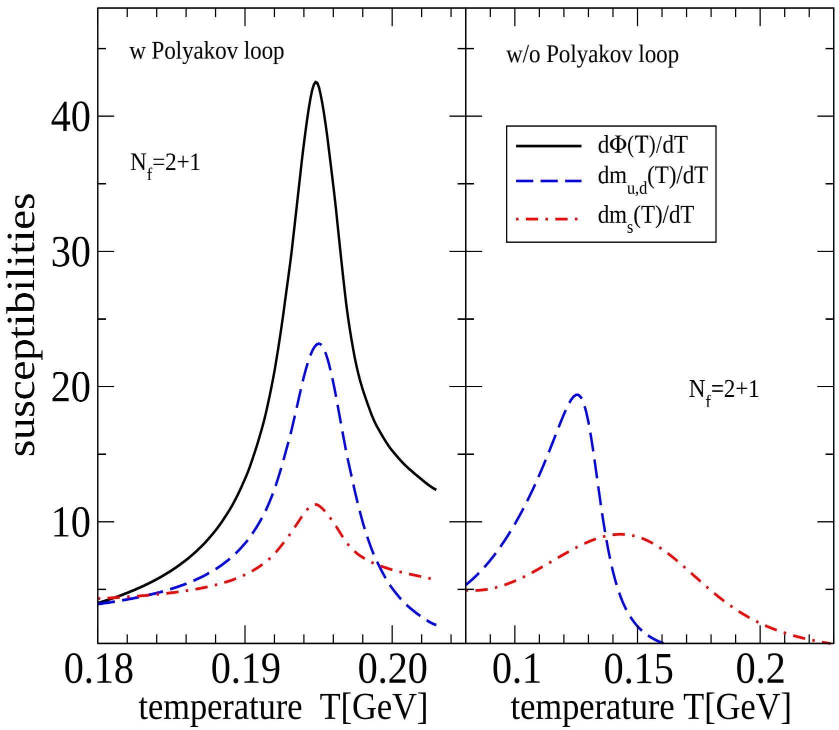
NONLOCAL POLYAKOV LOOP CHIRAL QUARK MODEL

2-flavor, rank-1, 4D separable
order parameters:



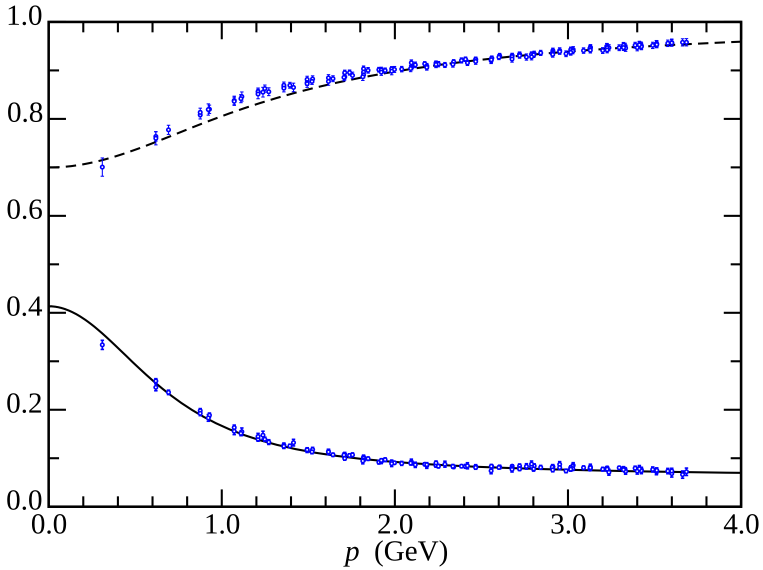
D.B., Buballa, Radzhabov, Volkov,
Yad. Fiz. 71 (2008); arXiv:0705.0384

3-flavor, rank-2, 4D separable
susceptibilities:



D.B., Horvatic, Klabucar, in prep.

Complex mass pole fit to Lattice propagator



Bhagwat, Pichowsky, Roberts, Tandy, Phys. Rev. **C68** (2003) 015203

$$S(p)^{-1} = i\not p A(p^2) + B(p^2) \; ,$$

$$M(p^2) = B(p^2)/A(p^2)$$

$$Z(p^2) = 1/A(p^2)$$

$S(p)$ sum of N pairs of complex conj. mass poles

$$S(p) = \sum_{i=1}^{N} \frac{1}{Z_2} \left\{ \frac{z_i}{i\not p + m_i} + \frac{z_i^*}{i\not p + m_i^*} \right\} = -i\not p \sigma_V(p^2) + \sigma_S(p^2)$$

Representation of the scalar amplitude

$$\sigma_S(p^2) = \sum_{i=1}^{N} Z_2^{-1} \left\{ \frac{z_i\, m_i}{p^2 + m_i^2} + \frac{z_i^*\, m_i^*}{p^2 + m_i^{*2}} \right\}$$

"Derivation" of the equivalent separable model (in Feynman-like gauge) $D_{\mu\nu}(p-q) = \delta_{\mu\nu}\, D(p,q)$ and

$$D(p,q) = f_0(p^2)\, f_0(q^2) + f_1(p^2)\, p\cdot q\, f_1(q^2)$$

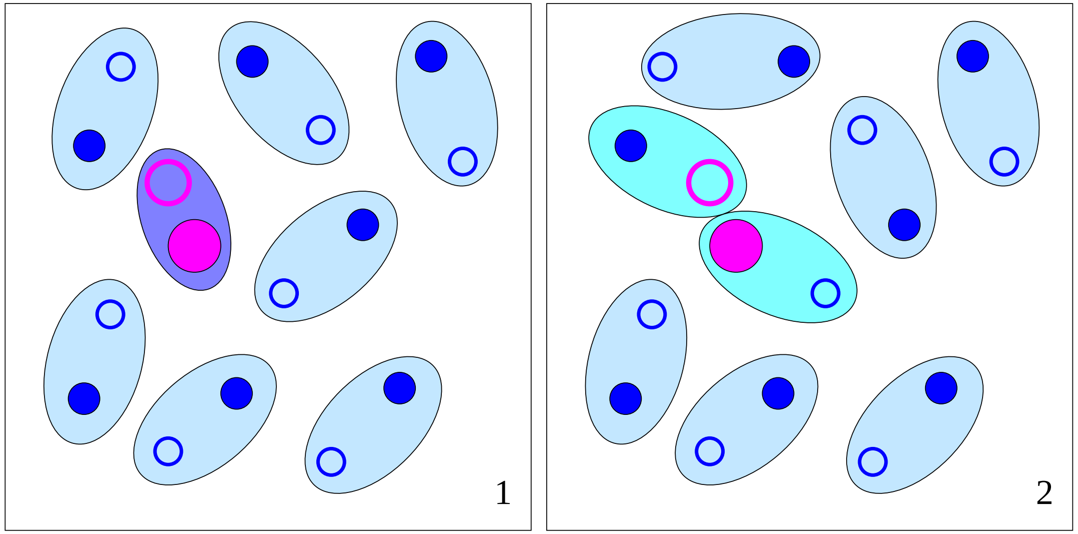
$$f_1(p^2) = \frac{A(p^2) - 1}{a} \quad ; \quad f_0(p^2) = \frac{B(p^2) - m_c}{b}$$

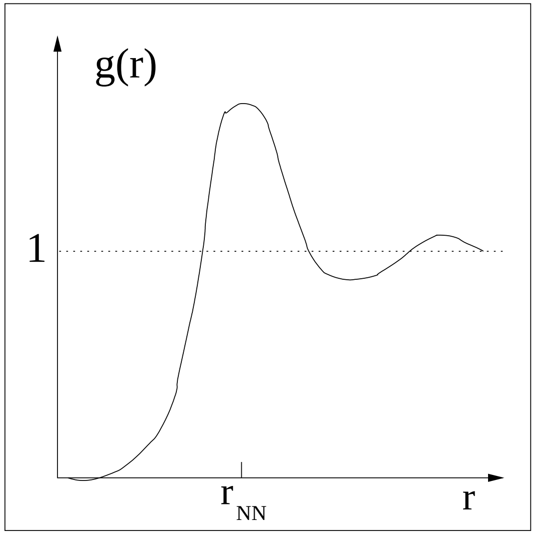
$$b^2 = \frac{16}{3} \int_q^{\Lambda} [B(q^2) - m_c] \sigma_s(q^2)$$

$$a^2 = \frac{8}{3} \int_q^{\Lambda} [A(q^2) - 1] \frac{q^2}{4} \sigma_v(q^2)$$

A snapshop of the sQGP

The Picture: String-flip (Rearrangement) $\Longleftrightarrow$ Pair correlation



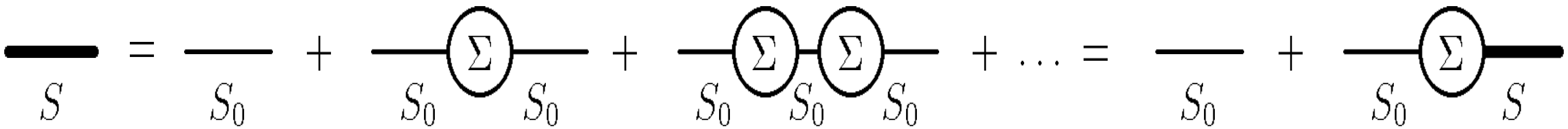


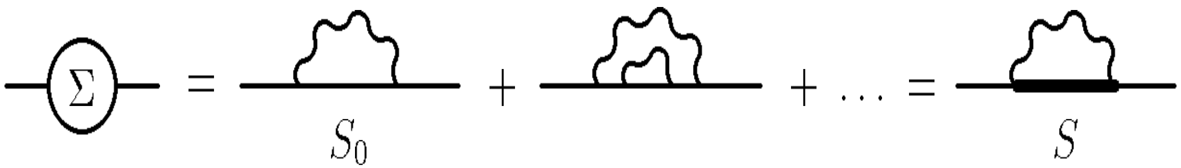
Horowitz et al. PRD (1985), D.B. et al. PLB (1985), Röpke, Blaschke, Schulz, PRD (1986)

Thoma,[hep-ph/0509154] Gelman et al., PRC 74 (2006)

- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

QUARKYONIC PHASE: DYSON-SCHWINGER EQ. PERSPECTIVE





Confining potential:

$$K^{ab}_{\mu\nu}(\vec{x}-\vec{y}) = g_{\mu 0} g_{\nu 0} \delta^{ab} V(|\vec{x}-\vec{y}|); \qquad \frac{\lambda^a \lambda^a}{4} V(r) = \sigma r; \qquad V(\vec{p}) = \frac{8\pi\sigma}{(\vec{p}^2+\mu^2_{\rm IR})^2}.$$

Self energy operator:

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma}\cdot\hat{\vec{p}})[B_p - p], \qquad A_p \cos\varphi_p - B_p \sin\varphi_p = 0, \qquad A_p = \frac{\sigma}{2\mu_{\rm IR}} \sin\varphi_p + A^f_p,$$

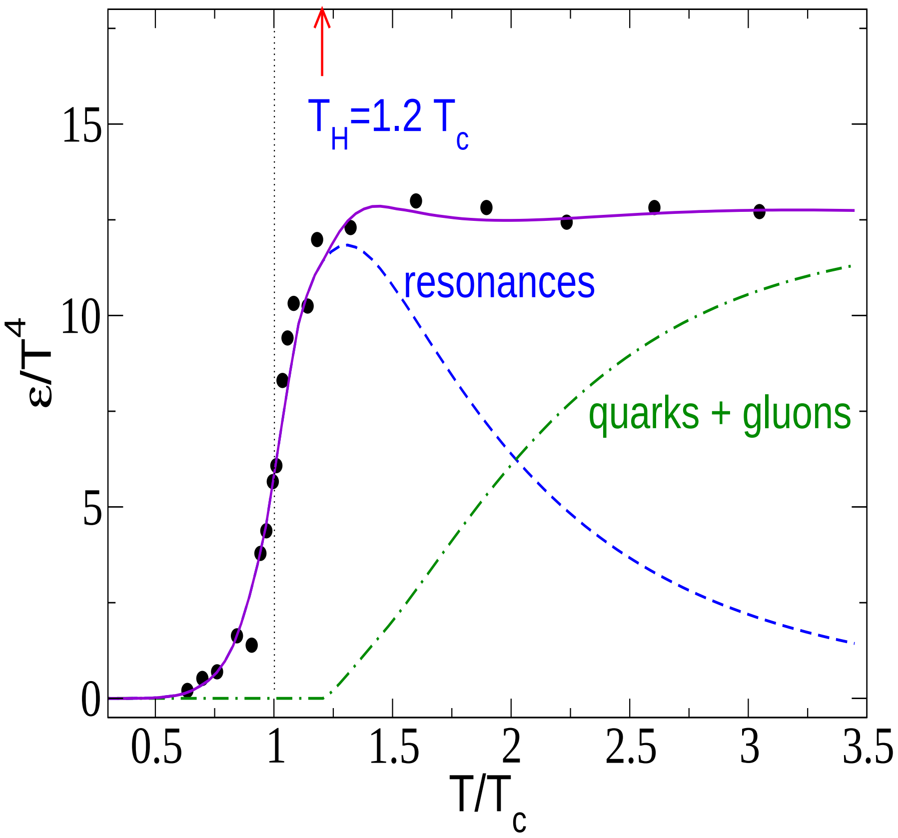
Diverging single-quark energy, compensated in color-singlet states:

$$\omega(p) = \sqrt{(A^2_p + B^2_p)} = \frac{\sigma}{2\mu_{IR}} + \omega_f(p), \qquad 2\frac{\sigma}{2\mu_{IR}} - \frac{\sigma}{\mu_{IR}} = 0.$$

Glozman, arXiv:0812.1101 [hep-ph]

LATTICE QCD EOS AND MOTT-HAGEDORN GAS

$$\varepsilon_{\rm R}(T,\{\mu_j\}) = \sum_{i=\pi,K,\ldots} \varepsilon_i(T,\{\mu_i\}) + \sum_{r=M,B} g_r \int_{m_r} dm \int ds\, \rho(m) A(s,m;T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+s}}{\exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)+\delta_r}$$



Hagedorn mass spectrum: $\rho(m)$

Spectral function for heavy resonances:

$$A(s,m;T) = N_s \frac{m\Gamma(T)}{(s-m^2)^2 + m^2\Gamma^2(T)}$$

Ansatz with Mott effect at $T = T_H = 180$ MeV:

$$\Gamma(T) = B\Theta(T-T_H)\left(\frac{m}{T_H}\right)^{2.5}\left(\frac{T}{T_H}\right)^{6}\exp\left(\frac{m}{T_H}\right)$$

No width below T_H: Hagedorn resonance gas
Apparent phase transition at $T_c \sim 150$ MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)
Prog. Part. Nucl. Phys. 53, 197 (2004)
Blaschke & Yudichev, in preparation
Bugaev, Petrov, Zinovjev, arXiv:0812.2189