Envelopes of neutron stars with strong magnetic fields

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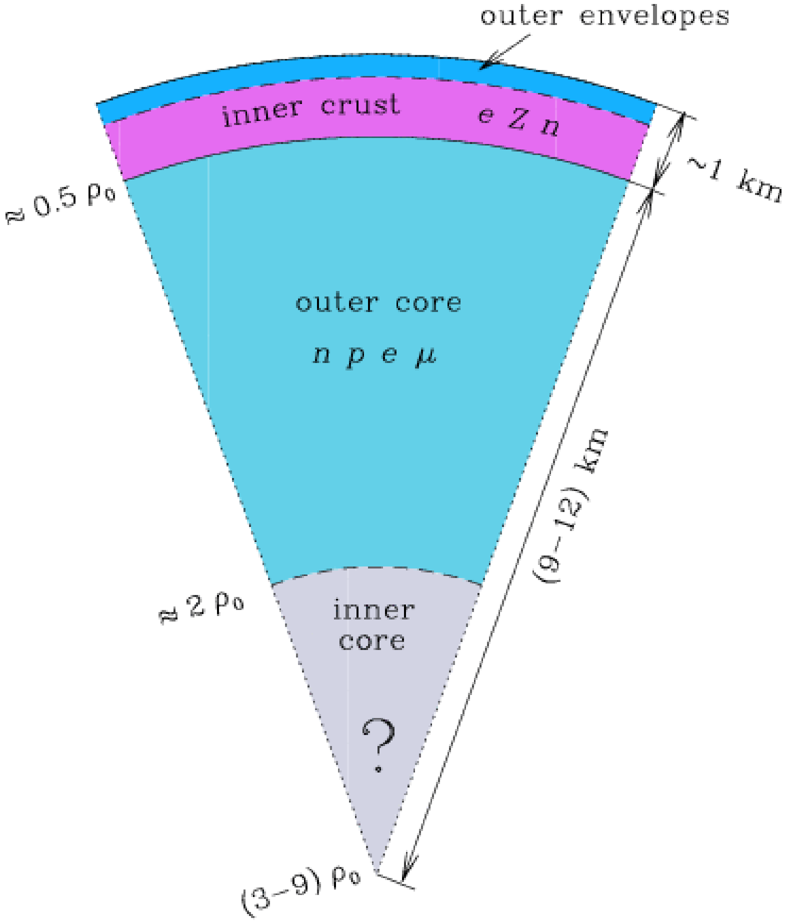
- ***Introduction***: neutron star structure and cooling; the importance of ***envelopes***
- The effects of ***magnetic fields*** on the electron heat conduction and cooling
- Challenges from ***superstrong*** magnetic fields
- ***Magnetars***: their thermal evolution and energy balance

Neutron stars – the densest stars in the Universe

Mass and radius: $M \sim 1.4\, M_\odot, \quad R \sim 10 \text{ km}$

$$(M_\odot = 1.989 \times 10^{33} \text{ g}, \quad R_\odot = 6.96 \times 10^5 \text{ km})$$

Schematic neutron star structure

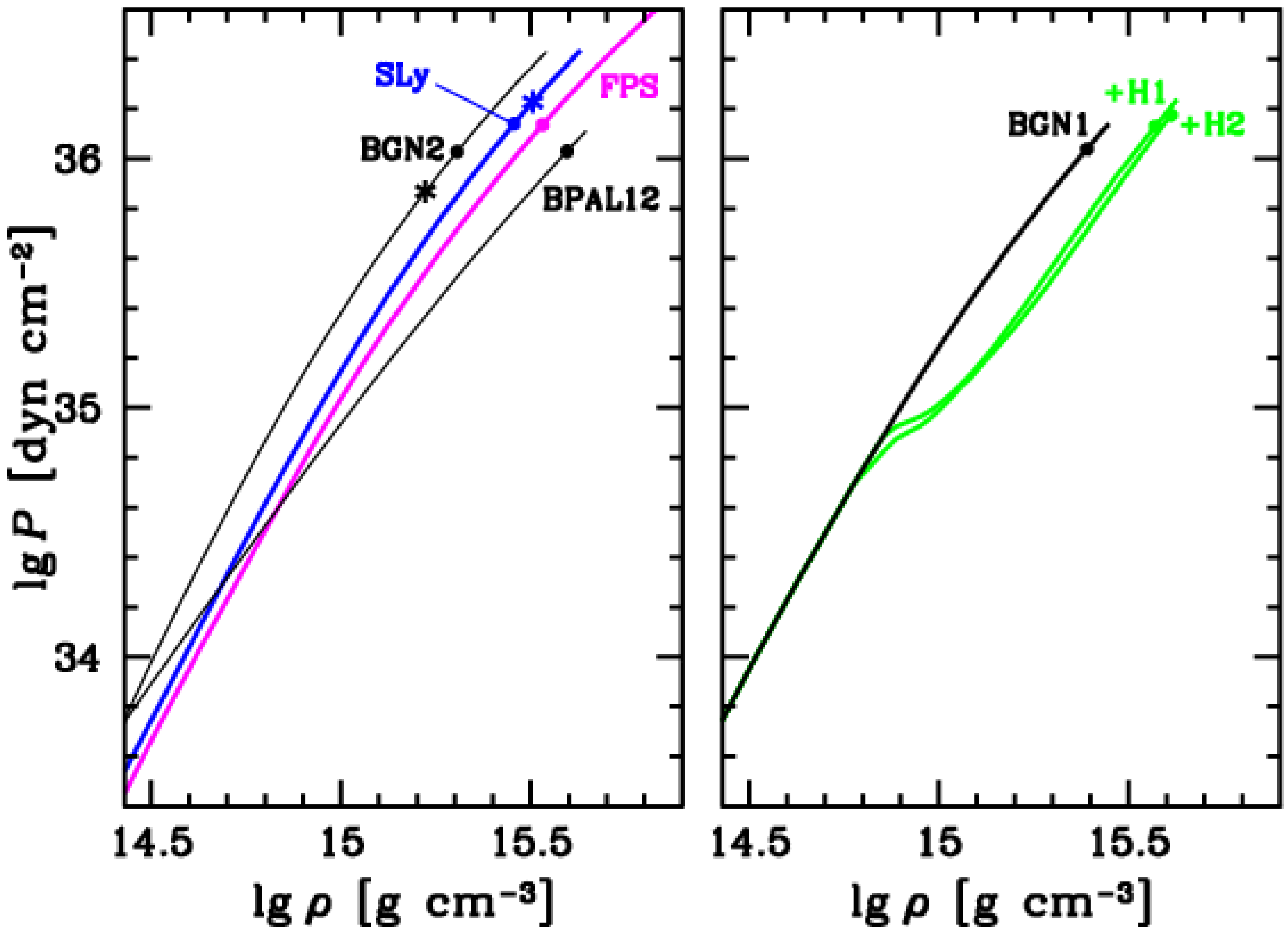


Gravitational acceleration:

$$g \sim GM/R^2 \sim 2 \times 10^{14} \text{ cm s}^{-2},$$

Average density:

$$\bar{\rho} \simeq 3M/(4\pi R^3) \simeq 7 \times 10^{14} \text{ g cm}^{-3}$$
$$\sim (2-3)\,\rho_0$$
$$(\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3})$$

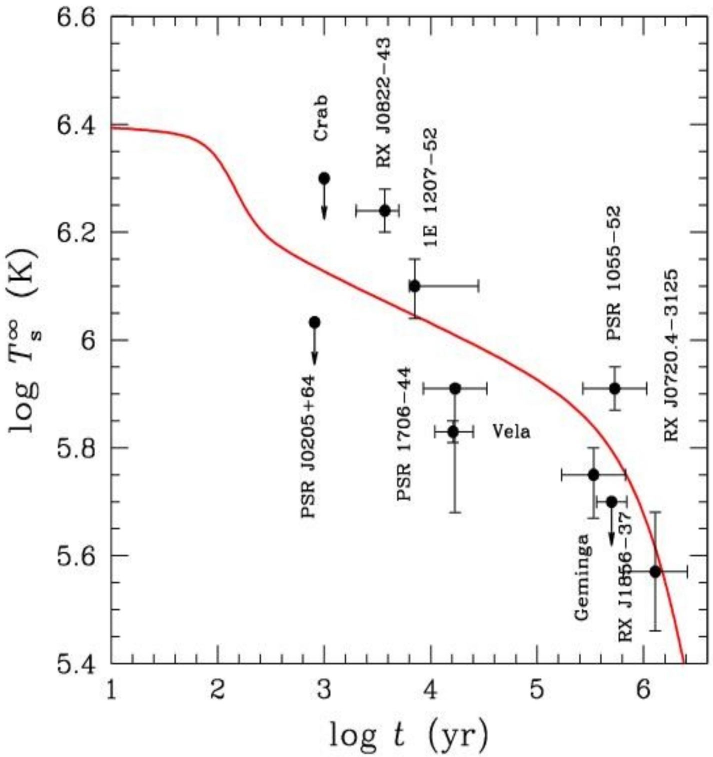


Examples of EOSs for the neutron star core.
Dots – stellar stability limit, asterisks – causal limit
(i.e., where speed of sound = speed of light).

[**Haensel, Potekhin, & Yakovlev,** ***Neutron Stars. 1. Equation of State and Structure***
(Springer, New York, 2007)]

Thermal evolution

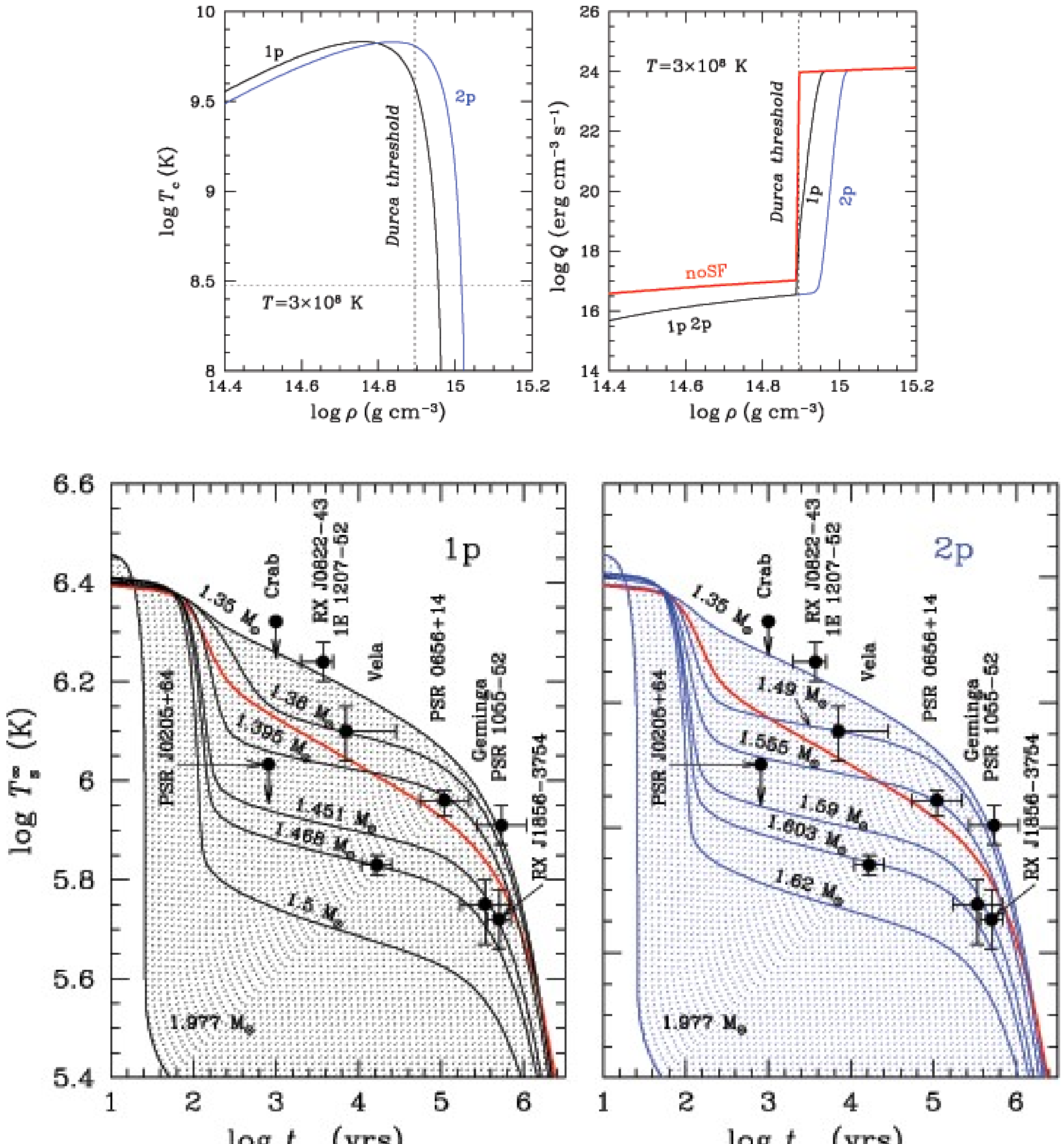
“Basic cooling curve” of a neutron star (no superfluidity, no exotica)



Neutron star cooling

[Yakovlev *et al*. (2005) *Nucl. Phys. A* **752**, 590c]

Cooling of neutron stars with proton superfluidity in the cores



The role and importance of the envelopes

➢ Relation between ***internal*** (core) temperature and ***effective temperature*** (surface luminosity)

- requires studying **thermal conduction** and **temperature profiles** in heat-blanketing envelopes

➢ Knowledge of the shape and features of the ***radiation spectrum*** at given effective temperature

- requires modeling neutron star **surface layers** and propagation of electromagnetic radiation in them

Solution of both problems relies on
modeling thermodynamic and kinetic properties
of ***outer neutron-star envelopes*** –
dense, strongly magnetized plasmas

Magnetic field affects thermodynamics properties and the heat conduction of the plasma, as well as radiative opacities

- Strong magnetic field B :

$$\hbar\omega_c = \hbar eB/m_c c > 1 \text{ a.u.}$$

$$B > m_e^2 c e^3/\hbar^3 = 2.35 \times 10^9 \text{ G}$$

- Superstrong field :

$$\hbar\omega_c > m_c c^2$$

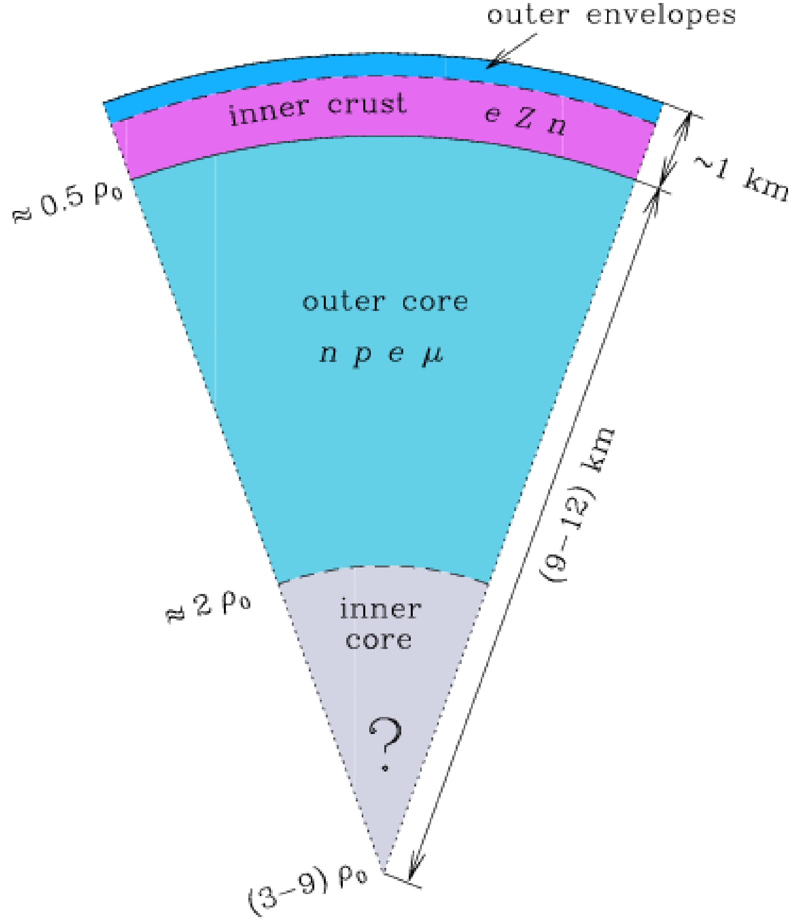
$$B > m_e^2 c^3/e\hbar = 4.4 \times 10^{13} \text{ G}$$

- Strongly quantizing :

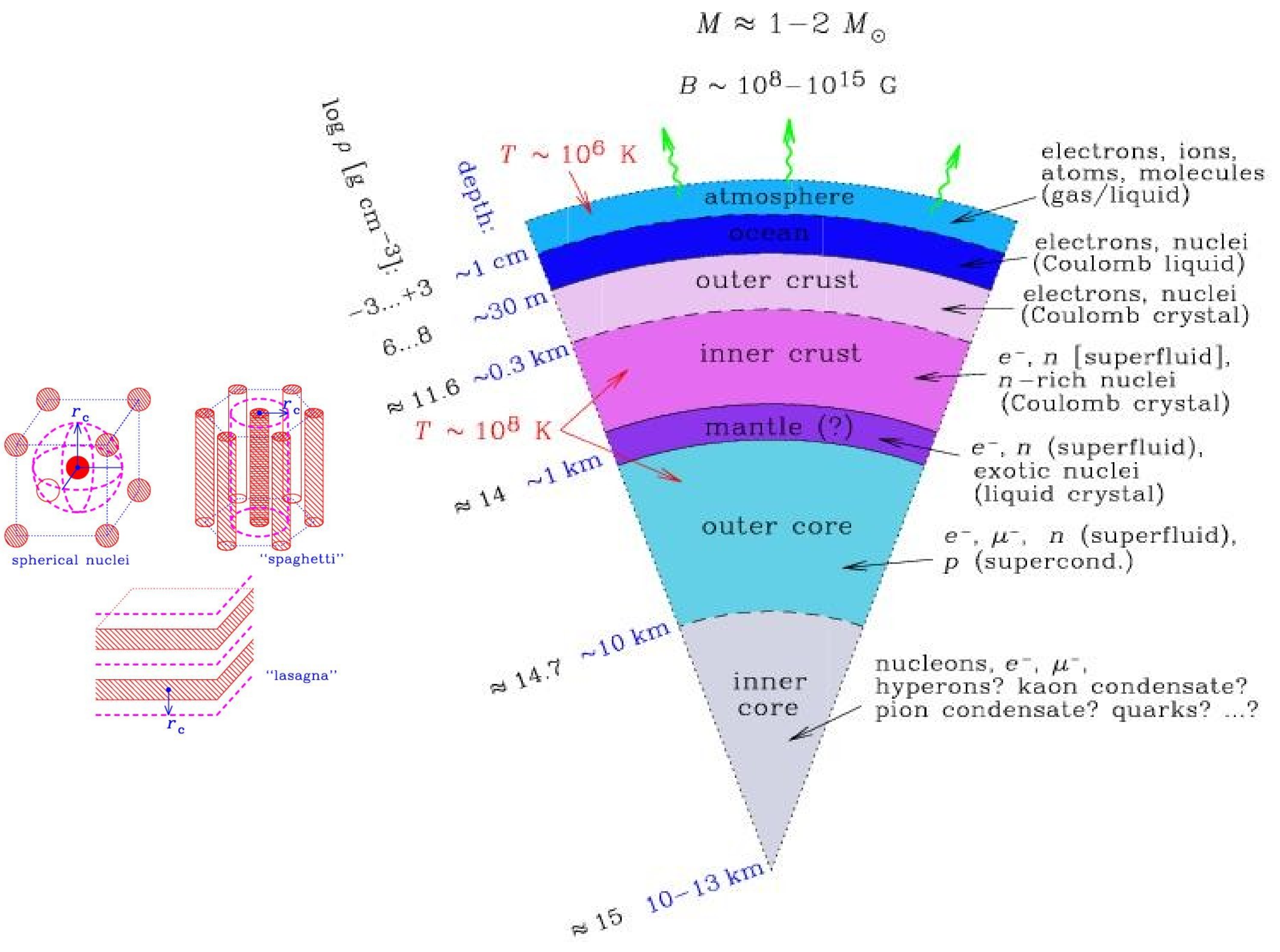
$$\rho < \rho_B = m_{\text{ion}} n_B \langle A\rangle/\langle Z\rangle \approx 7 \times 10^3 B_{12}^{3/2} (\langle A\rangle/\langle Z\rangle) \text{ g cm}^{-3}$$

$$T << T_B = \hbar\omega_c / k_B \approx 1.3 \times 10^8 B_{12} \text{ K}$$

Schematic neutron star structure

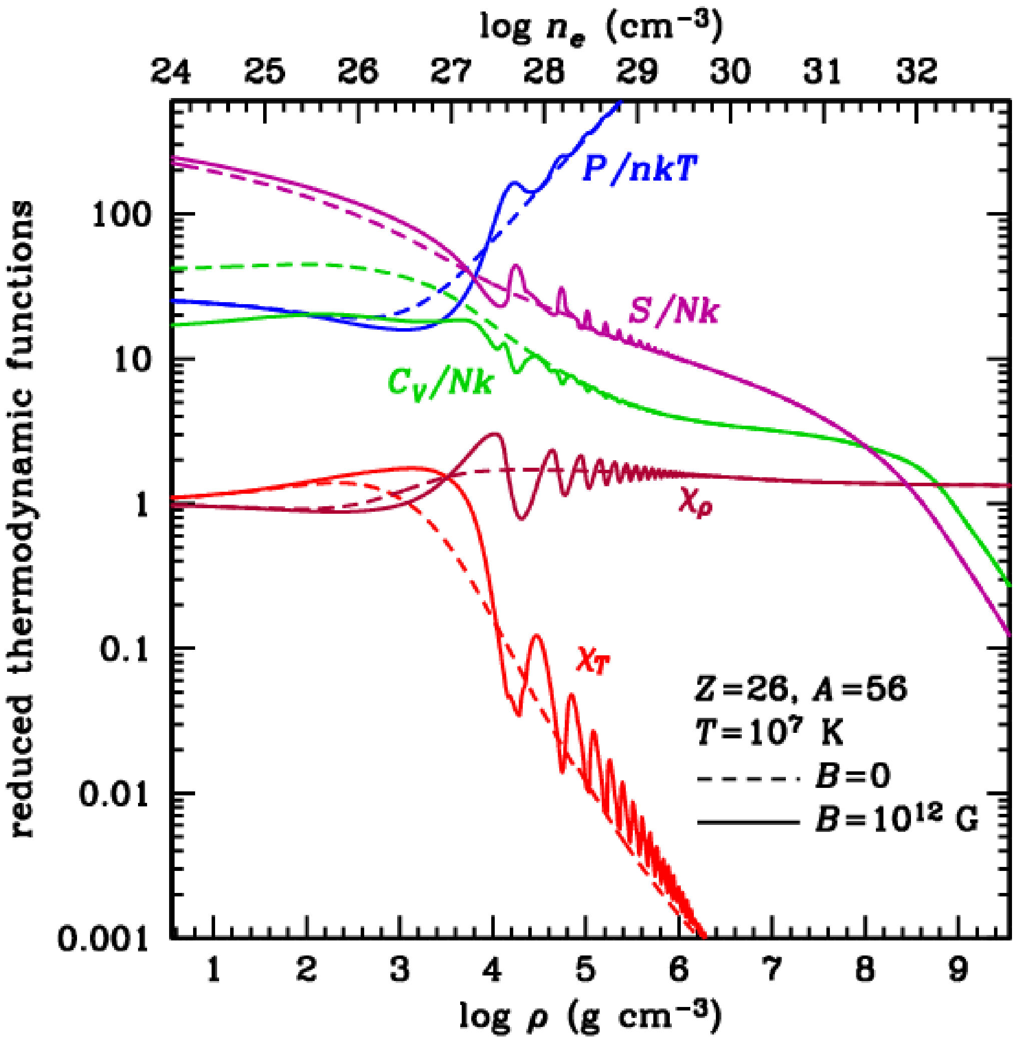


Neutron star structure in more detail



Blanketing envelopes of neutron stars

Plasma in quantizing magnetic field: Thermodynamic functions



Stellar heat conductivities
Basic data sources

W.B.Hubbard & M.Lampe (1966 – 1969)	***ei+ee*** degenerate and non-degenerate electrons; non-relativistic, classical ions. Tables for H, He, C and a few mixtures.
N.Itoh *et al*. (1976 – 1994)	***ei***, strongly degenerate electrons (arbitrary relativity), strongly coupled ions. Inaccurate treatment near the liquid/solid phase boundary.
D.G.Yakovlev et al. (1980 – 2001)	***ei***: (i) **liquid**: classical ions (strongly and weakly coupled) with a good structure factor; non-Born correction; (ii) **solid**: quantum treatment, account of multi-phonon processes. Allowance for **strong magnetic fields**. ***ee***: strongly degenerate electrons; inaccurate treatment at relativistic densities.
P.S.Shternin & D.G.Yakovlev (2006) S.Cassisi et al. (2007) A. Chugunov & P. Haensel (2007)	***ee***: improved at relativistic densities. ***ee***: extension to arbitrary degeneracy. ***Ie, ii***: ion thermal conduction.

http://www.ioffe.ru/astro/conduct/

Blanketing envelopes of neutron stars

Thermal conductivity

Basic estimates for thermal conductivities

In the "elementary theory" (with energy-independent effective frequency)

$$\varkappa = a\,\frac{n_e k^2 T}{m_e^* \nu}, \quad a = \left[\begin{array}{ll} 3/2 & (T \gg T_\mathrm{F}) \\ \pi^2/3 & (T \ll T_\mathrm{F}) \end{array}\right.$$

$$m_e^* = m_e \gamma_\mathrm{r}, \quad \gamma_\mathrm{r} = \sqrt{1 + x_\mathrm{r}^2}, \qquad x_\mathrm{r} = p_\mathrm{F}/m_e c = 0.01009\,(\rho Z/A)^{1/3}$$

$$T_\mathrm{F} = \frac{m_e c^2}{k}(\gamma_\mathrm{r} - 1) \qquad \left(\frac{m_e c^2}{k} = 5.93 \times 10^9\ \mathrm{K}\right)$$

Matthiessen rule: $\nu = \nu_{ei} + \nu_{ee}$

$$\nu_{ei} + \nu_{ee} \le \nu \le \nu_{ei} + \nu_{ee} + \delta\nu, \quad \delta\nu \ll \min(\nu_{ei}, \nu_{ee})$$

For non-degenerate electron gas:

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\,\frac{Z^2 e^4}{(kT)^{3/2}}\,n_i \Lambda_{ei}, \quad \Lambda_{ei} \sim \ln\frac{r_\mathrm{max}}{r_\mathrm{min}}$$

$$r_\mathrm{max}^{-2} = 4\pi(n_e + Z^2 n_i)e^2/kT, \quad r_\mathrm{min} = \max(\lambda_T,\, r_\mathrm{cl}), \qquad \lambda_T = \sqrt{\frac{2\pi\hbar^2}{m_e kT}}, \quad r_\mathrm{cl} = \frac{Ze^2}{kT}$$

$$\nu_{ee} = \frac{8}{3}\sqrt{\frac{\pi}{m_e}}\,\frac{e^4}{(kT)^{3/2}}\,n_e \Lambda_{ee}$$

For strongly degenerate electron gas:

Electron-ion scattering

[Potekhin, Baiko, Haensel, Yakovlev (1999) *A&A*, **346**, 345]

$$\nu_{ei} = \frac{4\pi Z_i^2 e^4}{p_\mathrm{F}^2 v_\mathrm{F}} n_i \Lambda_{ei} \qquad v_\mathrm{F} = \frac{p_\mathrm{F}}{m_e^*} = c\,\frac{x_\mathrm{r}}{\gamma_\mathrm{r}} = c\,\beta$$

Electron-electron scattering

[Shternin & Yakovlev (2006) *PRD*, **74**, 043004]

$$kT_\mathrm{p} = \hbar\omega_\mathrm{p} = \hbar\sqrt{4\pi e^2 n_e / m_e^*} \qquad y = \sqrt{3}\,T_\mathrm{p}/T = (571.6/T_6)\sqrt{\beta}\,x_\mathrm{r}$$

$$\nu_{ee} = \frac{m_e c^2}{\hbar}\frac{6\alpha_\mathrm{f}^{3/2}}{\pi^{5/2}} x_\mathrm{r} y \sqrt{\beta}\, I(\beta, y) = 1.66 \times 10^{17} x_\mathrm{r} y \sqrt{\beta}\, I(\beta, y)\ \mathrm{s}^{-1}$$

$$\begin{aligned} I(\beta, y) = {} & \frac{1}{\beta}\left(\frac{10}{63} - \frac{8/315}{1 + 0.0435y}\right)\ln\left(1 + \frac{128.56}{37.1y + 10.83y^2 + y^3}\right) \\ & + \beta^3\left(\frac{2.404}{B} + \frac{C - 2.404/B}{1 + 0.1\beta y}\right)\ln\left[1 + \frac{B}{A\beta y + (\beta y)^2}\right] \\ & + \frac{\beta}{1 + D}\left(C + \frac{18.52\beta^2 D}{B}\right)\ln\left[1 + \frac{B}{Ay + 10.83(\beta y)^2 + (\beta y)^{8/3}}\right] \end{aligned}$$

$$A = 12.2 + 25.2\,\beta^3 \qquad C = 8/105 + 0.05714\,\beta^4$$

$$B = A\exp[(0.123636 + 0.016234\,\beta^2)/C] \qquad D = 0.1558\,y^{1-0.75\,\beta}$$

Partially degenerate electron gas in magnetic field

Electron-ion scattering in arbitrary magnetic field

[Potekhin (1999) *A&A*, **351**, 787]

$$\vec{j}_e = \sigma \cdot \vec{E}^* - \alpha \cdot \nabla T, \quad \vec{j}_T = \tilde{\alpha} \cdot \vec{E}^* - \tilde{\kappa} \cdot \nabla T, \qquad \vec{E}^* = \vec{E} + \nabla \mu / e$$

$$\tilde{\alpha}_{ij}(\boldsymbol{B}) = k^2 T \alpha_{ji}(-\boldsymbol{B}) = k^2 T \alpha_{ji}(\boldsymbol{B})$$

$$\boxed{\varkappa = \tilde{\kappa} + k^2 T \alpha \cdot \sigma^{-1} \cdot \alpha}$$

$$\begin{bmatrix} \sigma_{ij} \\ \alpha_{ij} \\ \tilde{\kappa}_{ij} \end{bmatrix} = \int \begin{bmatrix} e^2 \\ e(\mu - \epsilon)/T \\ (\mu - \epsilon)^2/T \end{bmatrix} \frac{\mathcal{N}_B(\epsilon)}{m_e^*(\epsilon)} \tau_{ij}(\epsilon) \left(-\frac{\partial f^{(0)}}{\partial \epsilon} \right) \mathrm{d}\epsilon \qquad \mathcal{N}_B(\epsilon) = \frac{m_e \omega_c}{2(\pi \hbar)^2} \sum_{n=0}^{n_{\max}} g_n p_n(\epsilon)$$

$$p_n(\epsilon) = [(\epsilon/c)^2 - (m_e c)^2 - 2 m_e \hbar \omega_c n]^{1/2}$$

$$\tau_{zz} = \tau_\parallel, \quad \tau_{xx} = \frac{\tau_\perp}{1 + (\omega_g \tau_\perp)^2}, \quad \tau_{yx} = \frac{\omega_g \tau_\perp^2}{1 + (\omega_g \tau_\perp)^2} \qquad n_e = \int \mathcal{N}_B(\epsilon) \left(-\frac{\partial f^{(0)}}{\partial \epsilon} \right) \mathrm{d}\epsilon$$

Particular case: no magnetic field

$$\boxed{\varkappa = k^2 T (\sigma_2 - \sigma_1^2 / \sigma_0)} \qquad \sigma_n = \int \frac{\chi^n}{\nu_{ei}(\epsilon)} \frac{\mathcal{N}_0(\epsilon)}{m_e^*(\epsilon)} \frac{e^\chi}{(e^\chi + 1)^2} \mathrm{d}\chi \qquad \chi = \frac{\epsilon - \mu}{kT}$$

$$\mathcal{N}_0(\epsilon) = p^3 / (3\pi^2 \hbar^3) \qquad m_e^*(\epsilon) = \sqrt{m_e^2 + (p/c)^2}$$

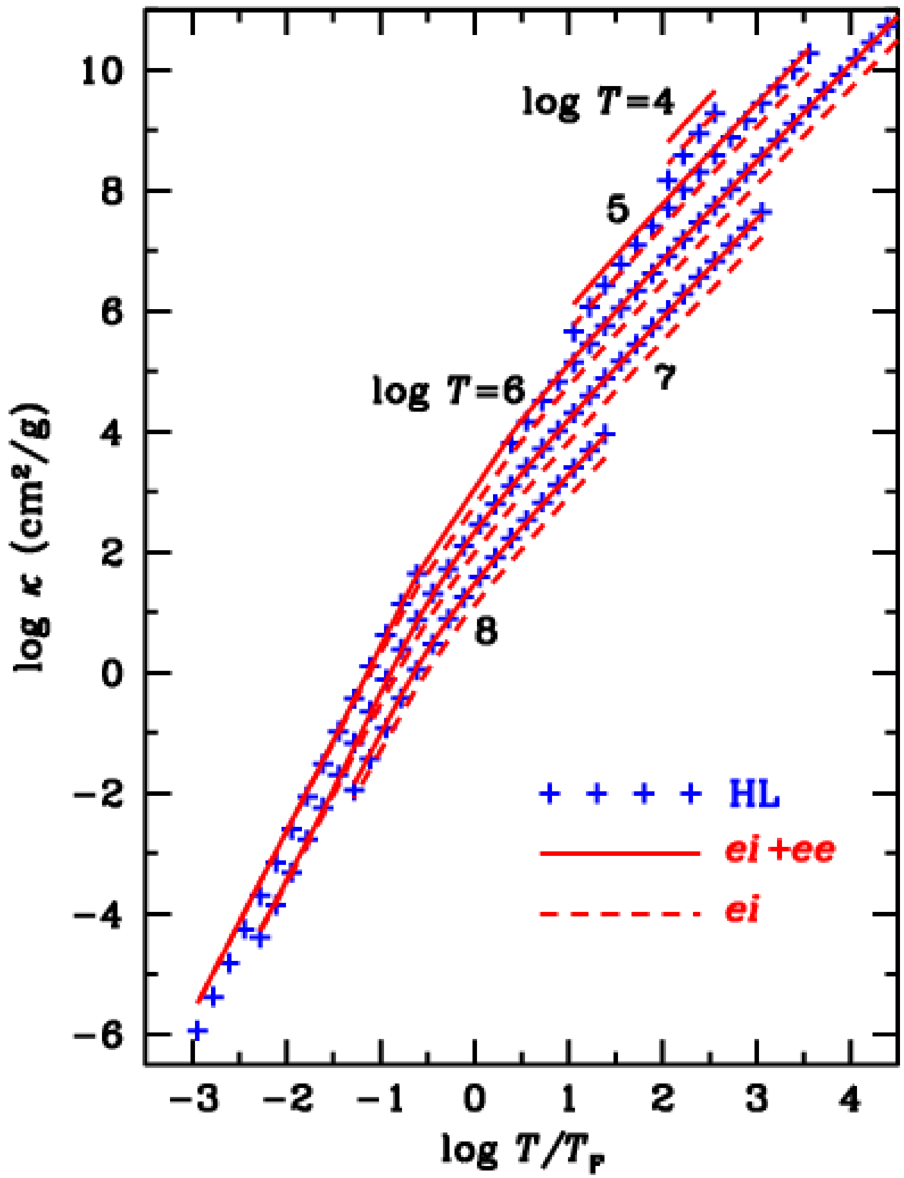
Electron-electron scattering: interpolation

[Cassisi, Potekhin, Pietrinferni, Catelan, & Salaris (2007) *ApJ* **661**, 1094]

$$\nu_{ee} = \nu_{ee}^{\rm deg} \frac{1+t^2}{1+t+bt^2\sqrt{T/T_{\rm F}}}$$

$$t = 25T/T_{\rm F}$$

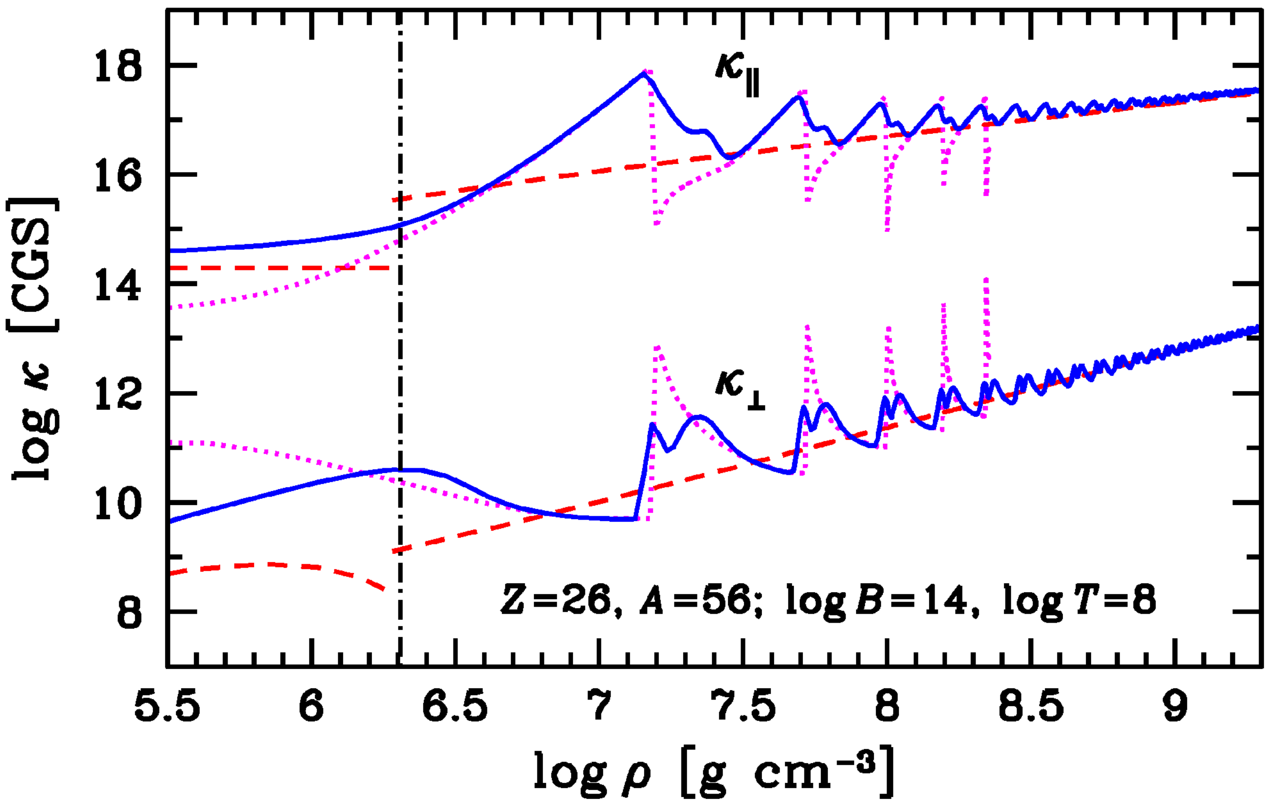
$$b = 135/\sqrt{32\pi^7} \approx 0.434$$



Conductive opacities for He as functions of degeneracy parameter compared to the tables (HL) of Hubbard & Lampe (1969) *ApJS*, **18**, 297

Blanketing envelopes of neutron stars

Plasma in quantizing magnetic field: Thermal conductivities



Solid – exact, dots – without *T*-integration, dashes – magnetically non-quantized

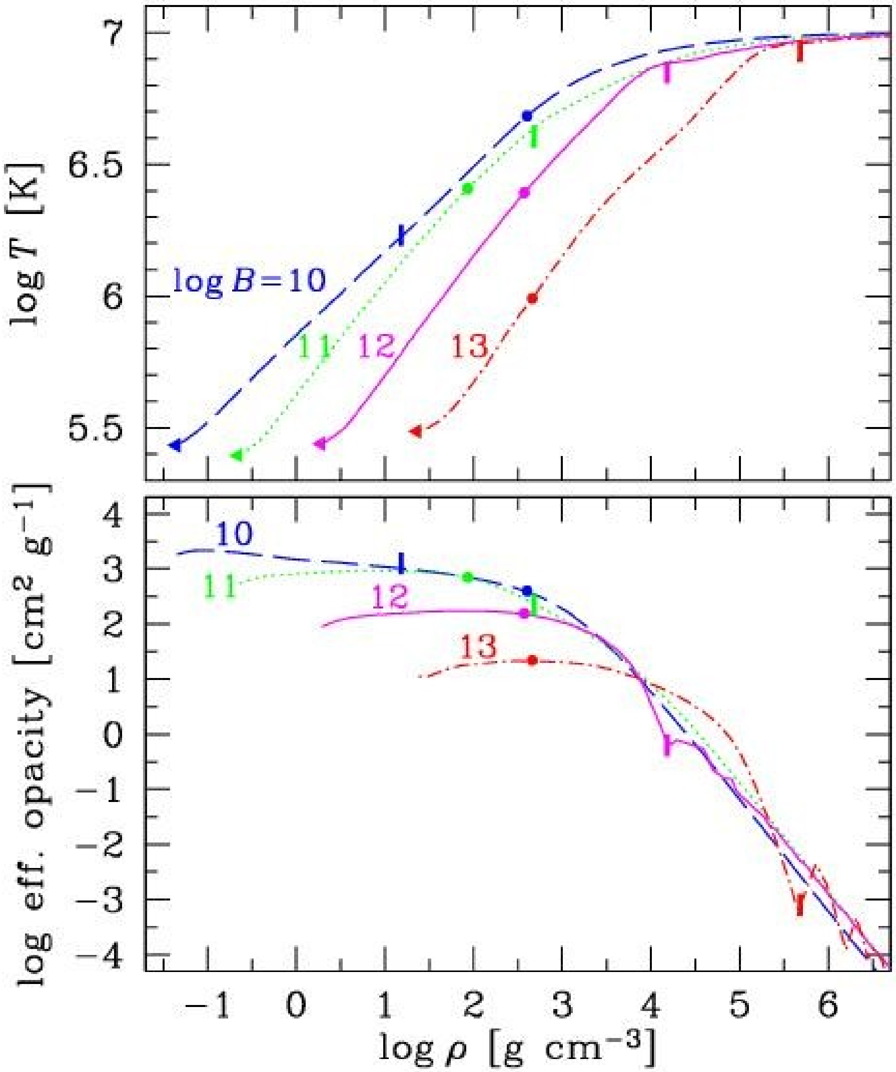
[Ventura & Potekhin (2001), in *The Neutron Star – Black Hole Connection*, ed. Kouveliotou *et al*. (Dordrecht: Kluwer) 393]

Summary and update: Cassisi, Potekhin, Pietrinferni, Catelan, & Salaris (2007) *ApJ* **661**, 1094

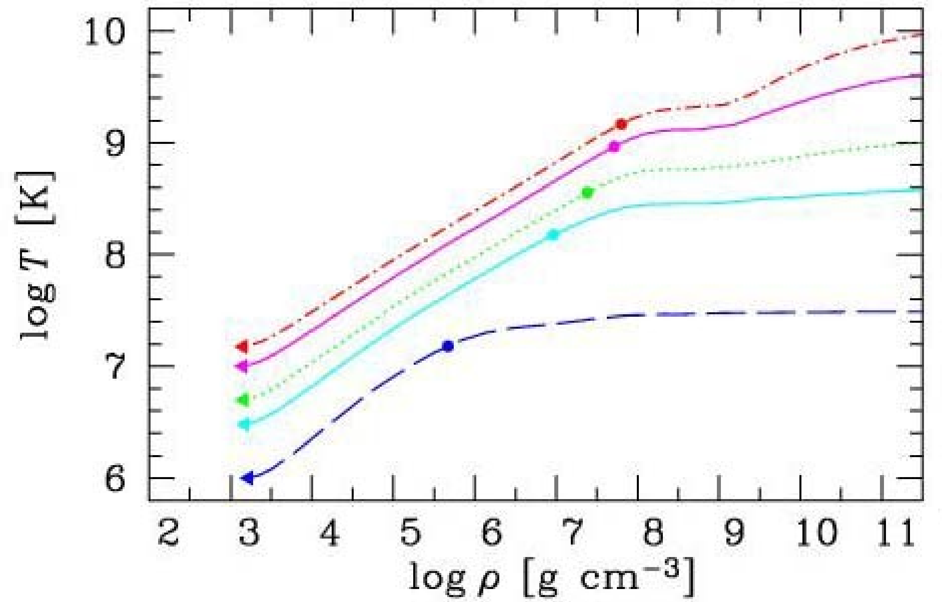
http://www.ioffe.ru/astro/conduct/

Blanketing envelopes of neutron stars

Thermal structure with magnetic field



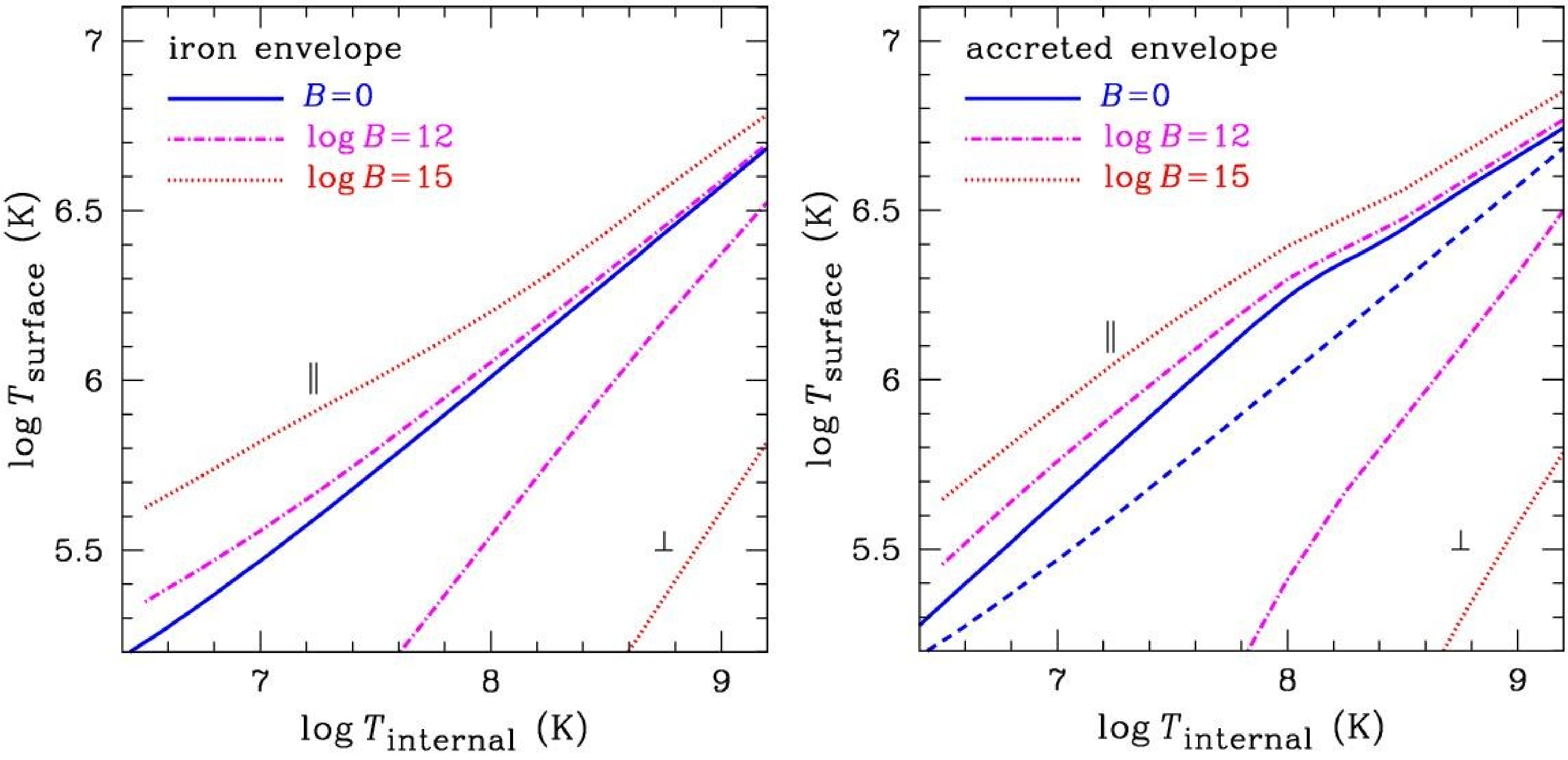
T_b=10^7 K, different B



B=10^{15} G, different T

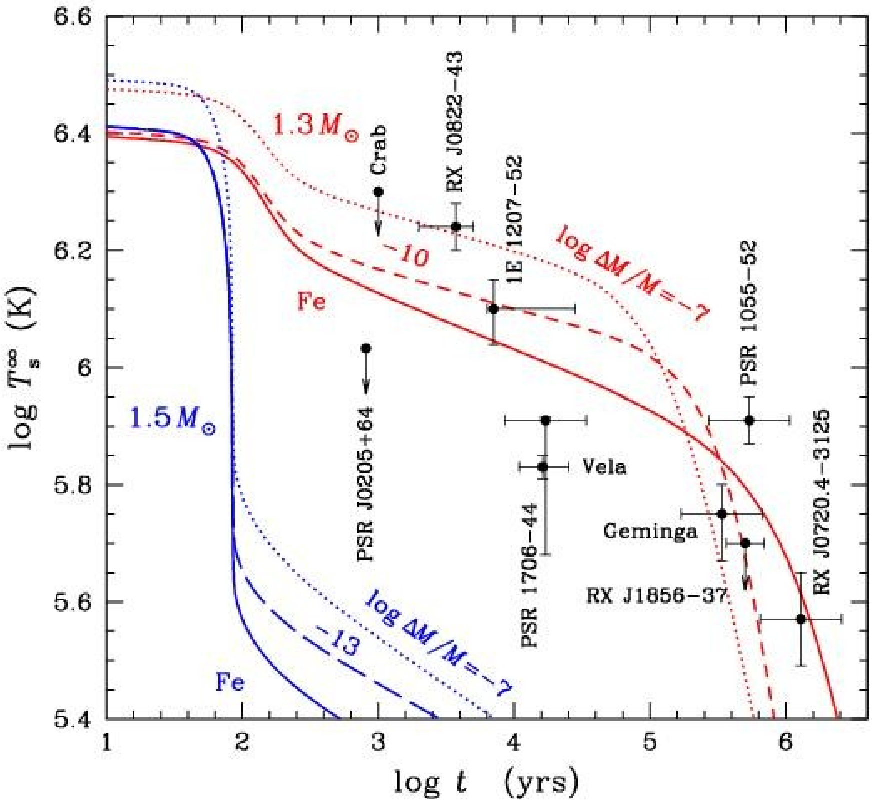
$T_s - T_b$

Temperature drops in magnetized envelopes of neutron stars

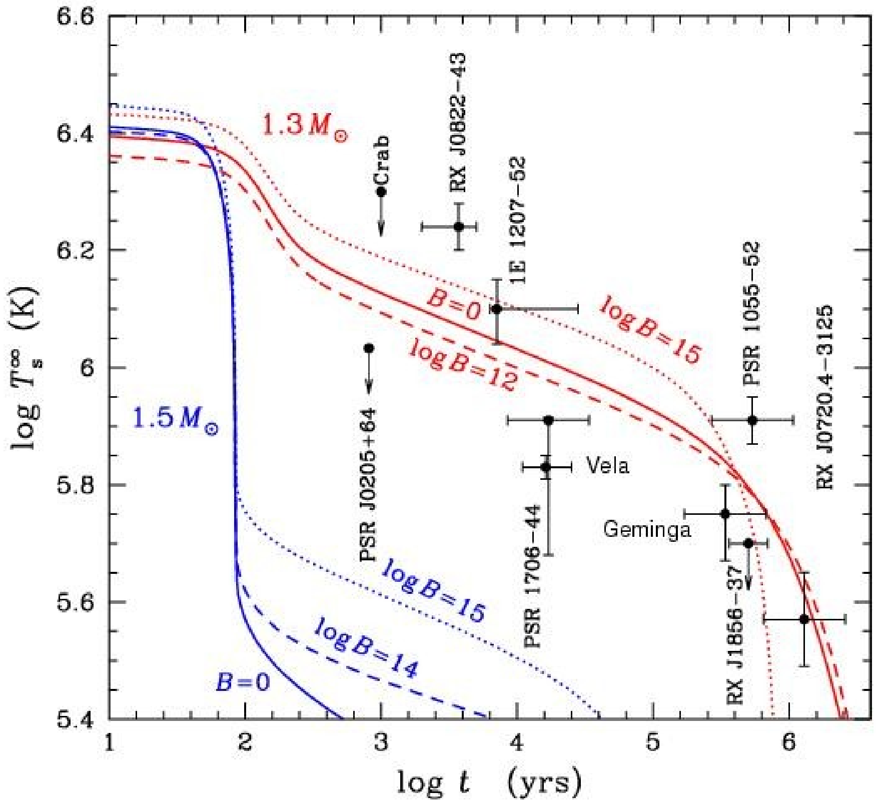


[based on Potekhin, Yakovlev, Chabrier, & Gnedin (2003) *ApJ* **594**, 404]

*Cooling of neutron stars with **accreted envelopes***

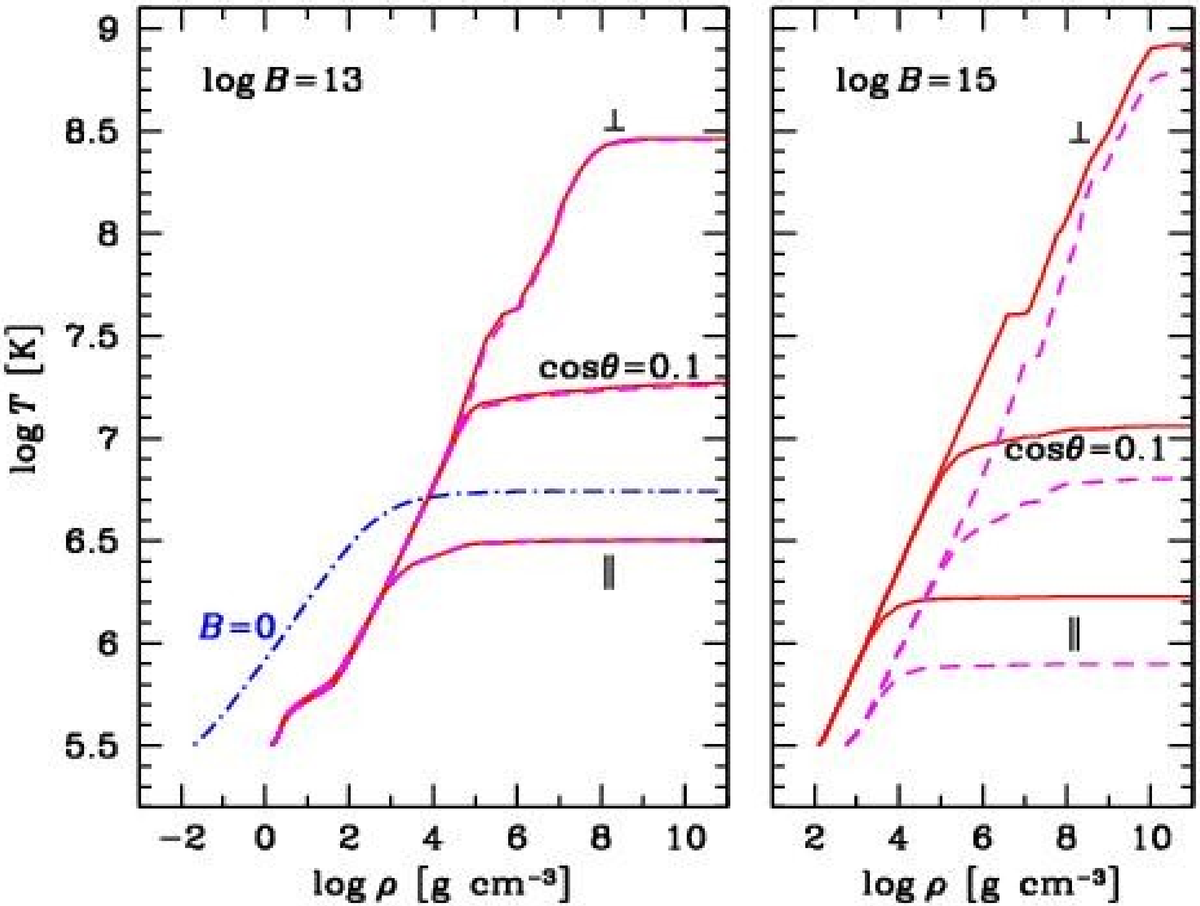


*Cooling of neutron stars with **magnetized envelopes***



[Chabrier, Saumon, & Potekhin (2006) *J.Phys.A: Math. Gen.* **39**, 4411; used data from Yakovlev *et al*. (2005) *Nucl. Phys. A* **752**, 590c]

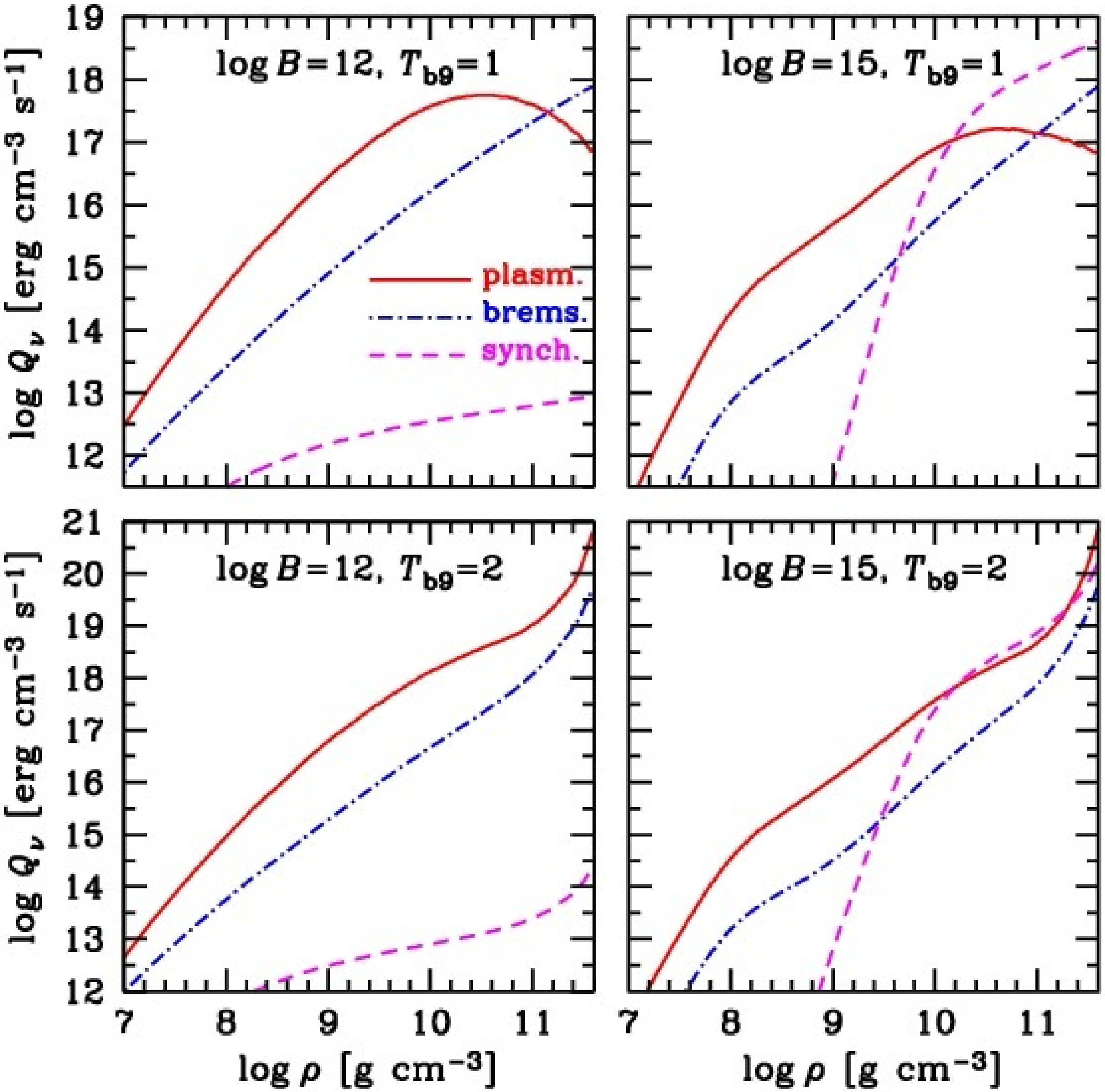
Superstrong fields: Energy transport below the plasma frequency may affect the temperature profile and T_s



Temperature profiles in the accreted envelope of a neutron star with "ordinary" (left panel) and superstrong (right) magnetic field, for the local effective temperature $10^{5.5}$ K, with (solid lines) and without (dashed lines) plasma-frequency cut-off

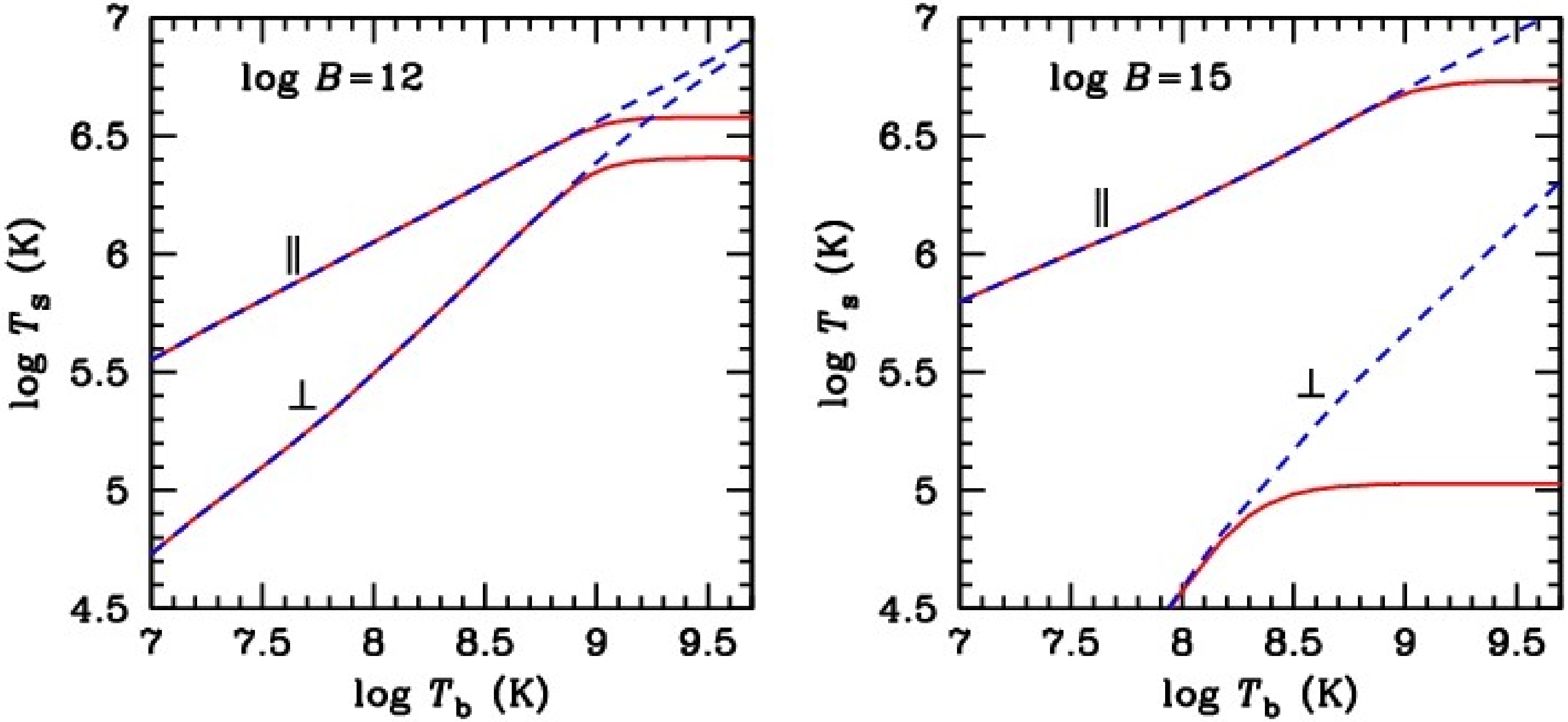
[Potekhin, Yakovlev, Chabrier, & Gnedin (2003) *ApJ* **594**, 404]

Neutrino emission rate in the outer crust



$T_s - T_b$

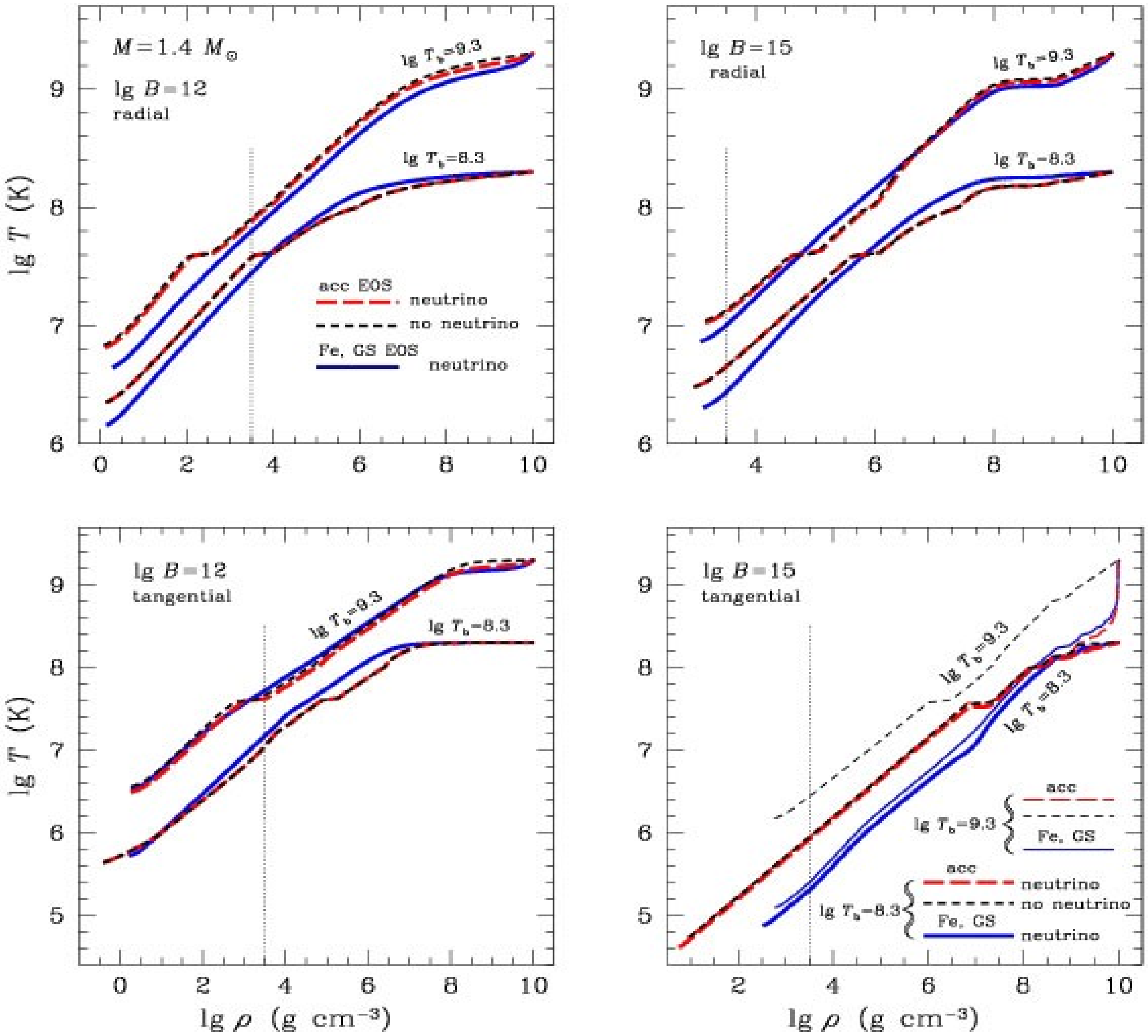
The effect of neutrino emission in the outer envelope



Effective temperature of the surface as a function of the internal temperature with account of the neutrino emission

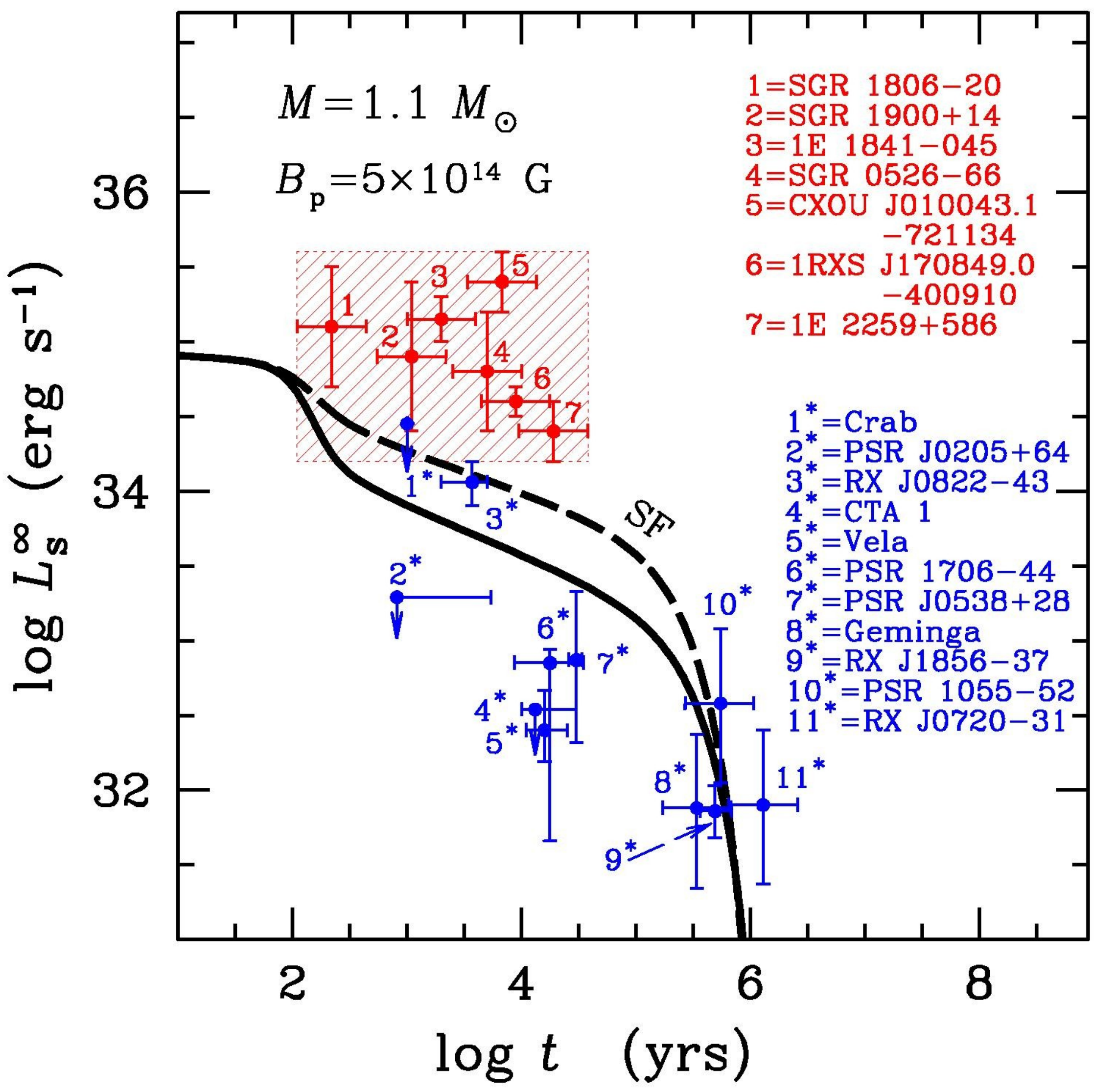
Temperature profiles in magnetized envelopes of neutron stars

The effects of neutrino emission, chemical composition, and magnetic fields



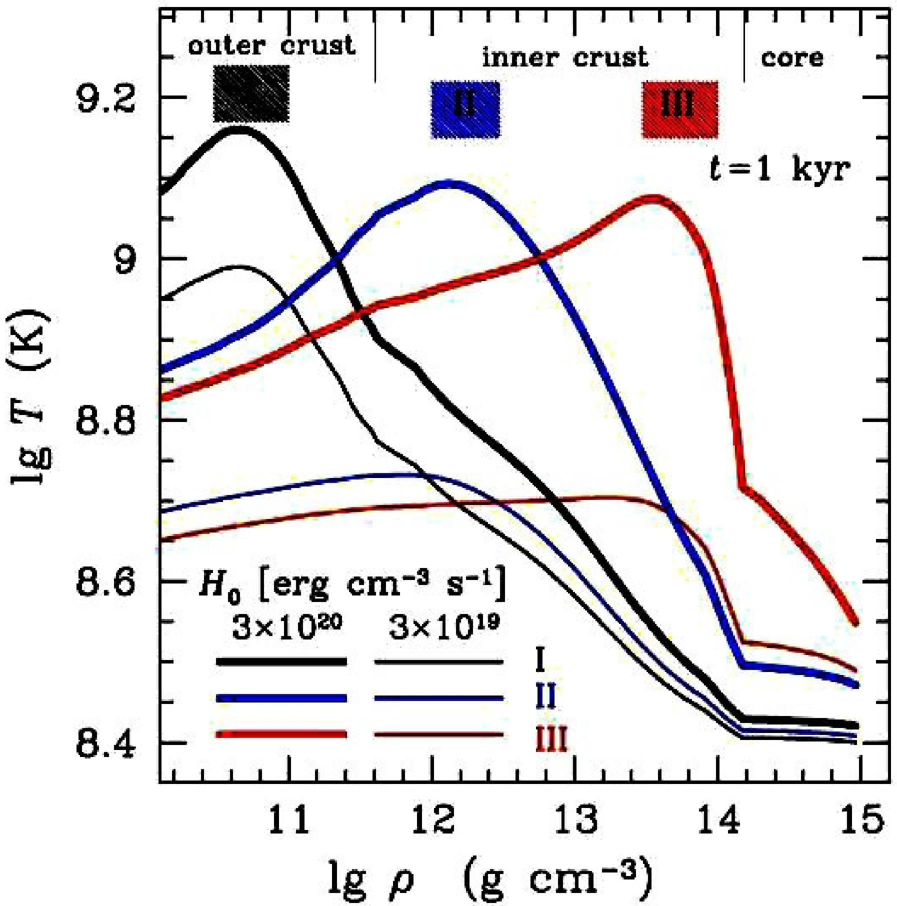
Magnetars versus ordinary neutron stars

The need for heating

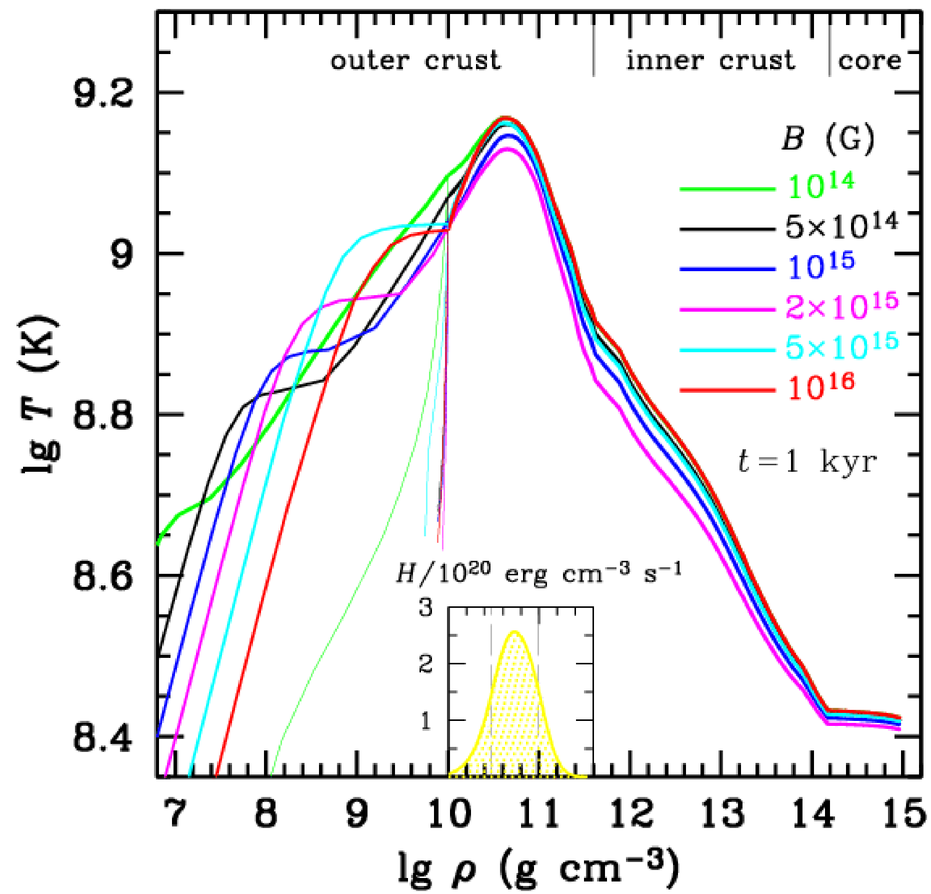


Test models of internal heating layers

Different layer positions and heating intensities

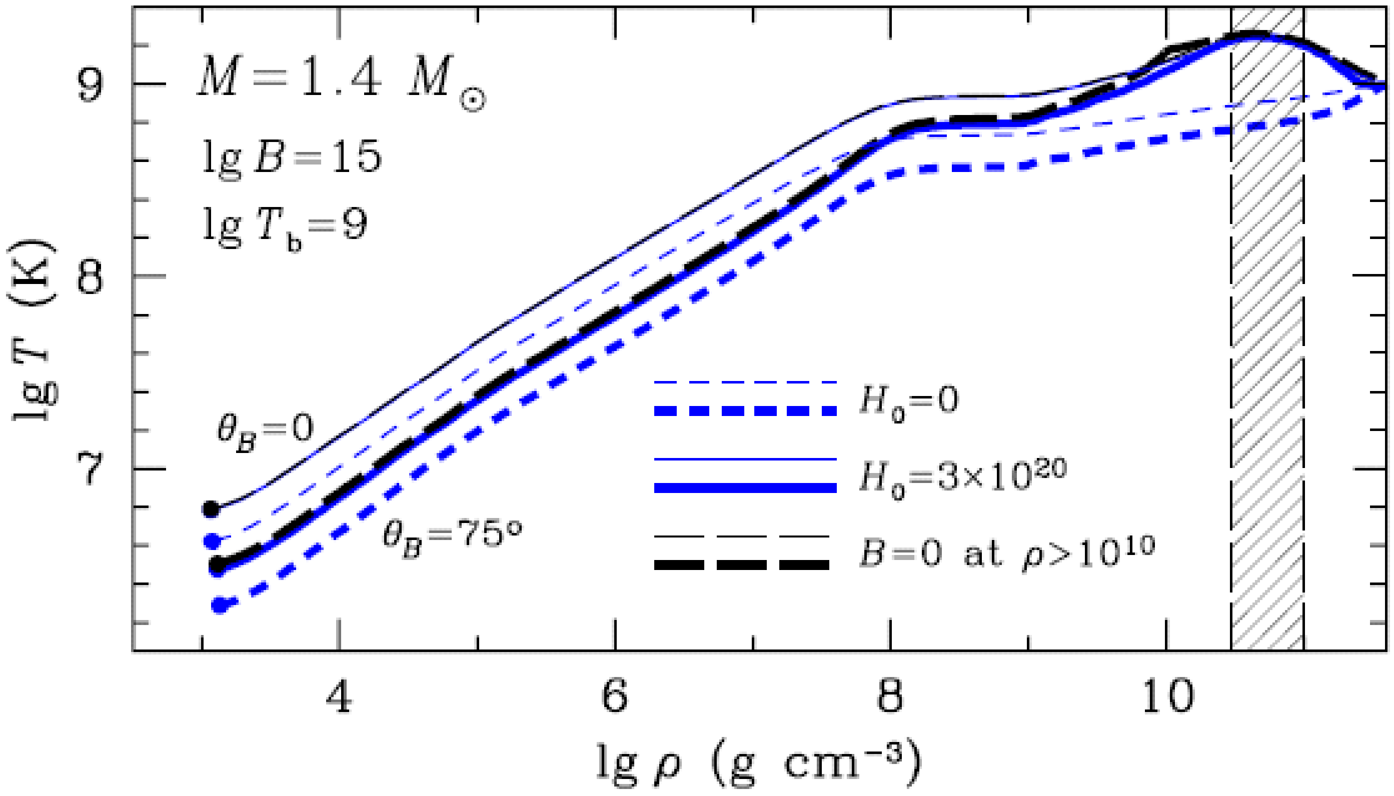


Different magnetic fields



Temperature profiles in magnetars

Thermal decoupling

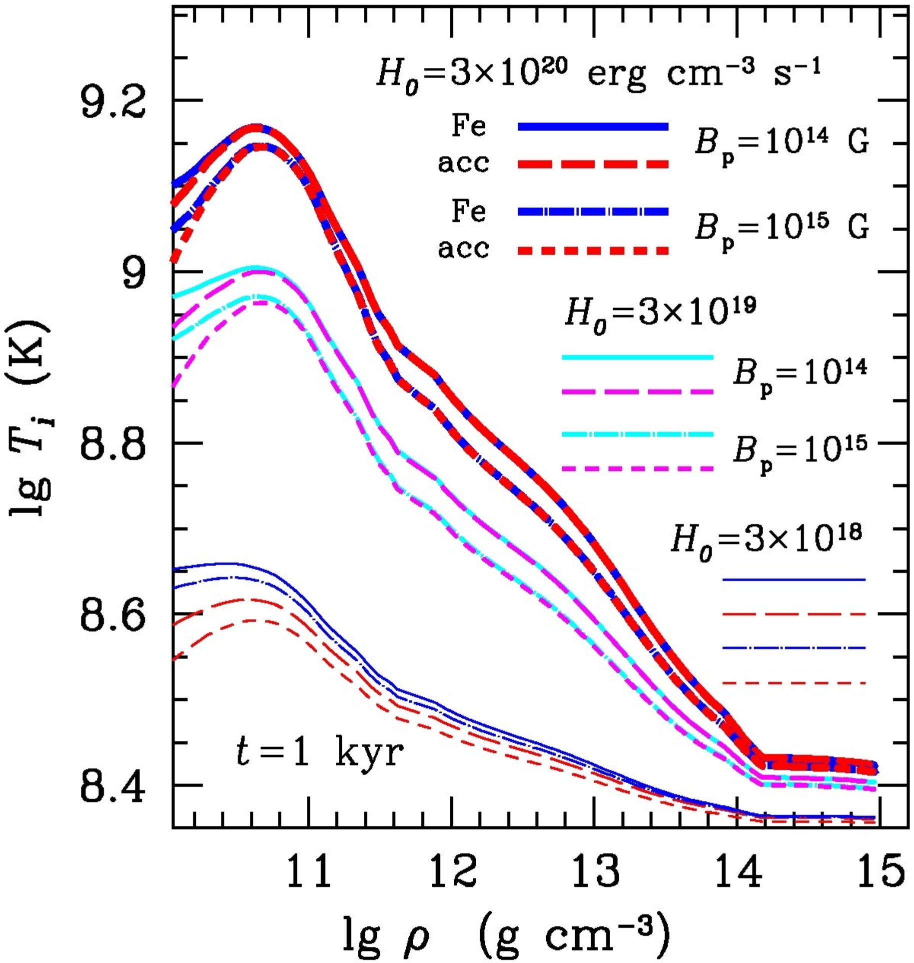


Temperature profiles within magnetars in different parts of the surface, with heating and without

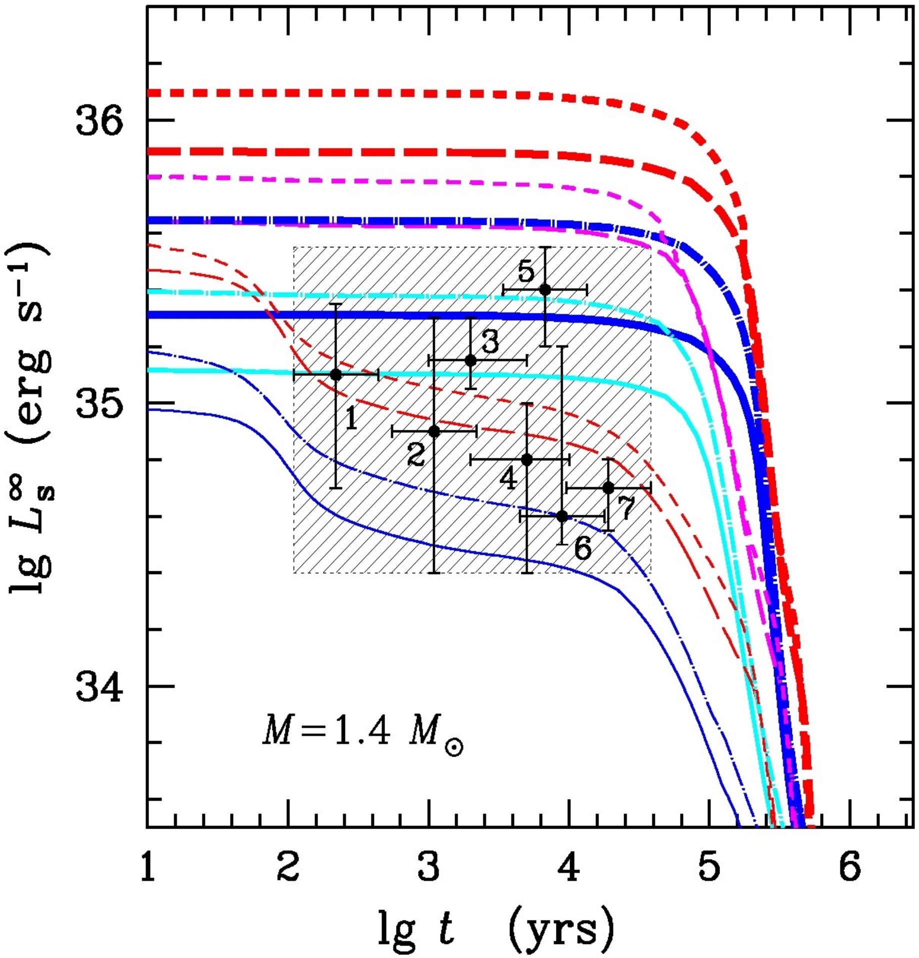
Thermal structure and cooling of magnetars

Different heating intensities, magnetic field strengths, envelope compositions

Thermal structure



Cooling curves



A.D.Kaminker, A.Y.Potekhin, D.G.Yakovlev, G.Chabrier, *Mon. Not. R. astr. Soc.* **395**, 2257 (2009)

ENERGY BUDGET AND HIGH THERMAL SURFACE LUMINOSITY

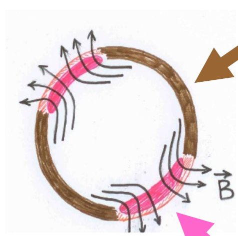
Models with spherical layer

- **Heating rate should not be too high; E_{max}~10^{50} erg ➔ W_{max}~3×10^{37} erg/s**
- **The heating rate must exceed the surface emission, optimal: L/W~0.01**
- **Photon surface luminosities should reach the magnetar level**

INDICATIONS OF HOT SPOTS

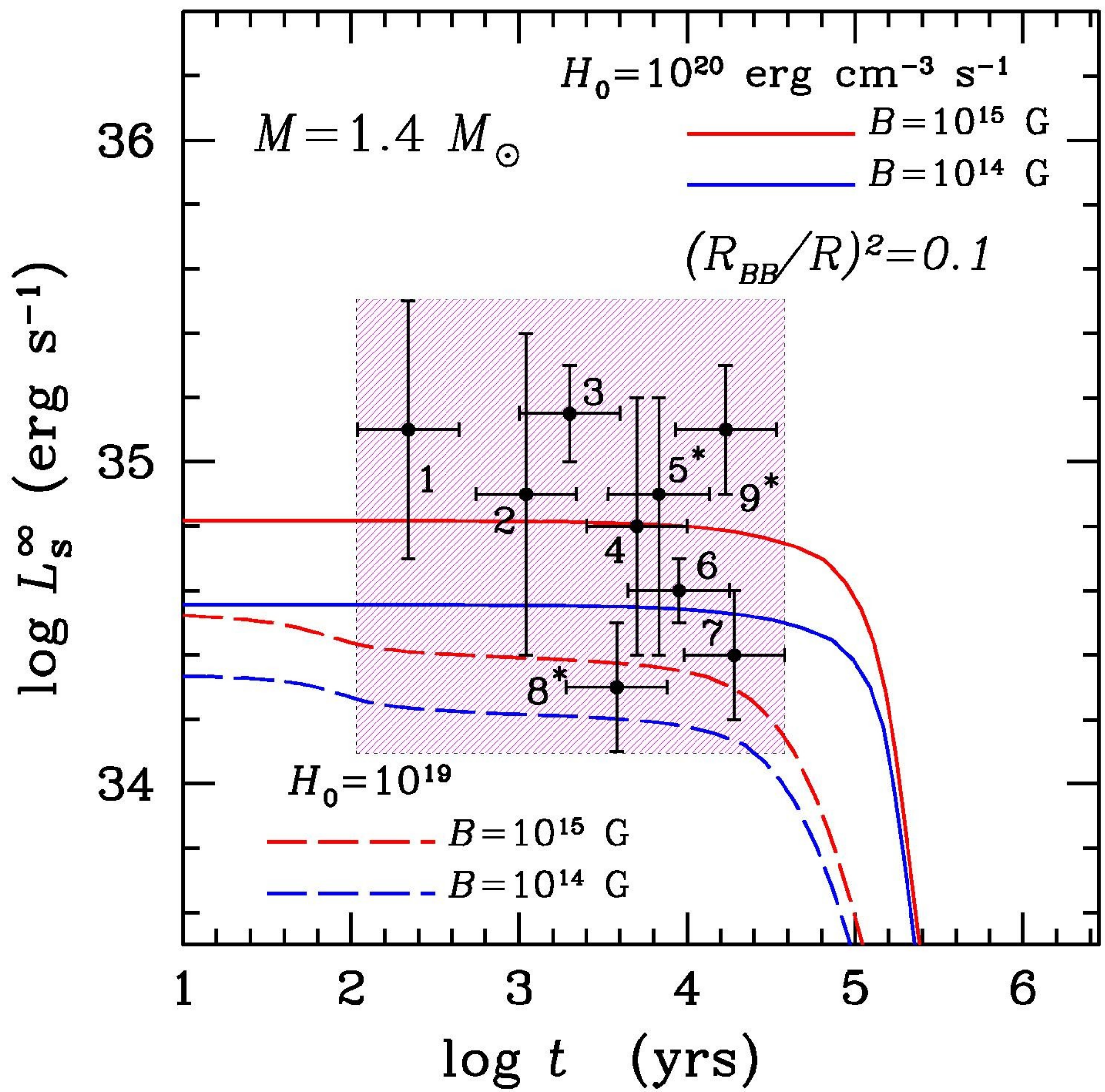
- **Pulsations detected in some of magnetars**
- **Discrepancy between observational estimates of temperature and luminosity**

Strong thermal insulation
Cold surface

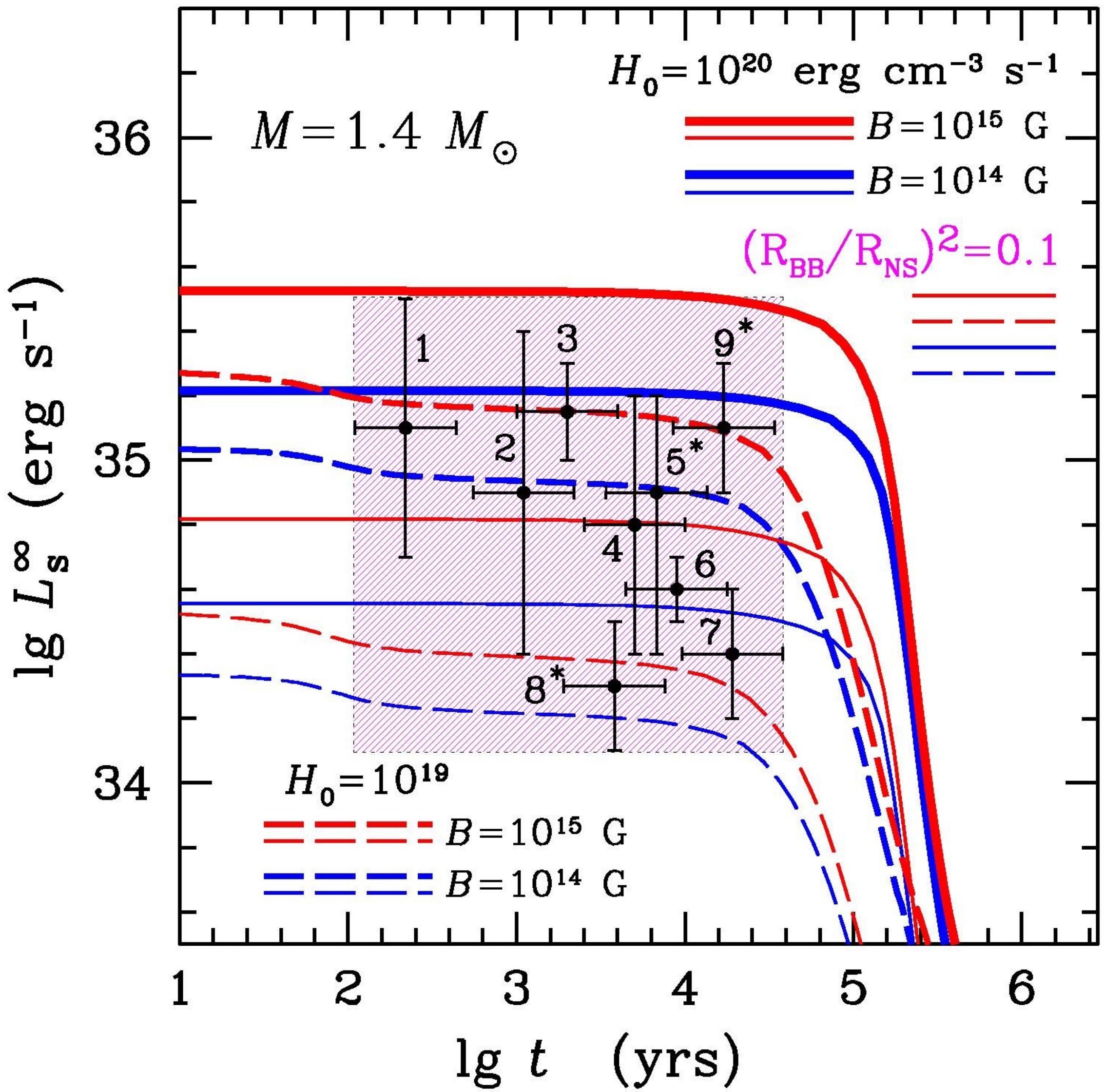


Low electric conductivity
Twisted B-lines
Strongest Ohmic dissipation
Heat source
Hot surface spots

HEAT RADIATION THROUGH MAGNETIC SPOTS



MAGNETIC SPOTS *vs.* ENTIRE SURFACE



Conclusions

- ***Quantizing magnetic fields*** strongly affect the EOS and kinetic coefficients of plasma in neutron-star envelopes.
- Magnetic fields make the temperature distribution highly anisotropic and can be important for evaluation of the effective temperature from observations.
- A ***superstrong*** magnetic field
  - on the average, makes the envelope more heat-transparent,
  - accelerates cooling at late epochs,
  - leads to theoretical uncertainties, which require further study.
- Reconciliation of crustal heating models with effective temperatures inferred form observations of some magnetars sensitively depends on the effects of superstrong magnetic fields and chemical composition of the outer envelopes.