Hydrodynamic description of first-order phase transitions in nuclear systems

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I-order phase transitions in:

nuclear systems:

- Liquid-gas and hadron-quark phase transitions in HIC

condensed matter:

any 1-order phase transition of the liquid-gas type

Dynamics of the first order phase transition:

Skokov, D.V. , arXiv 0811.3868, Pisma ZhETF 90 (2009) 245;
Nucl. Phys. A828 (2009) 401

We solve the system of non-ideal hydro equations describing non-trivial fluctuations (droplets/bubbles, aerosol) in d=2 space +1 time dimensions numerically for van der Waals-like EoS, and for arbitrary d in the vicinity of the critical point analytically.

Non-ideal hydrodynamics

$$mn\left[\partial_t u_i + (\mathbf{u}\nabla)u_i\right] = -\nabla_i P$$
$$+\nabla_k\left[\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right) + \zeta\delta_{ik}\mathrm{div}\mathbf{u}\right] \quad (8)$$
$$\partial_t n + \mathrm{div}(n\mathbf{u}) = 0, \quad (9)$$
$$T\left[\frac{\partial s}{\partial t} + \mathrm{div}(s\mathbf{u})\right] = \mathrm{div}(\kappa\nabla T)$$
$$+\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right)^2 + \zeta(\mathrm{div}\mathbf{u})^2. \quad (10)$$

Here η and ζ are shear and bulk viscosities; $\mathbf{u}$ is the velocity of the element of the fluid; s is the entropy density; κ is the thermal conductivity; d is the dimensionality of space.

typical time $t_\rho \sim R$

typical time $t_T \sim R^2$

R (t) is the typical size of evolving fluctuation

Dynamics of the phase transition is controlled by the slowest mode

$t_\rho > t_T$ for $R(t) < R_{fog}$: **Density evolution stage**

R_{fog} is typical seed size at which $t_\rho = t_T$

typical time of the heat transport $t_T = R^2 c_{V,r}/\kappa_r$

For seeds with sizes $R > R_{\rm fog}$, $t_T \propto R^2$ exceeds $t_\rho \propto R$ and growth of seeds is slown down. Thereby, number of seeds with the size $R \sim R_{\rm fog}$ grows with time.

$t_T > t_\rho$: Heat transport stage **(Fog stage)**

(for H-QGP phase transition R_{fog}~ 0.1-1 fm, for liquid-gas ~1-10 fm)

Supercooled gas; overheated liquid; aerosol-like mixture in spinodal region

Expand the Landau free energy in $\delta\rho = \rho - \rho_{\rm r}$ and δ T near the reference point, close to ρ_{cr}, T_{cr} *but outside fluctuation region, where Gi>1*

$$\delta F = \int \frac{d^3x}{\rho_{\rm r}} \left[\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda(\delta\rho)^4}{4} - \frac{\lambda v^2(\delta\rho)^2}{2} - \epsilon\delta\rho \right]$$

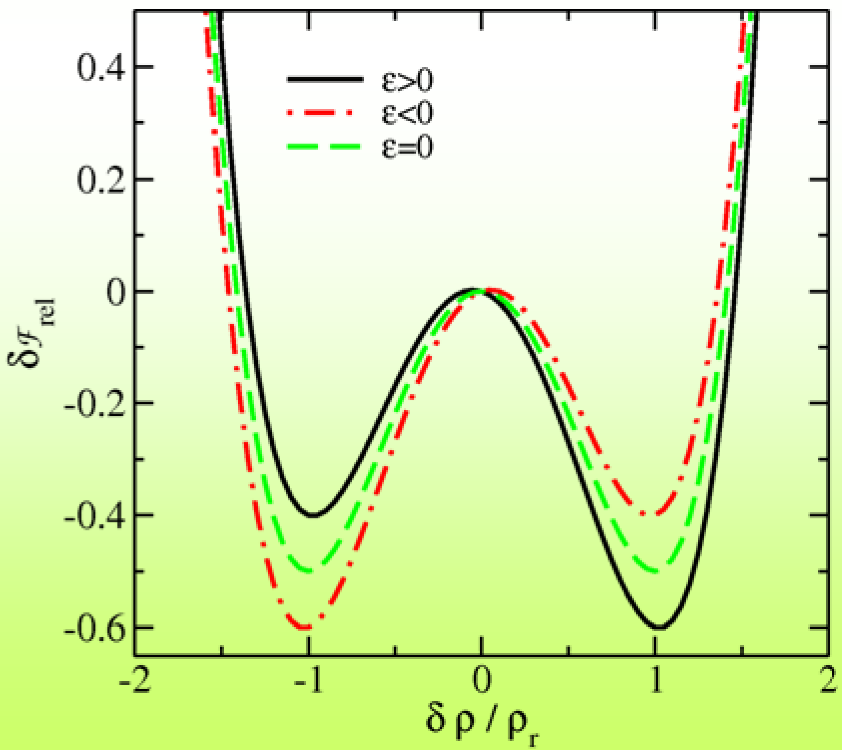
Surface term

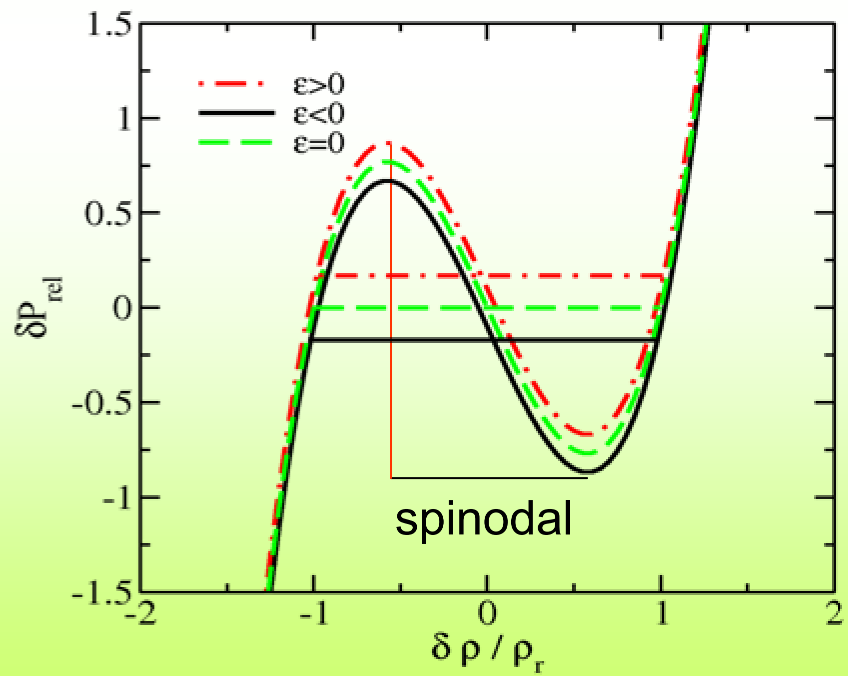
$v^2 \sim (T_{cr} - T)$

$$\delta P = \rho \left. \frac{\delta[F_L(T,\delta\rho)]}{\delta(\delta\rho)} \right|_T$$

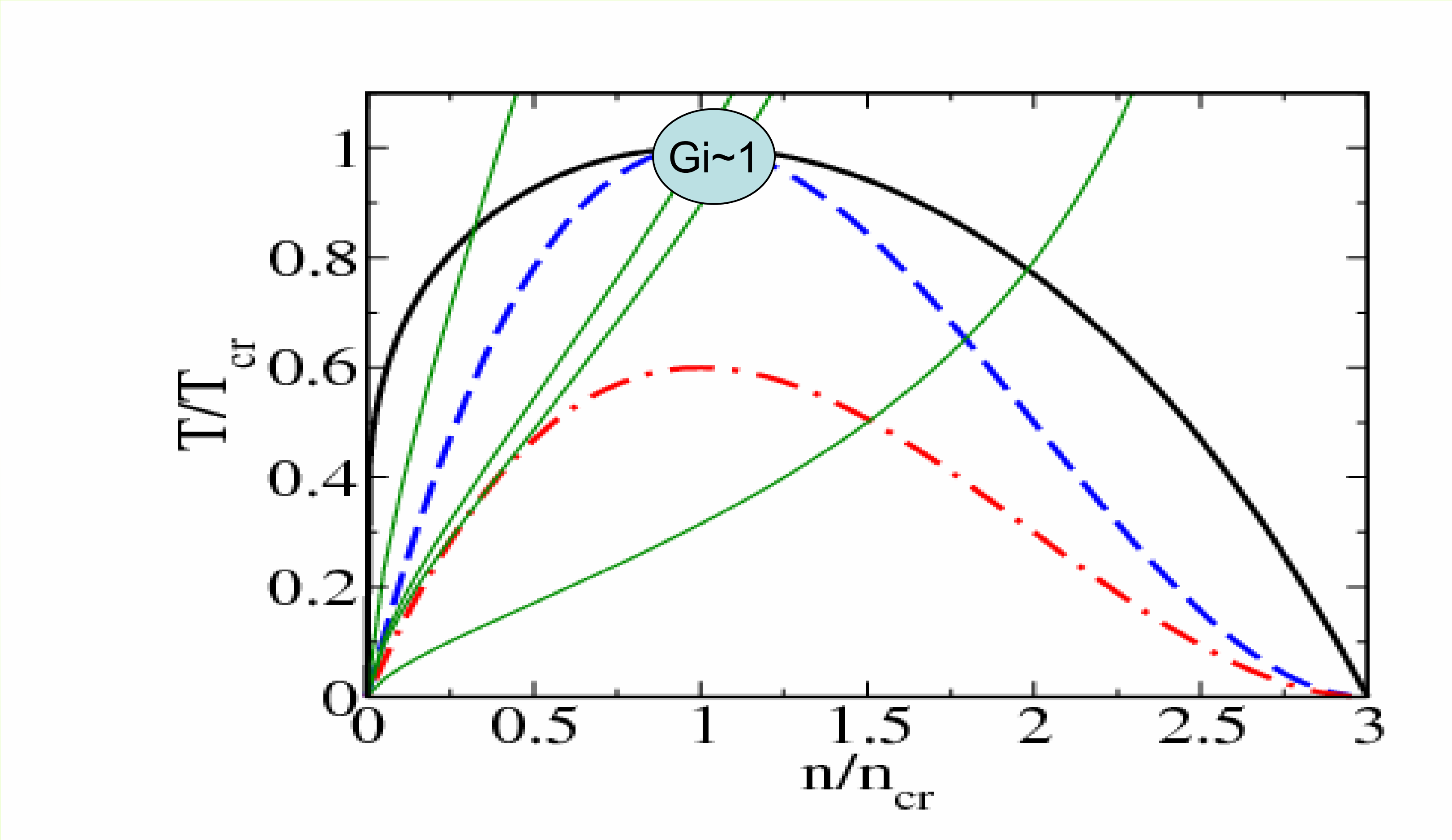
$$\delta\mathcal{F}_{rel} = \delta\mathcal{F}_L / \mathcal{F}_L(T_{cr}, \rho_{cr}) \qquad \delta P_{rel} = \delta P / P(T_{cr}, \rho_{cr})$$

at critical point





Constant entropy trajectories



- - - isothermal spinodal, - . - . - isoentropic spinodal, ▬ Maxwell construction

Some thermal models predict focusing of trajectories near CEP due to strong fluctuations

Dynamics of I order phase transition near CEP

Hydrodynamical equations give for $\delta\rho = \rho - \rho_{\rm r}$

viscosities

$$-\frac{\partial^2 \delta\rho}{\partial t^2} = \Delta\left[c\Delta\delta\rho + \lambda v^2 \delta\rho - \lambda(\delta\rho)^3 + \epsilon - \rho_{\rm r}^{-1}\left(\frac{4}{3}\eta_{\rm r} + \zeta_{\rm r}\right)\frac{\partial \delta\rho}{\partial t}\right]$$

In dimensionless variables

$$-\beta\frac{\partial^2\psi}{\partial\tau^2} = \Delta_\xi\left(\Delta_\xi\psi + 2\psi(1-\psi^2) + \widetilde{\epsilon} - \frac{\partial\psi}{\partial\tau}\right)$$

$$\delta\rho = v\,\psi, \quad \xi_i = x_i/l, \quad \tau = t/t_0$$

$$l = \left(\frac{2c}{\lambda v^2}\right)^{1/2}, \quad t_0 = \frac{2(\frac{4}{3}\eta_{\rm r} + \zeta_{\rm r})}{\lambda v^2 \rho_{\rm r}}, \quad \widetilde{\epsilon} = \frac{2\epsilon}{\lambda v^3} \quad \beta = \frac{c\rho_{\rm r}^2}{(\frac{4}{3}\eta_{\rm r} + \zeta_{\rm r})^2}$$

$$\boxed{v \propto |T - T_{\rm cr}|^{1/2}} \longrightarrow \boxed{t_0 \propto |T - T_{\rm cr}|^{-1}}$$

processes in the vicinity of the critical point prove to be very slow

Peculiarities of hydro- description

- Eq. is the 2-order in time derivatives -- beyond the Ginzburg-Landau description where:

$$- \rho_{\mathrm{r}}^{-2} \left(\tilde{d}\eta_{\mathrm{r}} + \zeta_{\mathrm{r}} \right) \frac{\partial \delta\rho}{\partial t} = \frac{\delta [F(T, \delta\rho)]}{\delta(\delta\rho)} |_T .$$

For a produced fluctuation two initial conditions should be fulfilled

$$\delta\rho(t=0, \vec{r}) = \delta\rho(0, \vec{r}), \qquad \frac{\partial \delta\rho(t, \vec{r})}{\partial t} |_{t=0} \simeq 0 \quad \Rightarrow$$

At least initial stage of fluctuation dynamics is not described by GL approximation

Flow-experiments at RHIC indicate on very low viscosity

Conformal theories show minimum $\eta/s \sim 1/4\pi$:

η/s ratio is extensively discussing

η/s does not appear in equations of motion for fluctuations

Dynamics of the density mode is controlled by another parameter β, which enters together with the second derivative in time. This parameter can be expressed in terms of the **surface tension** and the **viscosity** as

$$\beta = \frac{\sigma_0^2\, m}{32\, T_{\rm cr}\, [\frac{4}{3}\,\eta_{\rm r} + \zeta_{\rm r}]^2}$$

$$\sigma_0^2 = 32\, m\, \rho_{\rm cr}^2 T_{\rm cr}\, c$$

surface tension

The **larger viscosity** and the **smaller surface tension**,

the **more viscous** is the fluidity of seeds.

$\beta \ll 1$ regime of effectively viscous fluid

$\beta \gg 1$ regime of perfect fluid

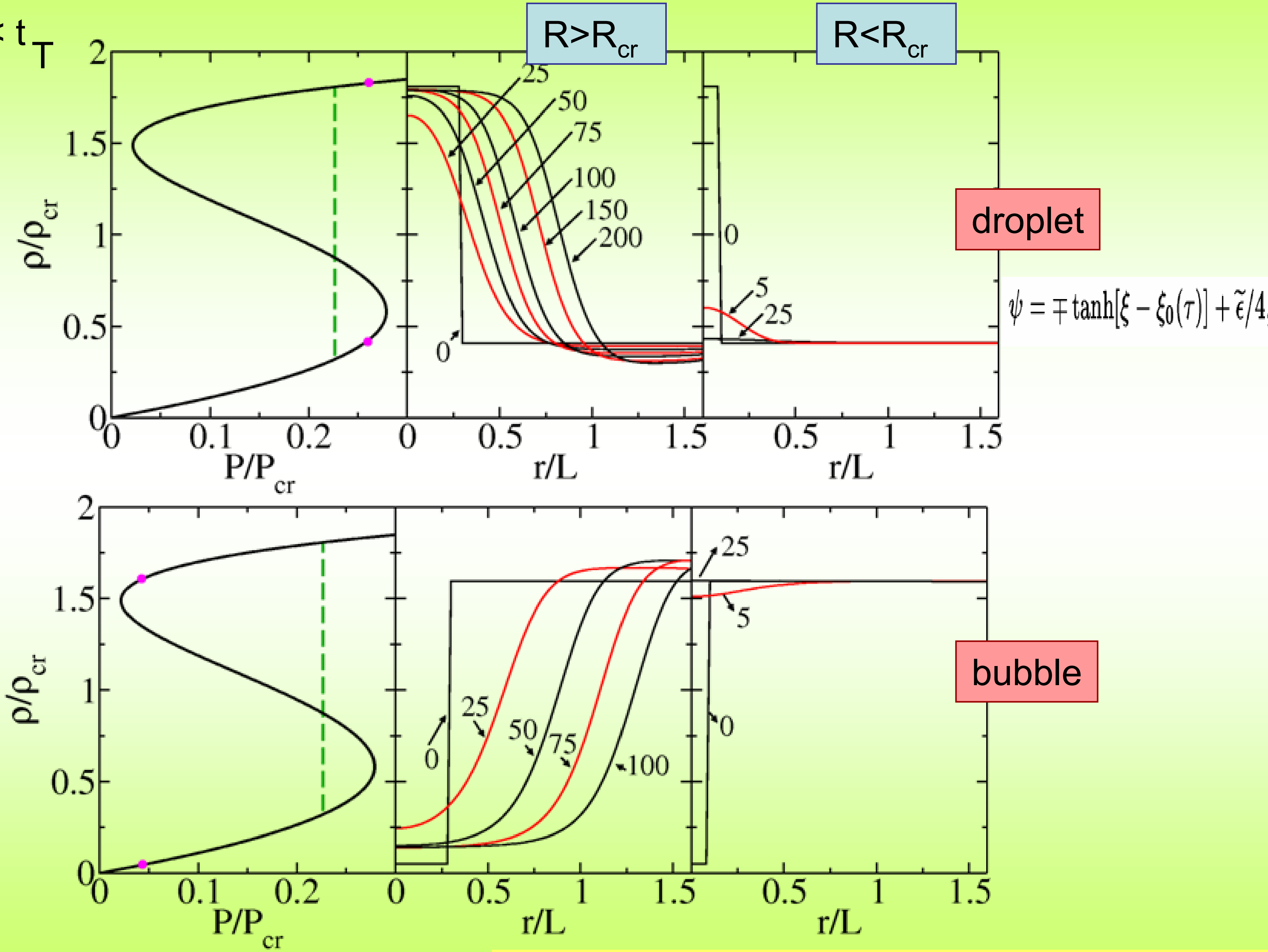
for liquid-gas phase transition $\beta \sim 0.01$;

for H-QGP phase transition: $\beta \sim 0.02\text{-}0.2$, even for $\eta/s \sim 1/4\pi$:

Effectively very viscous fluidity of density fluctuations!

Hadron-QGP phase transition: droplet/bubble evolution from metastable phases

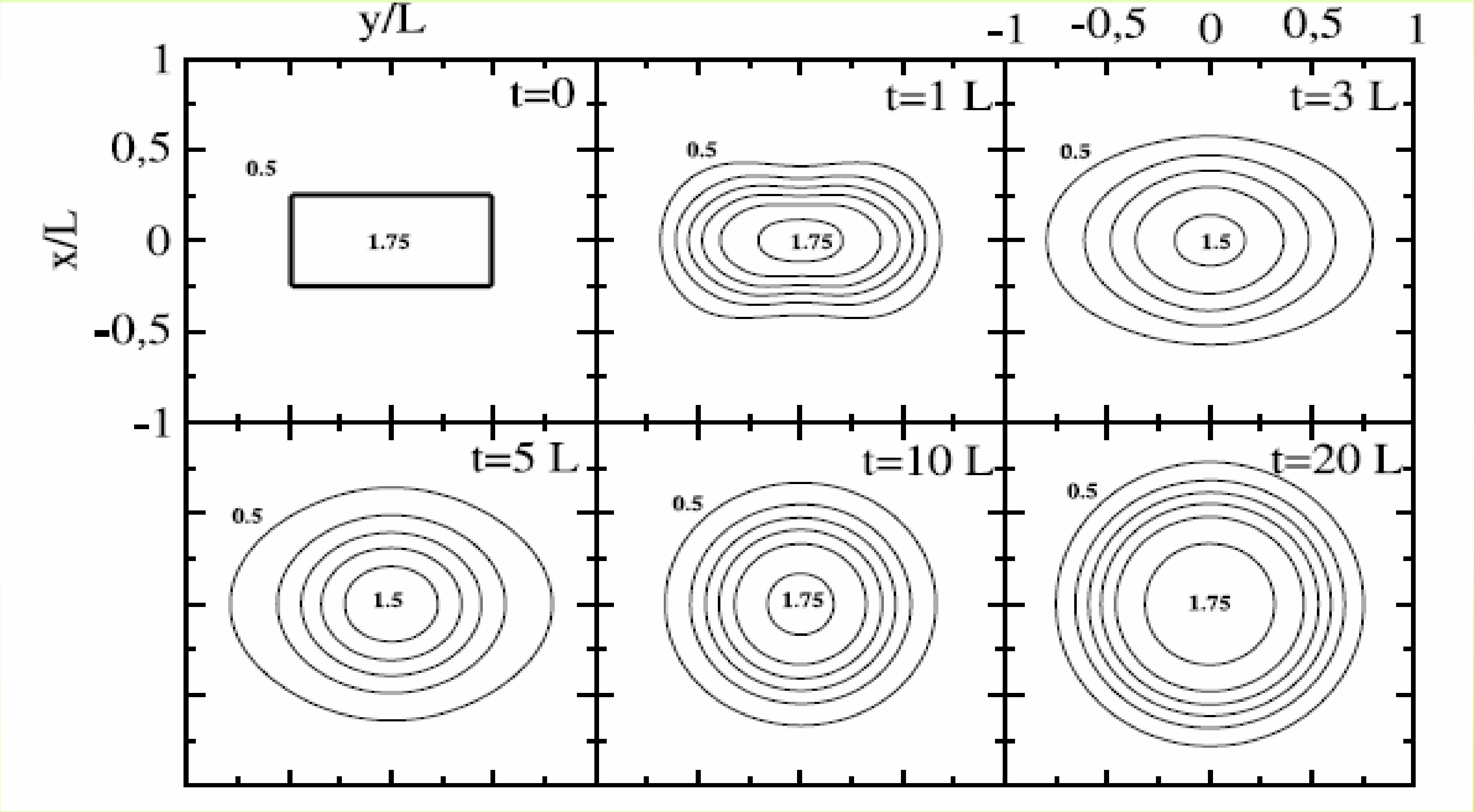
For $t_\rho < t_T$



$(T_{cr} - T)/T_{cr} = 0.15$; $T_{cr} = 162$ MeV; $L = 5$ fm; $\beta = 0.2$

Change of the seed shape with time

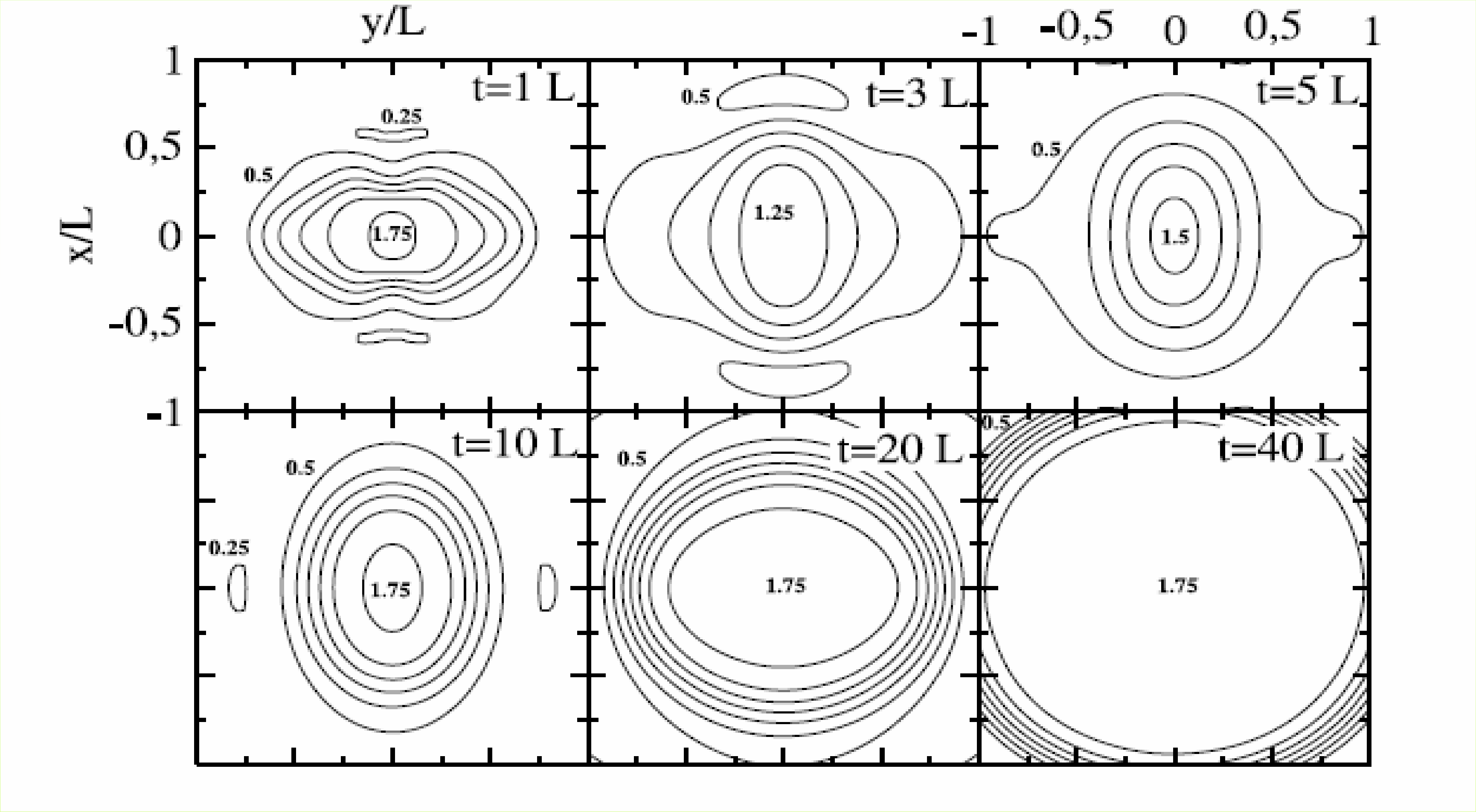
Isolines of the density n/n_{cr} with increment 0.25



Initially anisotropic droplet gets spherical form

$\beta = 0.1$

Change of the seed shape with time



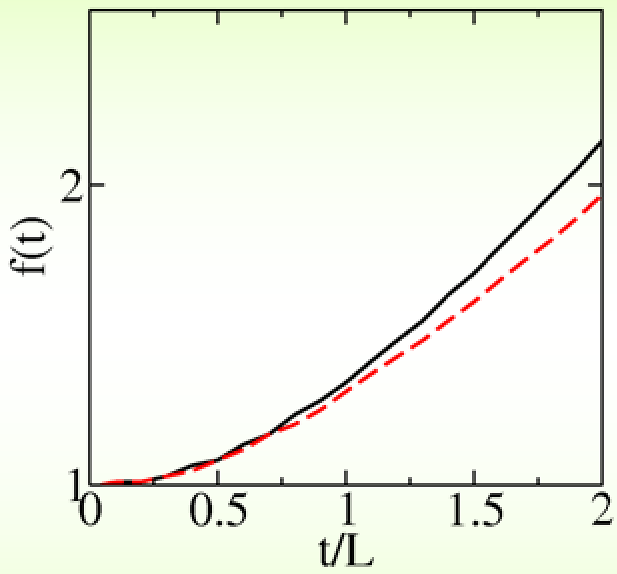
For almost perfect fluid dynamics is more peculier β=1000

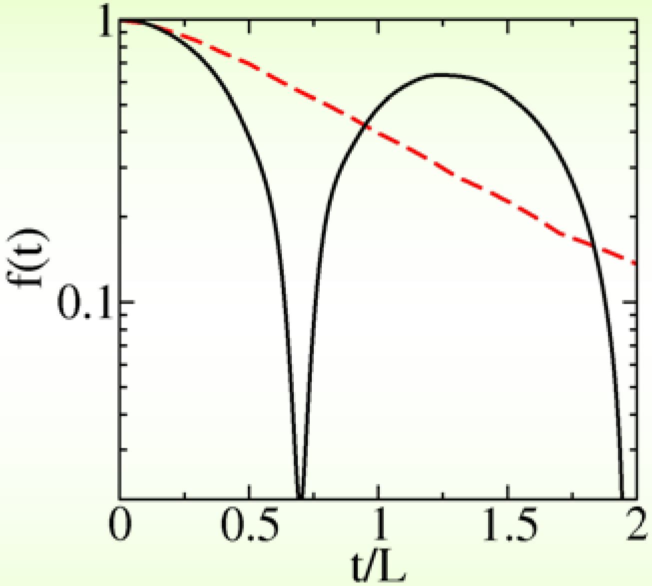
Hadron-QGP phase transition:
spinodal instability: aerosol-like mixture (**mixed phase**)

growing modes

$$\psi = -\frac{\tilde{\epsilon}}{2} + \mathrm{Re}\{\psi_0 e^{\gamma_\psi \tau + i\mathbf{k}\boldsymbol{\xi}}\}$$

oscillating modes





$\beta = 0.2$ — dash line $\quad \beta = 20$ — solid line

see also Randrup, PRC79 (2009) 024601

Far from CEP time evolution is sufficiently rapid –effect of warm Champagne

$$\gamma_\psi(k) = (-k^2 \pm \sqrt{k^4 + 8\beta k^2 - 4\beta k^4})/(2\beta)$$

Conclusions

The larger viscosity and the smaller surface tension the effectively more viscous is the fluidity

Anomalies in thermal fluctuations near CEP (which are under extensive discussion) may have not sufficient time to develop

Thus T_{cr} calculated in thermal models might be significantly higher than the value which may manifest in fluctuations in HIC

Heat transport effects may play important role

Effects of spinodal decomposition can be easier observed since they require a shorter time to develop

Thresholds in collision energy:

E_C: critical point

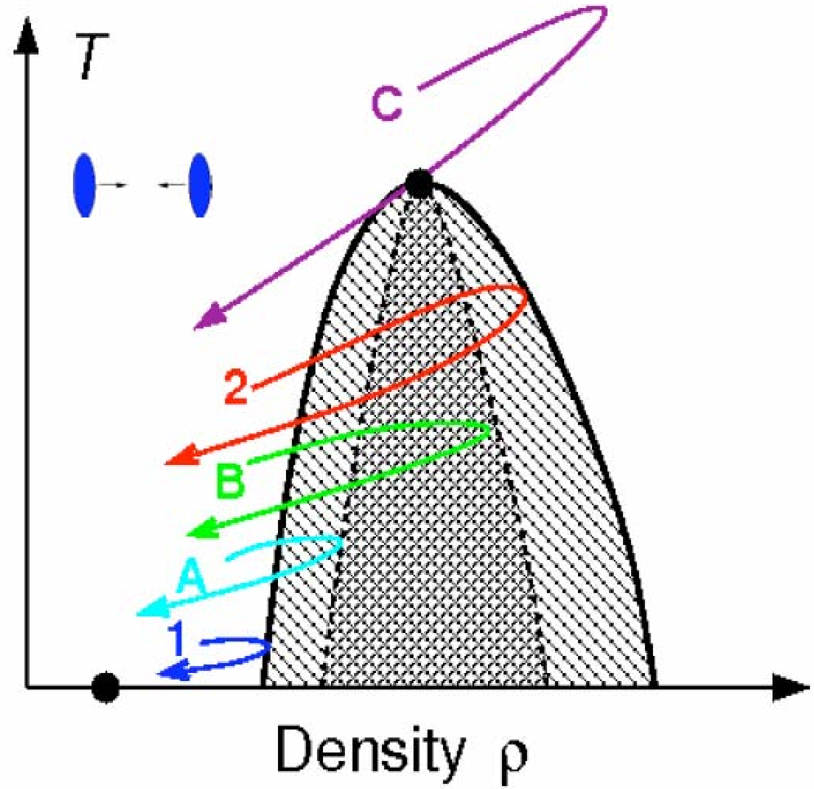
E_2: higher boundary

E_B: higher spinodal

E_A: lower spinodal

E_1: lower boundary

Schematic phase diagram:



Optimal collision energy range: $E_B < E < E_2$

Fig. from J.Randrup

Concluding:

- One may hope to observe non-monotonous behavior of different observables in HIC due to manifestation non-trivial fluctuation effects at monotonous increase of collision energies:

collision energy increase with a certain energy step will be possible at FAIR and NICA