Screening of the charged particle field in rare ionized gas.

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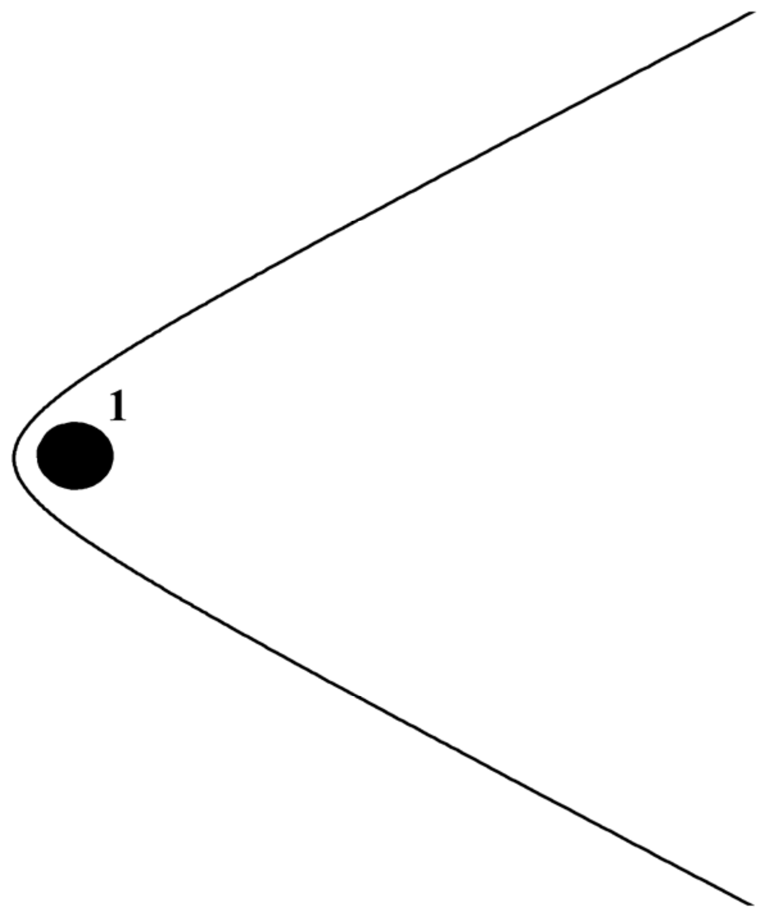
Contents.

Free ions in the particle field.



Distribution of free ions in a self-consistent particle field-1.

$$dP_i = dt = \frac{dR}{v_R} = \frac{dR}{\sqrt{1-\rho^2/R^2 - U(R)/\varepsilon}}$$

$$N_i \propto \frac{\int \rho d\rho dP_i}{4\pi R^2 dR} = N_o \int_0^{\rho(R)} \frac{\rho d\rho}{\sqrt{1-\rho^2/R^2 - U(R)/\varepsilon}}$$

$$N_i(R) = \frac{N_o}{2}\left[\sqrt{1-\frac{U(R)}{\varepsilon}} + \sqrt{1-\frac{\rho_c{}^2}{R^2}-\frac{U(R)}{\varepsilon}}\right]$$

$$\rho_c{}^2 = r_o{}^2\left[1-U(r_o)/\varepsilon\right]$$

$\boldsymbol{r_o}$ is a particle radius, $\boldsymbol{\varepsilon}$ is the ion energy far from the particle.

Distribution of free ions in a self-consistent particle field-2.

Averaging over the Maxwell distribution function of ion energies

$$f(\varepsilon) = N_o \frac{2\varepsilon^{1/2}}{\pi^{1/2} T_i^{3/2}} \exp\left(-\frac{\varepsilon}{T_i}\right)$$

The ion distribution in space

$$N_i(R) = N_o \sqrt{\frac{|U(r_o)|}{\pi T_i}}, \quad R - r_o << r_o$$

$$N_i(R) = N_o \sqrt{1 + \frac{4|U(R)|}{\pi T_i}}, \quad R >> r_o$$

Conclusion : Electrons do not contribute to the particle screening.

Coulomb interaction (no screening).

$$U(R)=\frac{Ze^2}{R},\quad Z=-\frac{r_oT_e}{e^2}\ln\left(\frac{T_eM}{T_im_e}\right)$$

$$\left|U(R_o)\right|=T_i\rightarrow R_o=\frac{|Z|e^2}{T_i}$$

R_o is a dimension of the particle field region

$$N_i(R)=N_o\sqrt{1+\frac{4R_o}{\pi R}},\ R>>r_o$$

Criterion : $N_oR_o^{\,3}<<|Z|$

Example of a gas discharge dusty plasma.

Ar, r_o=1μm, T_e=1 eV, T_i =400 K

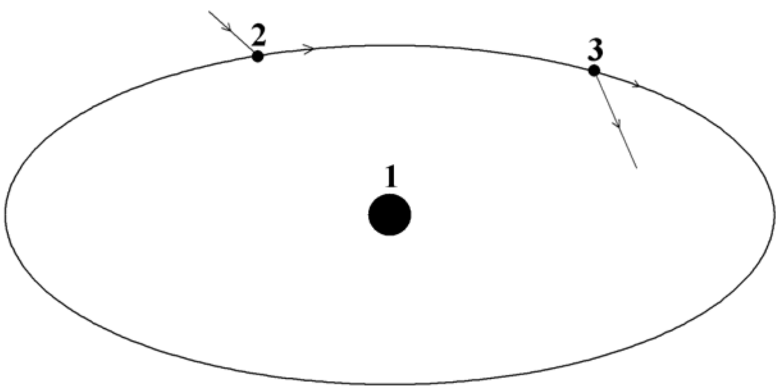
Weak shielding : $Z = -5 \cdot 10^3$, $R_o = 210\ \mu m$

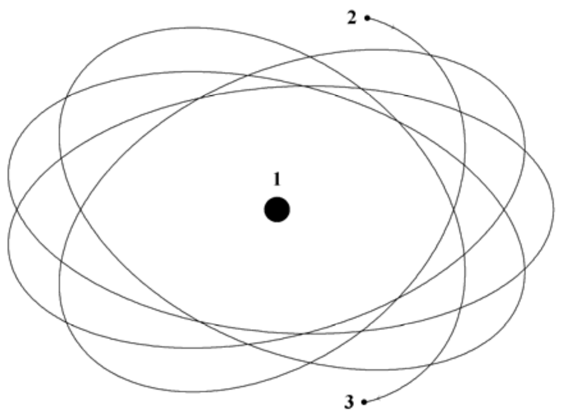
The shielding charge q = ✐ $4\pi R^2 dR\, N_i(R)$

Criterion of weak shielding (q<< |Z|:

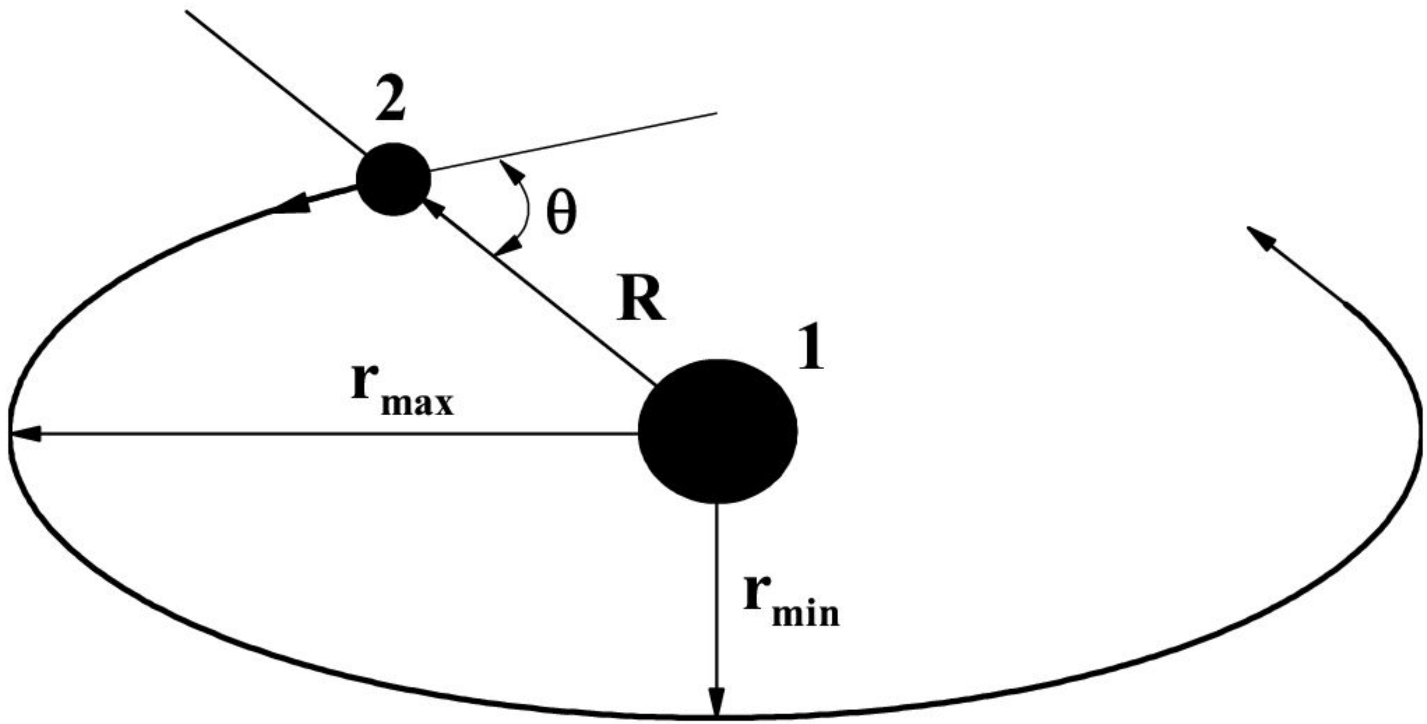
$N_o << 5 \cdot 10^8$ cm^{-3}

Trajectory of a trapped ion.





Parameters of ion capture by the particle field.



Capture in an elliptic orbit.

$$L^2(r_{\max}) = m^2 v^2 r_{\max}{}^2 = 2m\varepsilon r_{\max}{}^2$$

$$L^2(r_{\min}) = 2m|U(r_{\min})| r_{\min}{}^2 = 2m \frac{|Z|e^2}{r_{\min}} r_{\min}{}^2$$

$$r_{\min} = r_o \,,\, R_o = \frac{|Z|e^2}{\varepsilon}, \quad r_{\min} \geq r_o \rightarrow L(r_{\max}) \geq L(r_o)$$

Capture in an elliptic orbit is possible,
if $r_{max} \square (r_o R_o)^{1/2}$

Conservation laws in ion capture by the particle field.

$$A^{+} + A \rightarrow A + A^{+}$$

R is a point of resonant charge exchange, ε is the atom energy

The energy conservation :

$$E = U(R) + \varepsilon = Mv_R^2/2 + U(r) + L^2/(2Mr^2)$$

The momentum conservation :

$$L = Mv_\tau R = MvR\sin\theta$$

$$\sin\theta = \frac{r_{\min}}{R}\sqrt{\frac{\varepsilon + U(R) - U(r_o)}{\varepsilon}}$$

θ is the angle between vectors **v** and **R**.

Ion capture by the particle.

$$r_{\min} = r_o$$

The capture angle :

$$\sin\theta_o = \frac{r_o}{R}\sqrt{\frac{\varepsilon + U(R) - U(r_o)}{\varepsilon}}$$

Weak screening :

$$\sin\theta_o = \frac{r_o}{R}\sqrt{1 + \frac{R_o}{r_o} - \frac{R_o}{R}}$$

If $R \gg r_o$ and $R_o \gg r_o$

$$\sin\theta_o = \frac{\sqrt{R_o r_o}}{R}$$

Transition between elliptic orbits proceeds if $R > (r_o R_o)^{1/2}$

The number density of trapped ions.

The balance equation :

$$N_a \sigma_{res} N_i P_{tr} \mathrm{v}_\mathrm{i} = N_a \sigma_{res} N_{tr} \mathrm{v}_\mathrm{tr} (1 - p_{tr})$$

The ratio of the number densities for trapped and free ions :

$$|U(r_o)| >> |U(R)| >> \varepsilon$$

$$\frac{N_{tr}}{N_{free}} = \frac{\sqrt{2}\sqrt{1 - \frac{R_o r_o}{R^2}}}{1 - \sqrt{1 - \frac{R_o r_o}{R^2}}}, R_o >> R > \sqrt{R_o r_o}$$

The number density of trapped ions without particle field screening

$$N_{tr}(R) = N_o \sqrt{\frac{32}{\pi}} \frac{R^{3/2}}{r_o R_o^{1/2}}, R_o >> R >> \sqrt{R_o r_o}$$

Example of a gas discharge dusty plasma.

Ar, r_o=1μm, T_e=1 eV, T_i =400 K

Weak shielding ($q \sim 10eN_o R_o^4 / r_o$) :

$$Z = -5 \cdot 10^3, \; R_o = 210\ \mu m$$

Criterion of weak shielding ($N_{tr} R_o^4 / r_o \ll |Z|$)

$$N_o \ll 3 \cdot 10^5\ cm^{-3}$$

Self-consistent field of particle and free ions.

$$\frac{dz(R)}{dR} = -4\pi R^2 N_i(R), \quad z(R) = |Z| - q(R),$$

$$q(R) = \int_{r_o}^{R} dR\, 4\pi R^2 N_i(R),\; N_i(R) = N_o \sqrt{1 - \frac{4U(R)}{\pi T_i}},$$

$$U(R) = \int_{R}^{R_o} \frac{z(r)e^2}{r^2} dr,\; U(R) \approx -\frac{z(R)e^2}{R}$$

$$z(R) = \left(\sqrt{|Z|} - \frac{16\sqrt{\pi}}{5} N_o R^{5/2} \sqrt{\frac{e^2}{T_i}} \right)^2$$

Example of a gas discharge dusty plasma for screening by free ions.

Ar, r_o=1μm, T_e=1 eV, T_i =400 K, N_o= 1 ·10^{10}cm^{-3}

Weak shielding : $Z = -5 \cdot 10^3$, $R_o = |Z|e^2/T_i = 210\ \mu m$

The dimension of the particle field region *l*

According to definition z(*l*) = 0 :

$$l = \frac{0.66}{N_o^{2/5}}\left(\frac{|Z|T_i}{e^2}\right)^{1/5}, \quad l = 43\mu m$$

If $|U(l)| = T_i\ , \quad l = 40\mu m$

For **N_o = 1 . 10^9 cm^{-3}** l=108 μm and 99μm

Self-consistent field of particle and trapped ions.

$$\frac{dz(R)}{dR} = -4\pi R^2 N_{tr}(R), \quad z(R) = |Z| - q(R),$$

$$q(R) = \int_{r_o}^{R} dR\, 4\pi R^2 N_{tr}(R)$$

$$z(R) = |Z| \left[1 - \left(\frac{R}{l} \right)^{9/2} \right]^2, \quad l >> \sqrt{r_o R_o}$$

$$l = 1.05 \left(\frac{|Z|^2 r_o{}^2 R_o}{N_o{}^2} \right)^{1/9}, N_{\max} \sim \frac{l}{r_o} N_i$$

Example of a gas discharge dusty plasma for screening by trapped ions.

Ar, r_o=1μm, T_e=1 eV, T_i =400 K,
$N_o=1 \cdot 10^{10} cm^{-3}$

Weak shielding : $Z = -5 \cdot 10^{3}$, $R_o = |Z|\, e^2/T_i = 210\ \mu m$

The dimension of the particle field region *l*
according to definition z(*l*) = 0 $l = 33\mu m$

$$N_{\max} \sim 10^{11} cm^{-3}$$

Peculiarities of particle screening.

The equilibrium particle charge Z is almost independent of the screening degree.

The role of trapped ions is the higher, the less is the ion number density far from the particle.

These effects are stronger for a nonequilibrium plasma $T_e >> T_i$.

The similarity law.

$$\frac{l_{free}}{r_o} = \frac{const}{(N_o r_o^2)}, \qquad \frac{l_{trap}}{r_o} = \frac{const}{(N_o r_o^2)}$$

The similarity law : $N_o r_o^2$ =const

The contribution of free and trapped ions is identical at N_o=7 10^9 cm^{-3} for r_o=1μm and at N_o =7 10^7 cm^{-3} for r_o =10μm

Trapped ions disappear at N_o =10 ^{11} cm^{-3} for r_o =1μm and at N_o =10^9 cm^{-3} for r_o =10μm

The number density of free and trapped ions 1.

The field dimension is determined by trapped ions :

$$N_i(R) = N_o \sqrt{1 + \frac{4R_o}{\pi R}\left[1 - \left(\frac{R}{l}\right)^{9/2}\right]^2}$$

$$N_{tr}(R) = N_i(R)\,\varphi(x)\left(1 - \frac{R}{l}\right)$$

$$x = \frac{R^2}{r_o R_o},\ \varphi(x) = \sqrt{2}\left(\sqrt{x^2 - x} + x - 1\right)$$

The number density of free and trapped ions 2.

The field dimension is determined by free ions :

$$N_i(R) = N_o\sqrt{1+\frac{4R_o}{\pi R}\left[1-\left(\frac{R}{l}\right)^{5/2}\right]^2}$$

$$N_{tr}(R) = N_i(R)\,\varphi(x)\left(1-\frac{R}{l}\right)$$

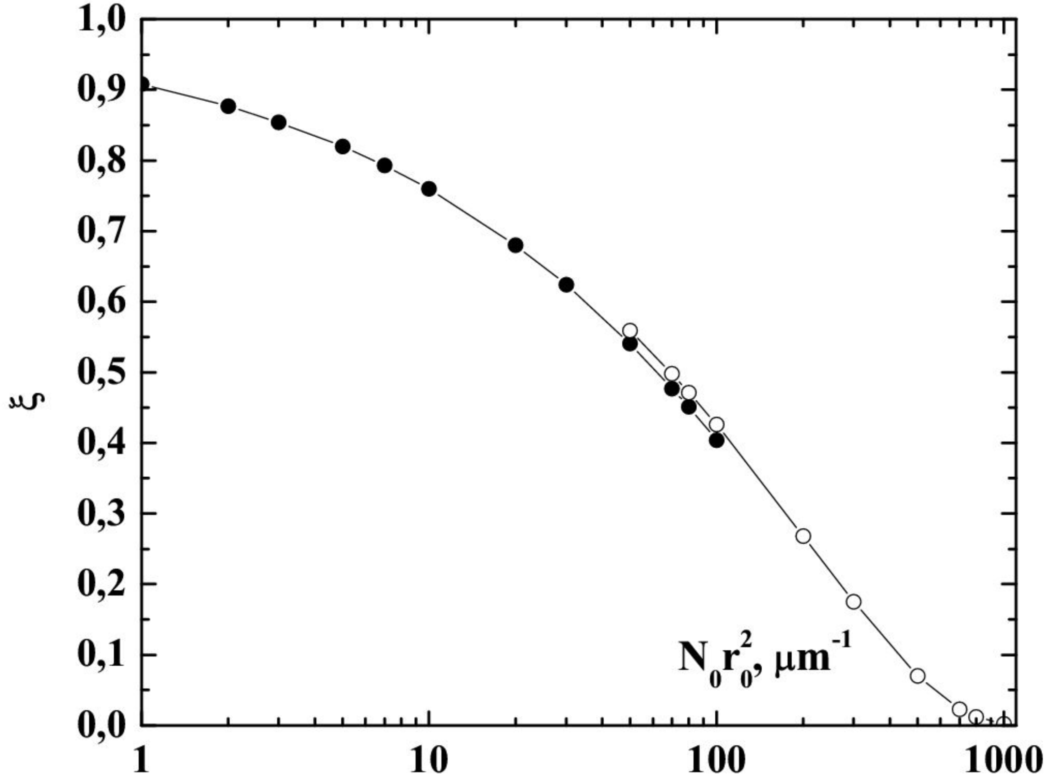
$$x = \frac{R^2}{r_o R_o},\ \varphi(x) = \sqrt{2}\left(\sqrt{x^2-x}+x-1\right)$$

Screening charge of ions.

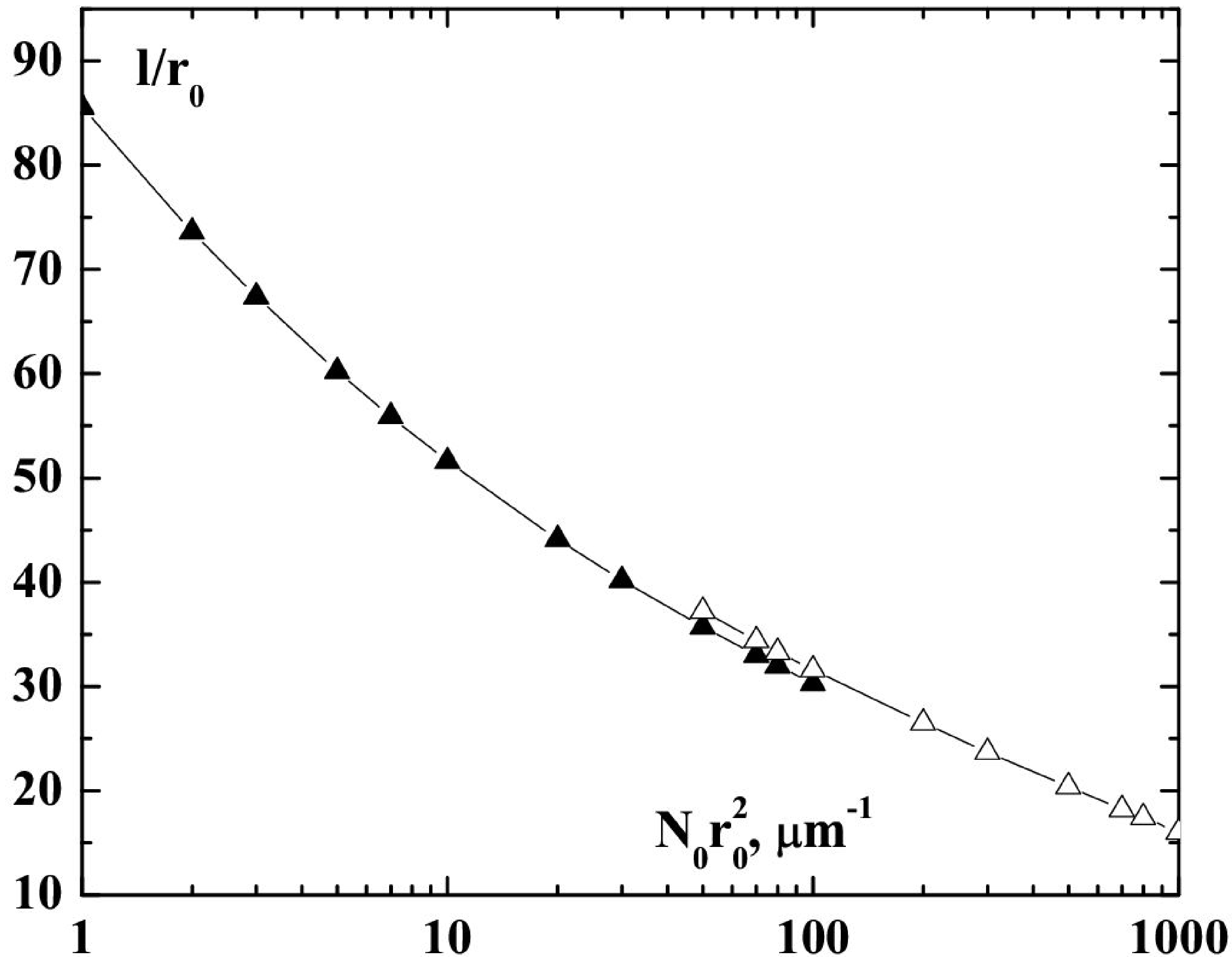
$$Q = \int_0^l 4\pi R^2 N_i(R)\,dR, \quad q = \int_{\sqrt{r_o R_o}}^{l} 4\pi R^2 N_{tr}(R)\,dR,$$

The definition of particle field region $l : Q + q = |Z|$

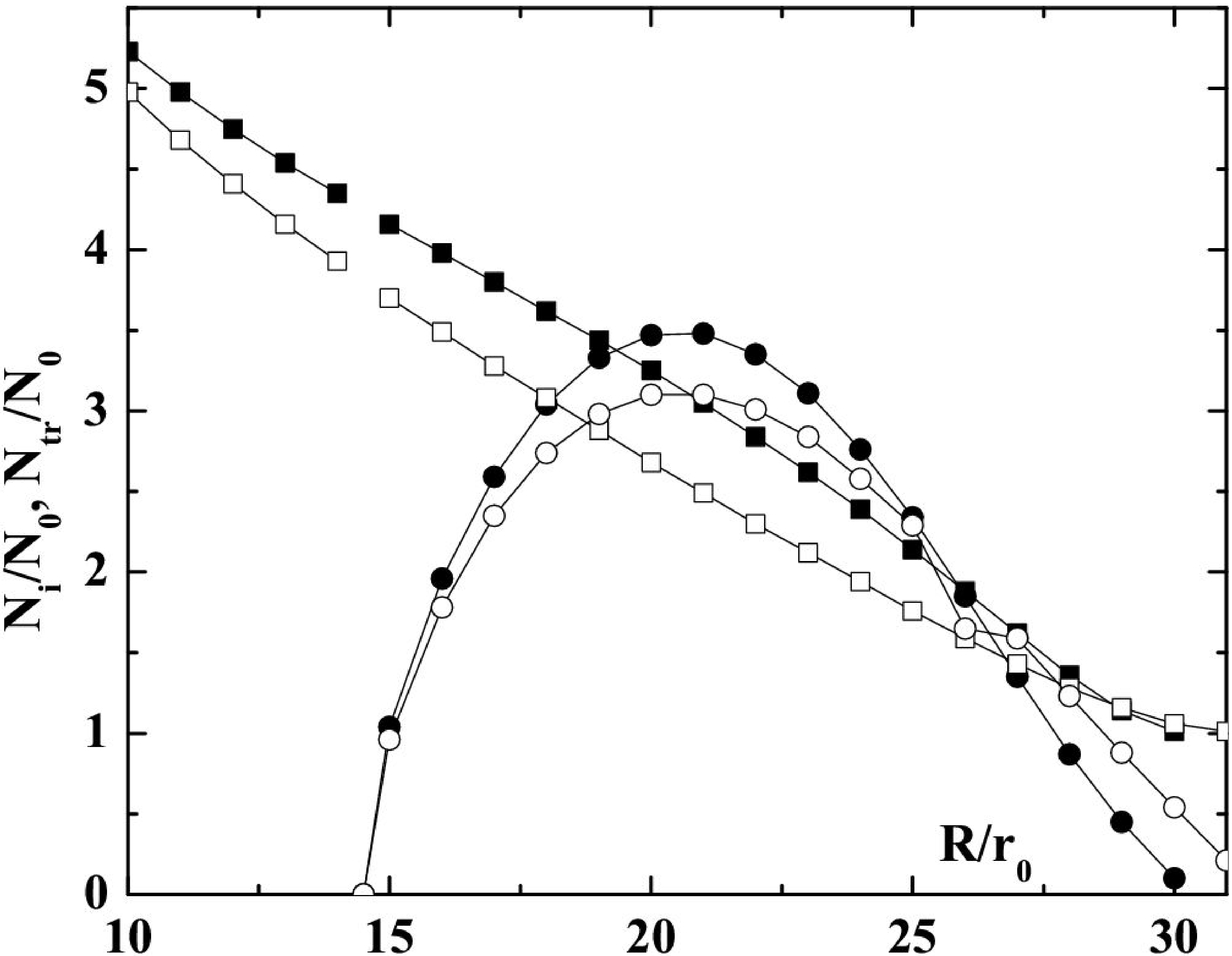
The part of screening due to trapped ions $: \xi = \dfrac{q}{q+Q}$



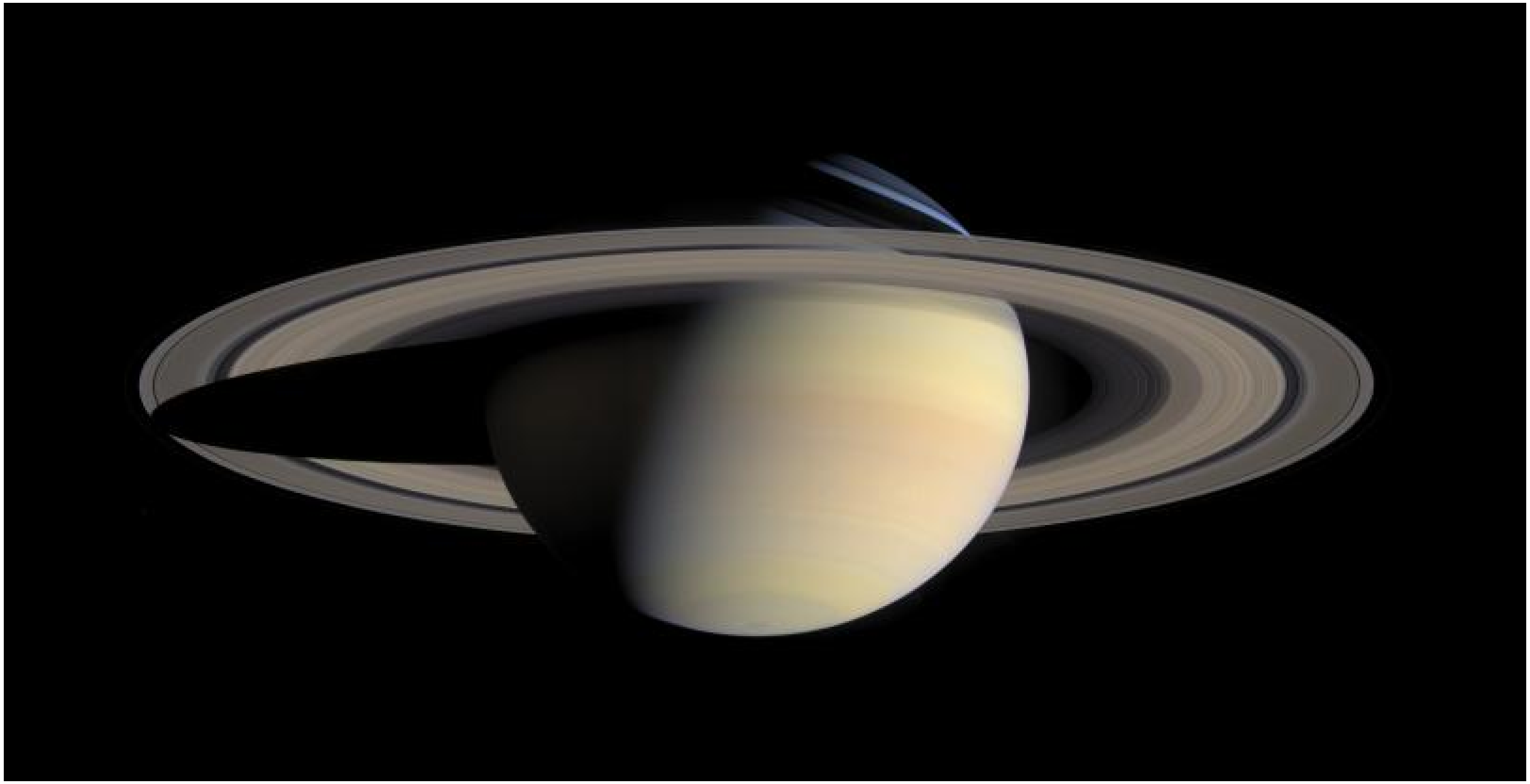
Dimension of region of the particle field.



The number density of free and trapped ions.



Saturn rings.



E and F-rings of Saturn.

The source of E-ring is Enceladus, the source for F-ring of Saturn are Prometheus and Pandora.

Solar wind $N_o \sim 0.1\ cm^{-3}$ **, for Saturn rings**

$N_o \sim 10\text{-}100\ cm^{-3}$, $N_{part} \sim 30\ cm^{-3}$, $T_e \sim 30eV$

Under equilibrium conditions $|Z|=2\cdot 10^5$ **,** $N_e \sim 10^6\ cm^{-3}$

Charge equilibrium conditions are violated for a dusty plasma of Saturn rings.

Comet tail.

Solar wind $N_o \sim 0.1\ cm^{-3}$, comet tails $N_o \sim 10^3 - 10^4\ cm^{-3}$, $T_e \sim 10^4$ K

Thank you !