Coulomb and Surface Effects on "Pasta" Structures in Nuclear Matter

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Collaboration with

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- Mixed phase at first-order phase transitions in nuclear matter.
- Its non-uniform structures.
- Its equation of state.

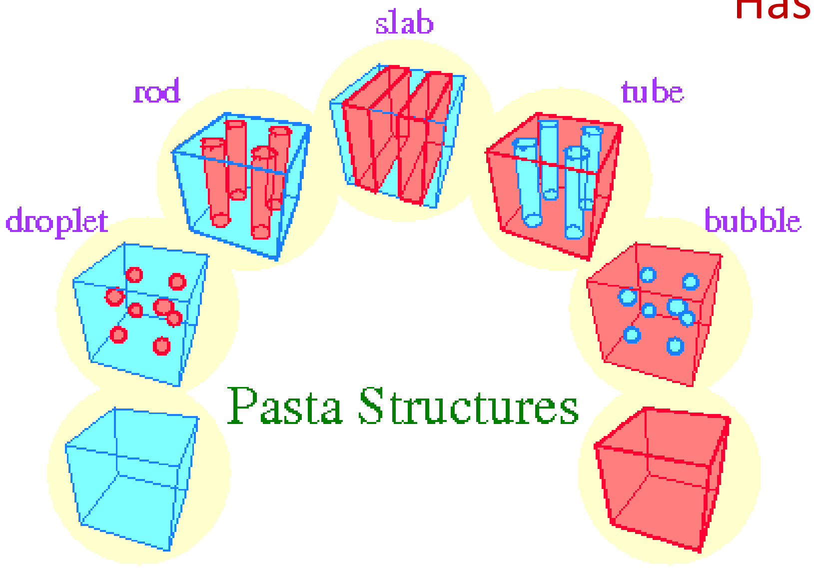
Phase transitions in *nuclear matter*

Liquid-gas, neutron drip, meson condensation, hyperon mixture, quark deconfinement, color super-conductivity, etc.

Some of them are the first-order → **mixed phase**

In the mixed phase with charged particles, non-uniform "Pasta" structures are expected.

[Ravenhall *et al*, PRL 50(1983)2066, Hashimoto *et al*, PTP 71(1984)320]



Optimum structure to minimize the (free) energy of the system.

Depending on the density, geometrical structure of mixed phase changes from droplet, rod, slab, tube and to bubble configuration.

(I) Low density nuclear matter

- Collapsing stage of supernova explosion (non β–equil)

  → Liquid-gas phase transition

  T=0 (not realistic)

  electron gas + nuclear liquid

  T>0 (1~several MeV)

  nuclear gas + nuclear liquid

- Neutron star crust (β–equil)

  → Liquid-gas phase transition (Neutron drip)

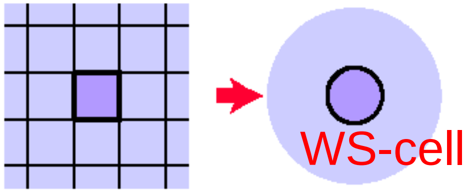
  T=0

  neutron liquid + nuclear liquid

Numerical calculation of mixed-phase

• Assume regularity in structure: divide whole space into equivalent and neutral cells with a geometrical symmetry (3D: sphere, 2D : cylinder, 1D: plate).

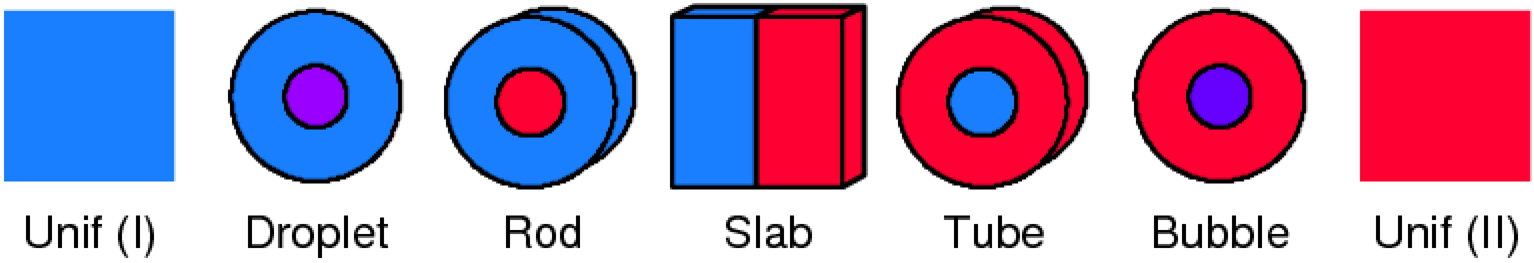
→ Wigner-Seitz cell approx.



• Give a geometry (Unif/Dropl/Rod/...) and a baryon density ρ_B.

• Solve the field equations numerically. Optimize the cell size (choose the free-energy-minimum).

• Choose an free-energy-minimum geometry among 7 cases (Unif (I), droplet, rod, slab, tube, bubble, Unif (II)).



Field equations to be solved

Relativistic Mean Field (RMF) model:

Lorentz-covariant Lagrangian L with baryon densities, meson fields (σ, ω, ρ), electron density and the Coulomb potential is determined.

Local density approx:

Local density approximation for baryons and electron → Thomas Fermi

Consistent treatment for potentials and densities:

→ Coulomb screening by charged particles
[T.M. et al,PRC72(2005)015802; Rec.Res.Dev.Phys,7(2006)1]

$$\mathrm{L} = \mathrm{L}_N + \mathrm{L}_M + \mathrm{L}_e,$$

$$\mathrm{L}_N = \bar{\Psi}\left[i\gamma^\mu \partial_\mu - m_N^* - g_{\omega N}\gamma^\mu \omega_\mu - g_{\rho N}\gamma^\mu \vec{\tau}\vec{b}_\mu - e\frac{1+\tau_3}{2}\gamma^\mu V_\mu \right]\Psi$$

$$\mathrm{L}_M = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}_\mu\vec{R}^\mu,$$

$$\mathrm{L}_e = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \bar{\Psi}_e\left[i\gamma^\mu\partial_\mu - m_e + e\gamma^\mu V_\mu \right]\Psi_e, \qquad (F_{\mu\nu} \equiv \partial_\mu F_\nu - \partial_\nu F_\mu)$$

$$m_N^* = m_N - g_{\sigma N}\sigma, \qquad U(\sigma) = \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 + \frac{1}{4}c(g_{\sigma N}\sigma)^4$$

EOM for fields

Chemical equilibrium fully consistent with all the density distributions and fields.
Ground state properties of nuclei and nuclear matter are well reproduced.

From $\partial_\mu \left[\frac{\partial \mathrm{L}}{\partial(\partial_\mu \phi)} \right] - \frac{\partial \mathrm{L}}{\partial \phi} = 0, \quad (\phi = \sigma, \omega_\mu, R_\mu, V_\mu, \Psi),$

$$-\nabla^2 \sigma(\boldsymbol{r}) + m_\sigma^2 \sigma(\boldsymbol{r}) = -\frac{dU}{d\sigma}(\boldsymbol{r}) + g_{\sigma N}(\rho_n^{(s)}(\boldsymbol{r}) + \rho_p^{(s)}(\boldsymbol{r})),$$

$$-\nabla^2 \omega_0(\boldsymbol{r}) + m_\omega^2 \omega_0(\boldsymbol{r}) = g_{\omega N}(\rho_p(\boldsymbol{r}) + \rho_n(\boldsymbol{r})),$$

$$-\nabla^2 R_0(\boldsymbol{r}) + m_\rho^2 R_0(\boldsymbol{r}) = g_{\rho N}(\rho_p(\boldsymbol{r}) - \rho_n(\boldsymbol{r})),$$

$$\nabla^2 V_C(\boldsymbol{r}) = 4\pi e^2 \rho_{\mathrm{ch}}(\boldsymbol{r}), \qquad (\rho_{\mathrm{ch}}(\boldsymbol{r}) \equiv \rho_p(\boldsymbol{r}) - \rho_e(\boldsymbol{r}))$$

For Fermions, we employ Thomas-Fermi approx. at finite T

$$f_{i=n,p}(\boldsymbol{r}; \boldsymbol{p}, \mu_i) = \left(1 + \exp\left[\left(\sqrt{p^2 + m_N^*(\boldsymbol{r})^2} - \nu_i(\boldsymbol{r})\right)/T\right]\right)^{-1},$$

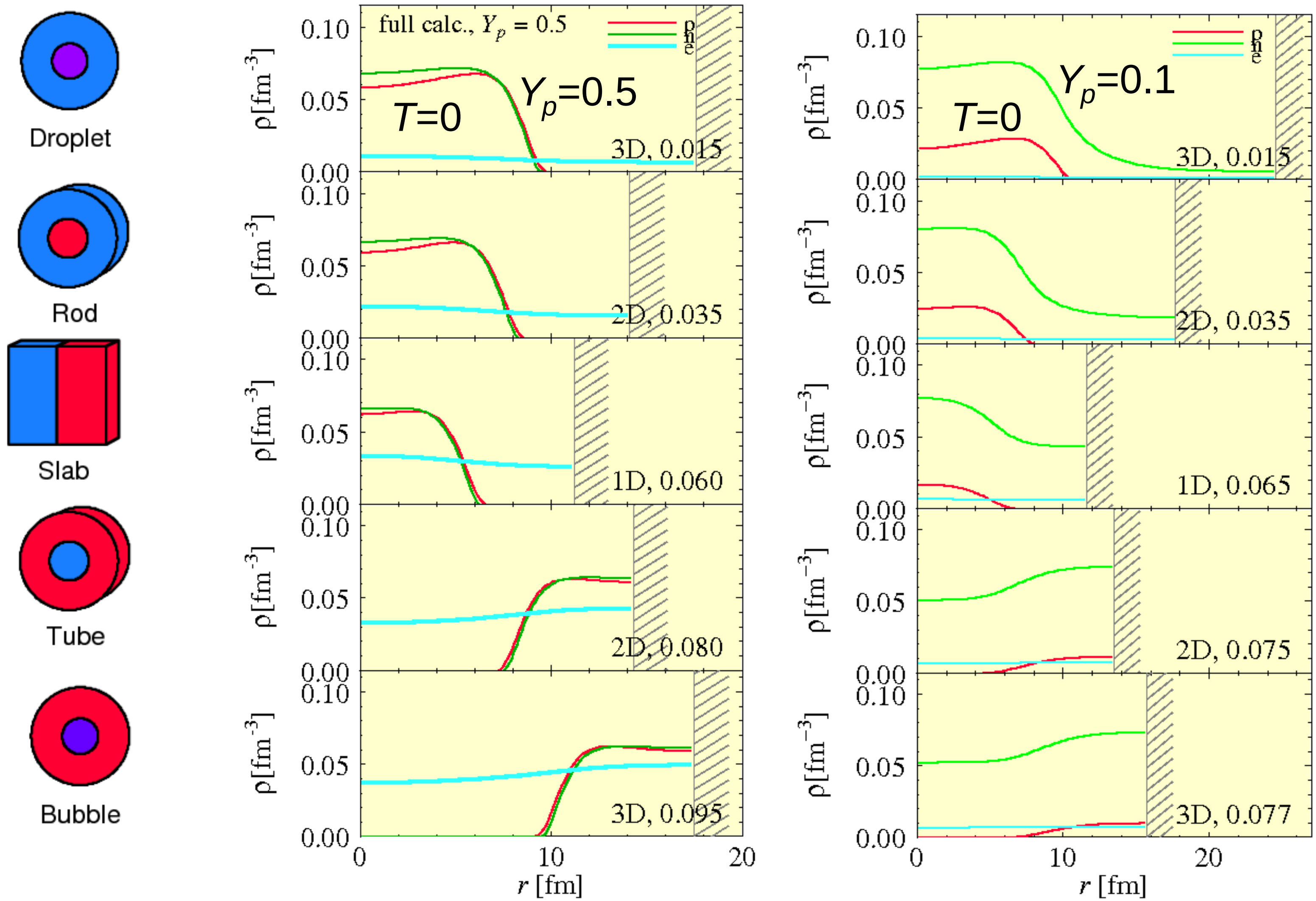
$$f_e(\boldsymbol{r}; \boldsymbol{p}, \mu_e) = \left(1 + \exp\left[\left(p - (\mu_e - V_C(\boldsymbol{r}))\right)/T\right]\right)^{-1}$$

$$\rho_e(\boldsymbol{r}) = 2\int_0^\infty \frac{d^3 p}{(2\pi)^3} f_e(\boldsymbol{r}; \boldsymbol{p}, \mu_e), \quad \rho_{i=p,n}(\boldsymbol{r}) = 2\int_0^\infty \frac{d^3 p}{(2\pi)^3} f_i(\boldsymbol{r}; \boldsymbol{p}, \mu_i)$$

$$\mu_n = \nu_n(\boldsymbol{r}) + g_{\omega N}\omega_0(\boldsymbol{r}) - g_{\rho N} R_0(\boldsymbol{r}), \quad \mu_p = \nu_p(\boldsymbol{r}) + g_{\omega N}\omega_0(\boldsymbol{r}) + g_{\rho N} R_0(\boldsymbol{r}) - V_C(\boldsymbol{r}),$$

Pasta of low density nuclear matter (with fixed Y_p)

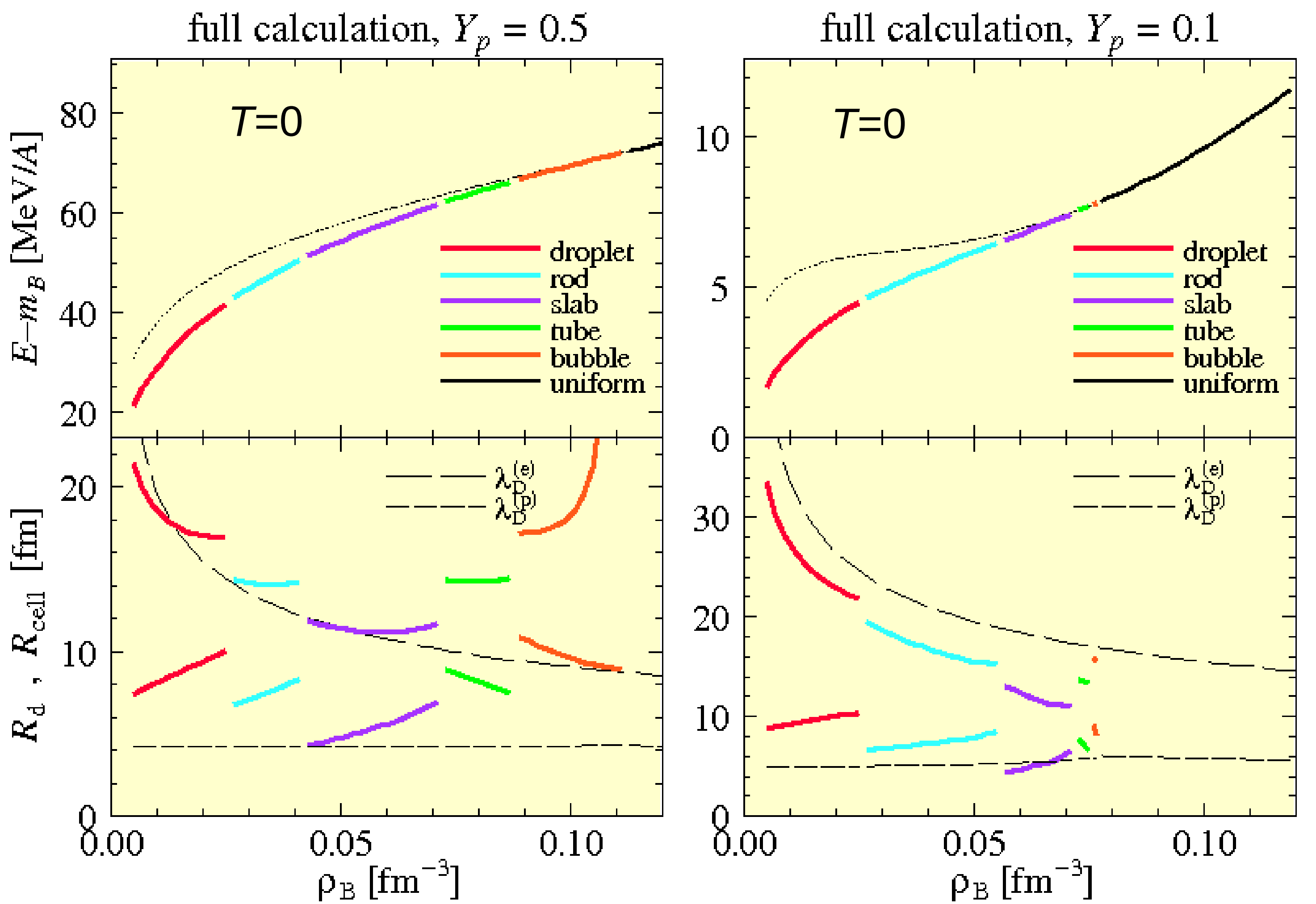
Density profiles in WS cell



EOS and the structure size

Pasta structures (droplet, rod,…) appear with the change of density.

Pasta → energy gain



Structure size

Strong surface tension and weak Coulomb

→ large *R*

Extreme case

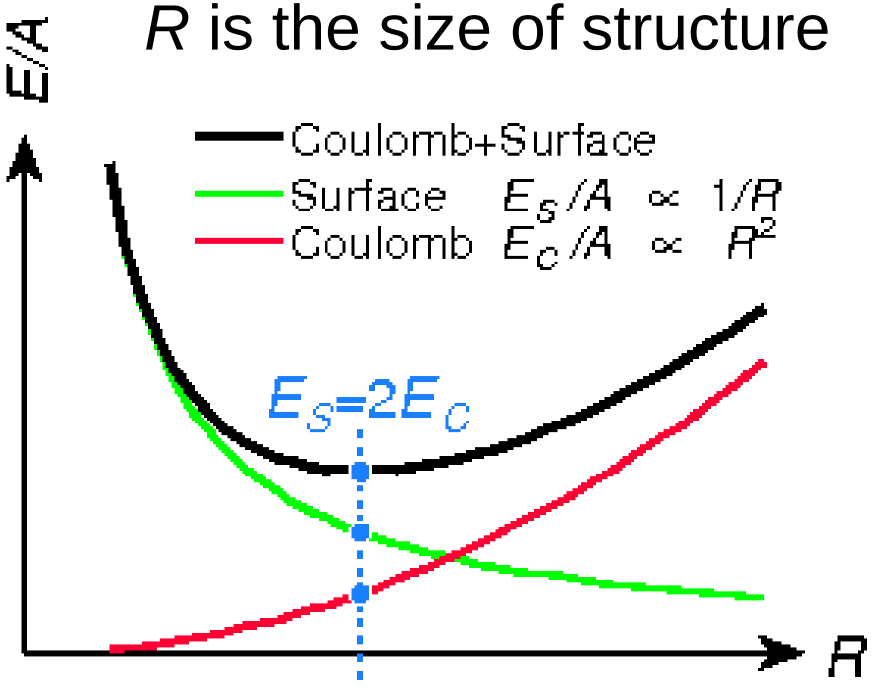
→ no minimum.

(pasta unstable)

[Voskresensky et al, PLB541(2002)93; NPA723(2003)291]

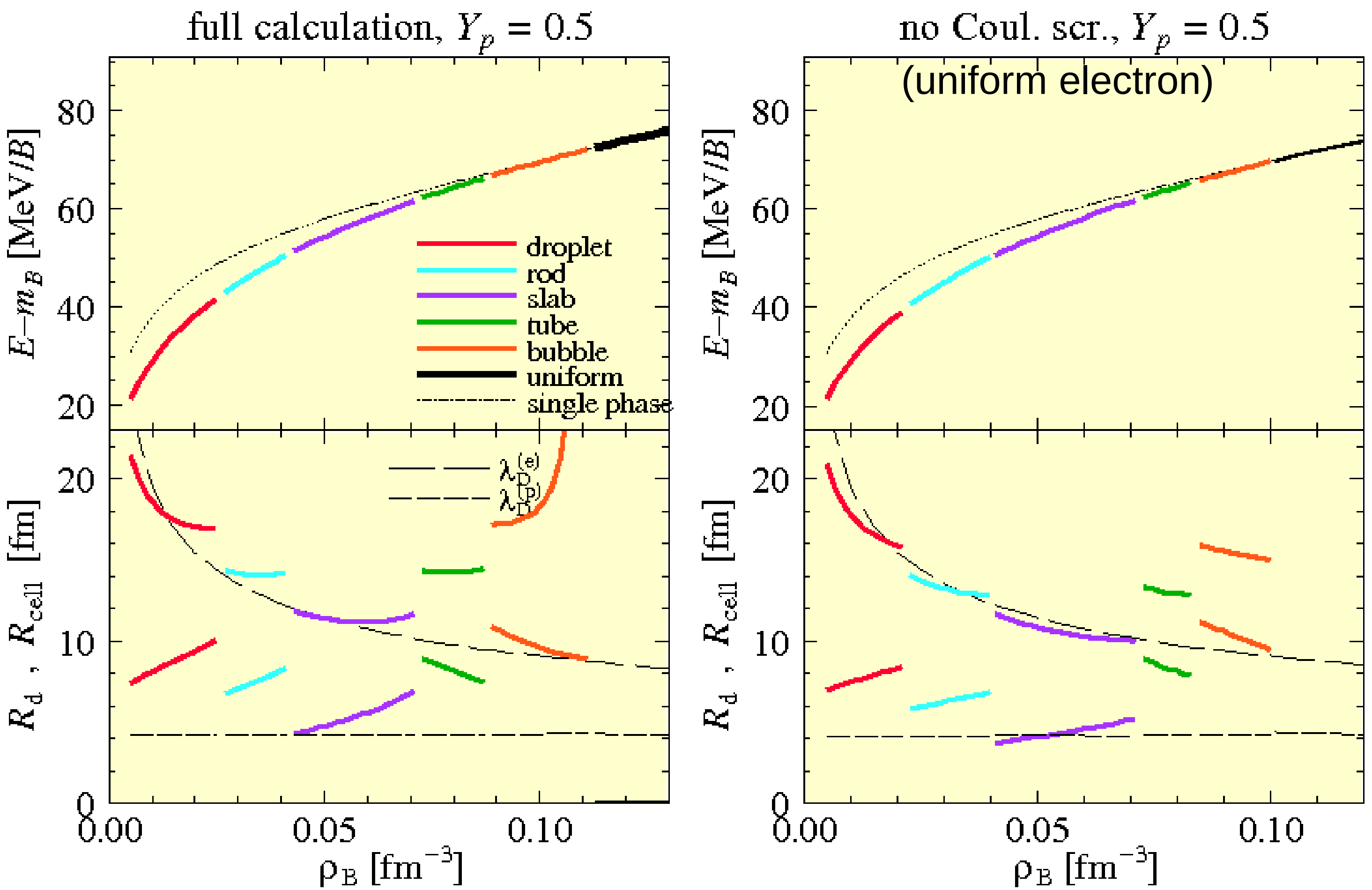
Dependence of *E/A* on *R*.

R is the size of structure



Coulomb screening effects

Compare different treatments of Coulomb int.



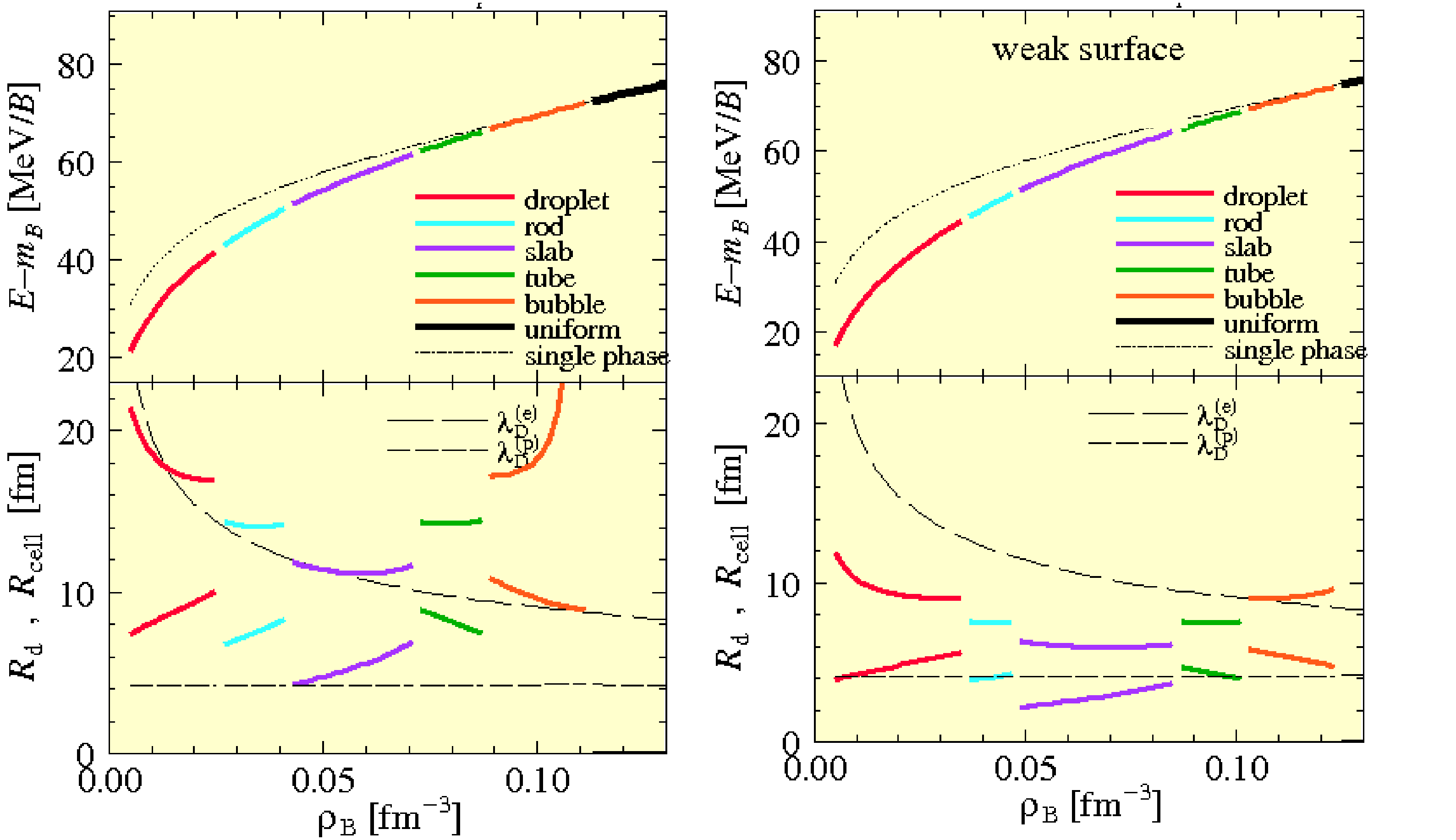
Smaller structure size for “no Coulomb screening” calc.

→ Coulomb screening enlarges the structure size.

Surface tension effects

Compare different nuclear surface tension

Normal surface tension | Weak surface



Smaller structure size for weak surface tension.

→ Stronger surface tension enlarges structure size.

Mixed phase at higher densities

- Kaon condensation
  [PRC 73(2006)035802, RecentResDevPhys7(2006)1]

- Hadron-quark transition
  [PRD 76(2007)123015, PLB 659(2008)192]

(III) Hadron-quark phase transition

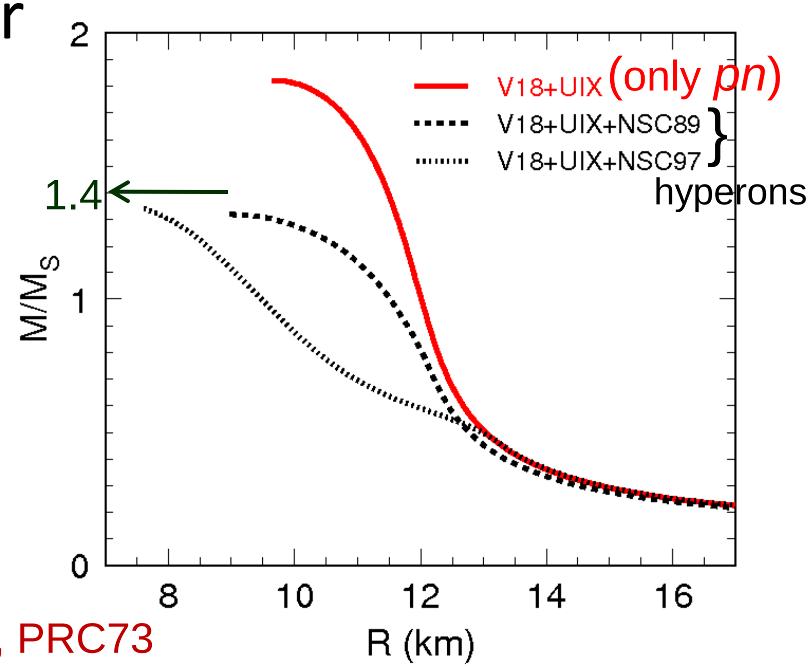
At 2-3 ρ_0, hyperons are expected to appear.

→ Softening of EOS

→ Maximum mass of neutron star becomes less than 1.4 solar mass.

→ Contradicts the obs >1.5 M_{sol}

Possibility to resolve this problem by introducing quark phase in neutron stars.



[Schulze et al, PRC73 (2006) 058801]

Coupled equations

to get density profile, energy, pressure, etc of the system

$$\mu_u + \mu_e = \mu_d = \mu_s\,, \quad \mu_n = \mu_u + 2\mu_d\,, \quad \mu_p + \mu_e = \mu_n = \mu_\Lambda = \mu_\Sigma - \mu_e$$

$$\mu_i = \frac{\partial \varepsilon(\mathbf{r})}{\partial \rho_i(\mathbf{r})} \quad (i = u, d, s, p, n, \Lambda, \Sigma^-, e)$$

$$\varepsilon(\mathbf{r}) = \varepsilon_B(\mathbf{r}) + \varepsilon_e(\mathbf{r}) + \left(\nabla V_C(\mathbf{r})\right)^2 / 8\pi e^2$$

$$\varepsilon_B(\mathbf{r}) = \begin{cases} \varepsilon_H(\mathbf{r}) & (\texttt{hadron phase: BHF model}) \\ \varepsilon_Q(\mathbf{r}) & (\texttt{quark phase: MIT bag model}) \end{cases}$$

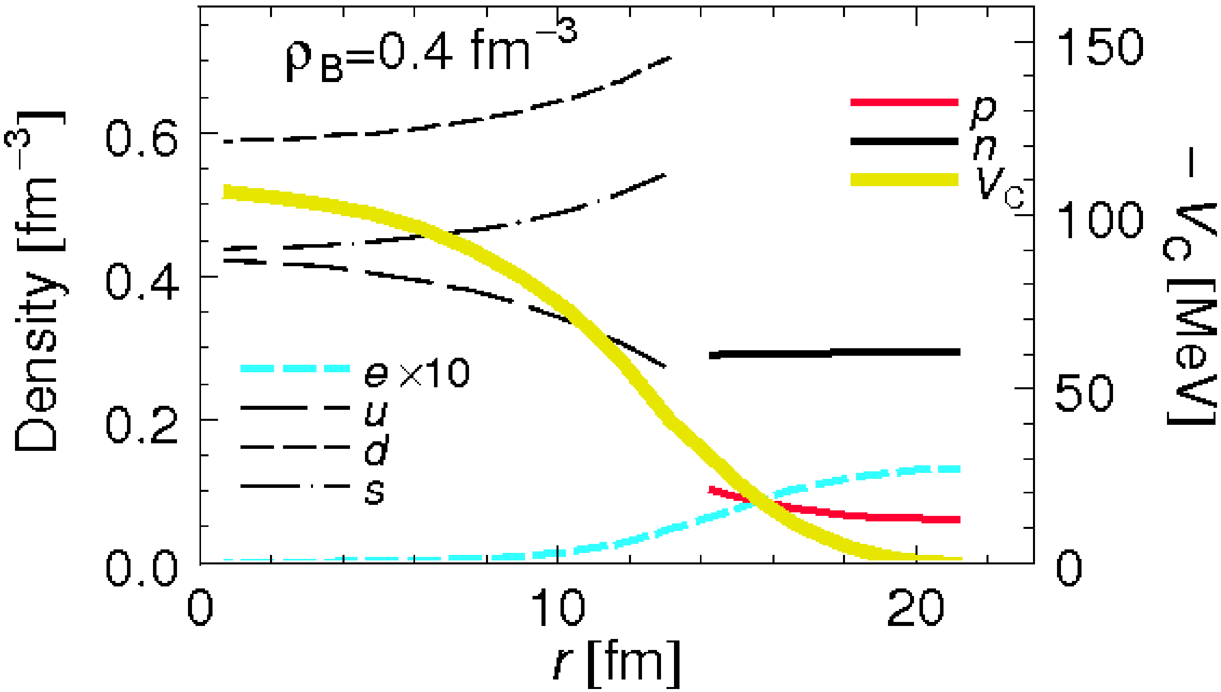
$$\varepsilon_e(\mathbf{r}) = \left(3\pi\rho_e(\mathbf{r})\right)^{4/3}$$

$$E/A = \frac{1}{\rho_B V}\left[\int_V d^3r\,\varepsilon(\mathbf{r}) + \tau S\right] \quad \begin{pmatrix} \rho_B = \texttt{average baryon density} \\ S = \texttt{Q-H bondary area} \\ V = \texttt{cell volume} \end{pmatrix}$$

$$\int_V d^3r\left[\rho_p(\mathbf{r}) - \rho_\Sigma(\mathbf{r}) + \frac{2}{3}\rho_u(\mathbf{r}) - \frac{1}{3}\rho_d(\mathbf{r}) - \frac{1}{3}\rho_s(\mathbf{r}) - \rho_e(\mathbf{r})\right] = 0 \quad (\texttt{total charge})$$

$$\frac{1}{V}\int_V d^3r\left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r}) + \rho_\Lambda(\mathbf{r}) + \rho_\Sigma(\mathbf{r}) + \frac{1}{3}\rho_u(\mathbf{r}) + \frac{1}{3}\rho_d(\mathbf{r}) + \frac{1}{3}\rho_s(\mathbf{r})\right] = \rho_B \quad (\texttt{given})$$

Hadron-quark droplet



Rearrangement of charge

Quark phase is negatively charged.

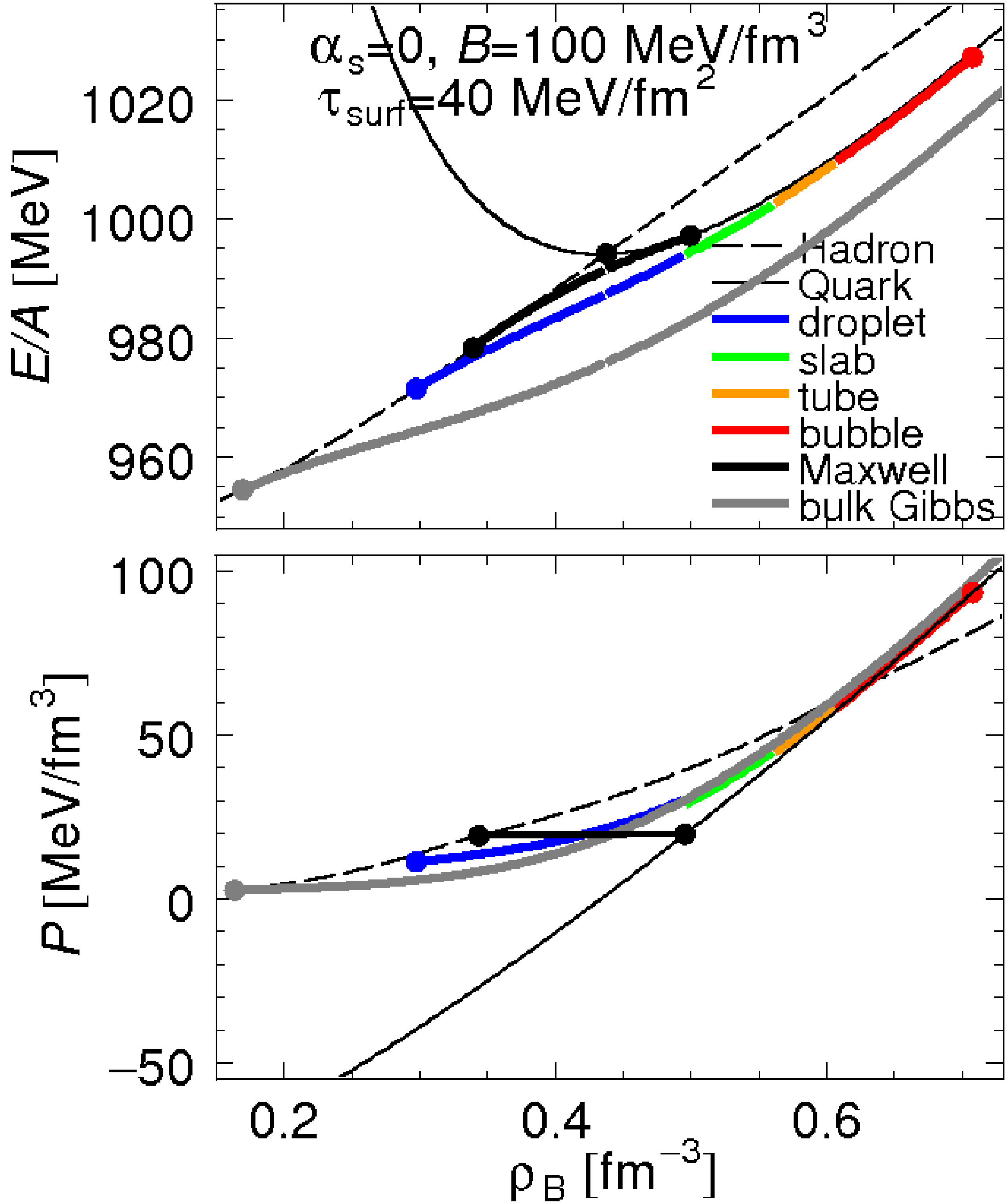
→ *u* quarks are attracted and *ds* quarks repelled.

Same happens to *p* in the hadron phase.

Localization of *e* into hadron phase.

EOS of matter

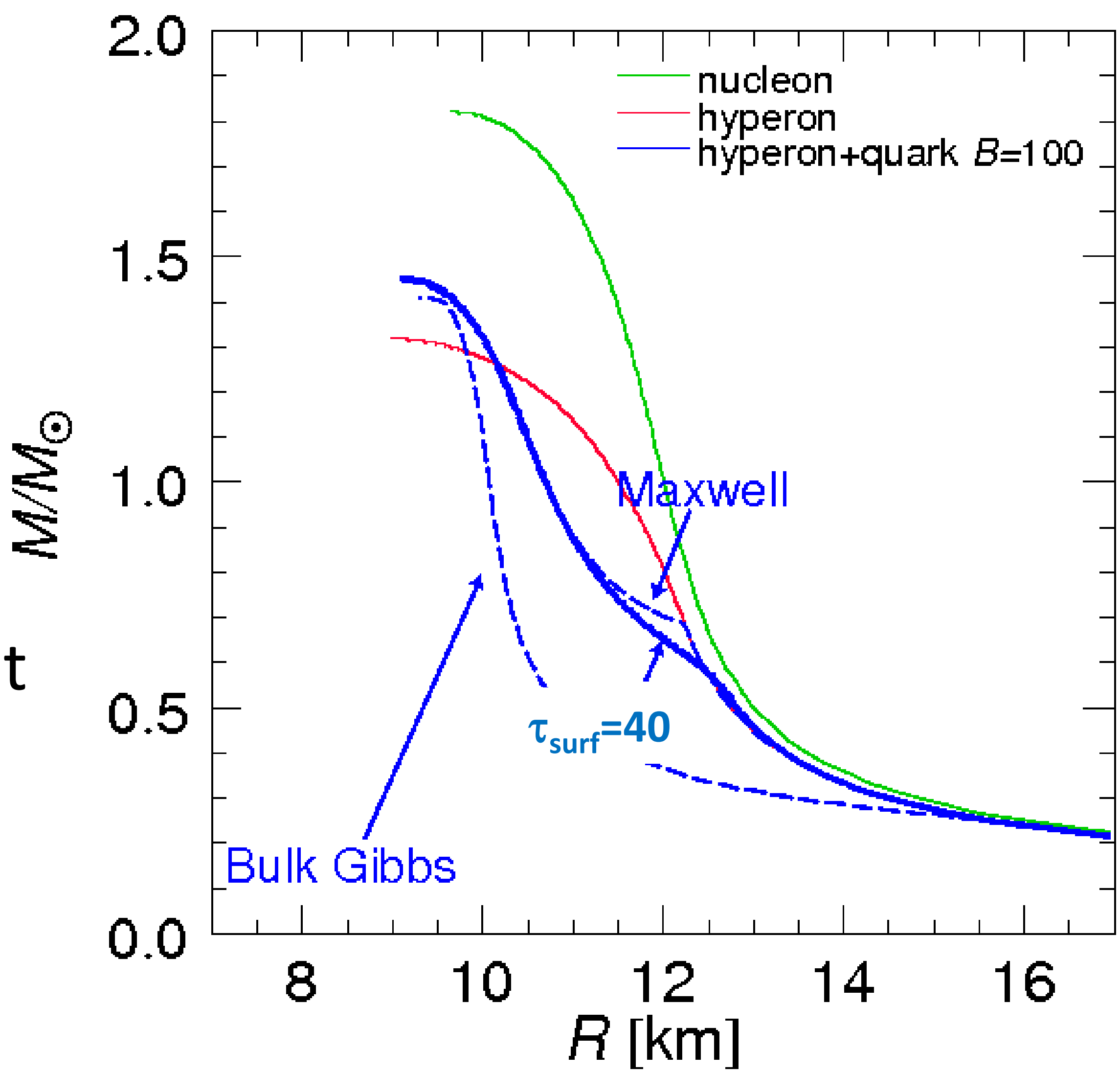
Full calculation is close to the Maxwell construction (local charge neutral).
Far from the bulk Gibbs calculation (neglects the surface and Coulomb).



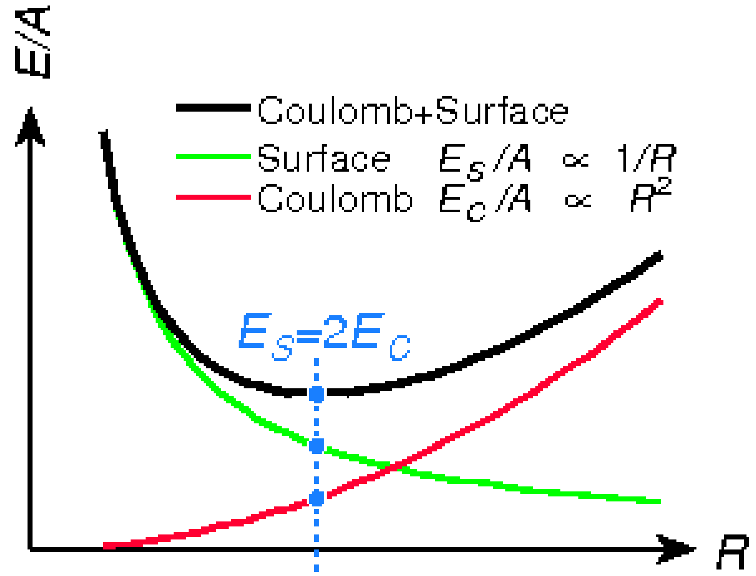
Mass-Radius relation of compact stars

Full calc yields the neutron star mass very close to that of the Maxwell constr.

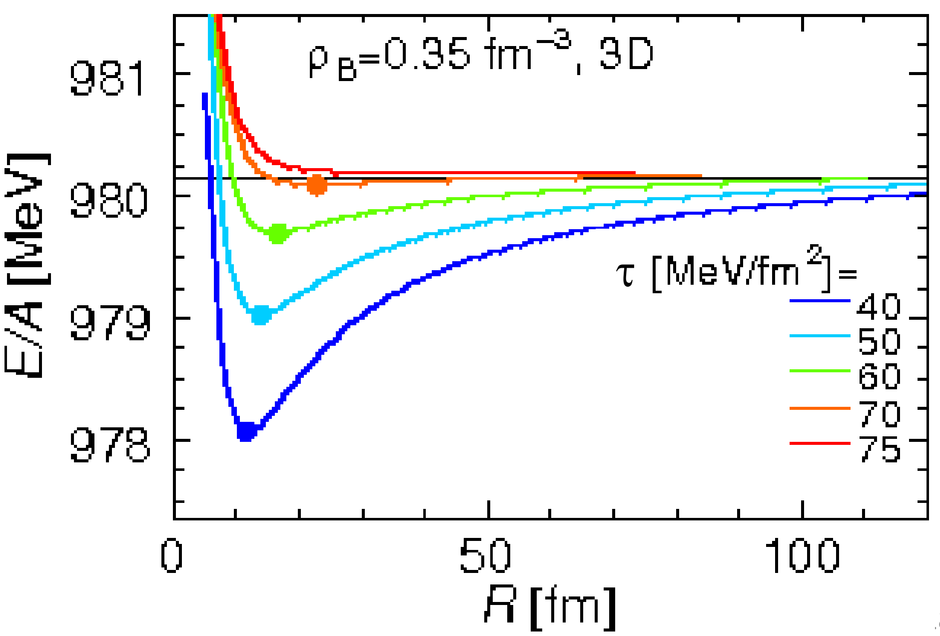
The maximum mass are not very different for three cases.



R and τ dependence of E/A.



Large τ → large R
Unstable for very largle τ



Equation of state of mixed phase

- **Maxwell construction** :

Assumes local charge neutrality (violates the Gibbs cond.) and neglects surface tension.

- **Bulk Gibbs calculation** :

Respects the balances of μ_i between 2 phases but neglects surface tension and the Coulomb interaction.

- **Full calculation**: includes everything.

Strong surface → Large R,
charge-screening → effectively
local charge neutral.
→ close to the Maxwell construction.

Weak surface → Small R → Coulomb ineffective
→ close to the bulk Gibbs calc.

Summary

- We have studied ``Pasta'' structures of low density matter (liquid-gas mixed phase) and high density matter (meson condensation and hadron-quark mixed phase).
- Coulomb screening and stronger surface tension enlarges the structure size.
- If surface tension is strong, Maxwell construction is effectively valid.