The Confining Color String Model and the Surface Tension of Quark Gluon Bags

K.A. Bugaev

Bogolyubov ITP, Kiev, Ukraine

Moscow, December 1-2, 2010

Astro & Cosmic QGP Searches Programs

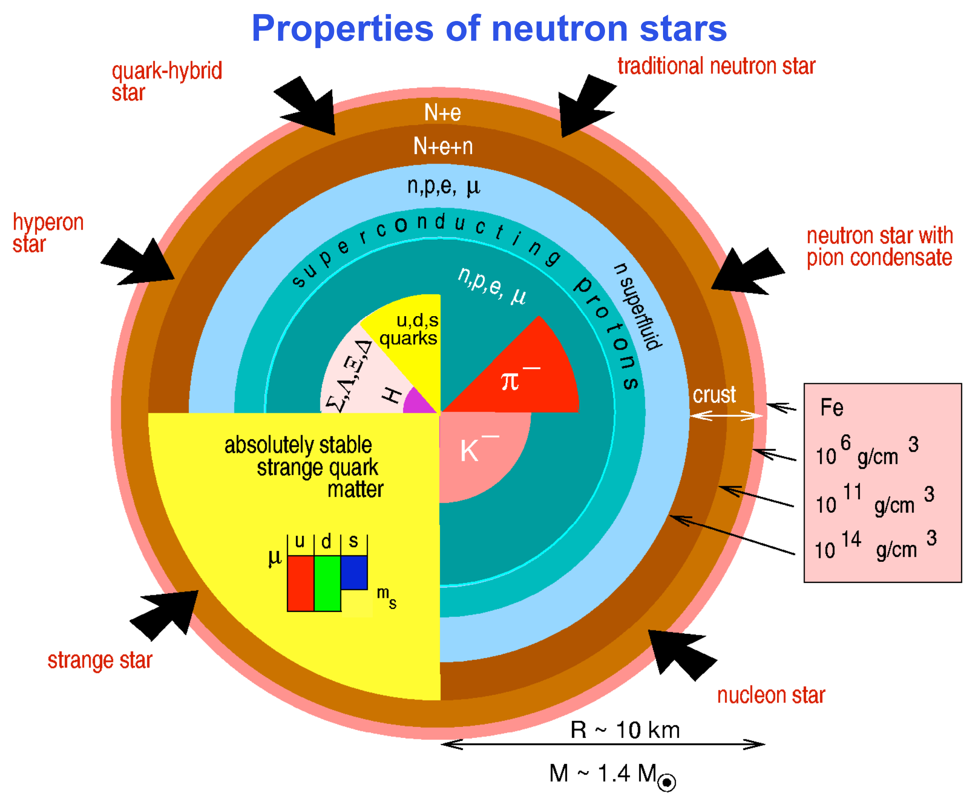
* Quark (core) stars, neutron stars, **stable strange stars, ...**

E. Witten, PRD 30 (1984) suggested that quark matter can survive till present time

Alcock & Olinto, PRD 39 (1989) showed that for surf. tension > $(178\ \mathrm{MeV})^3$ then quark matter would not be boiled away

These findings initiated the research on QGP surface tension.

Unfortunately, the QGP surface tension is UNKNOWN despite many efforts and model results!

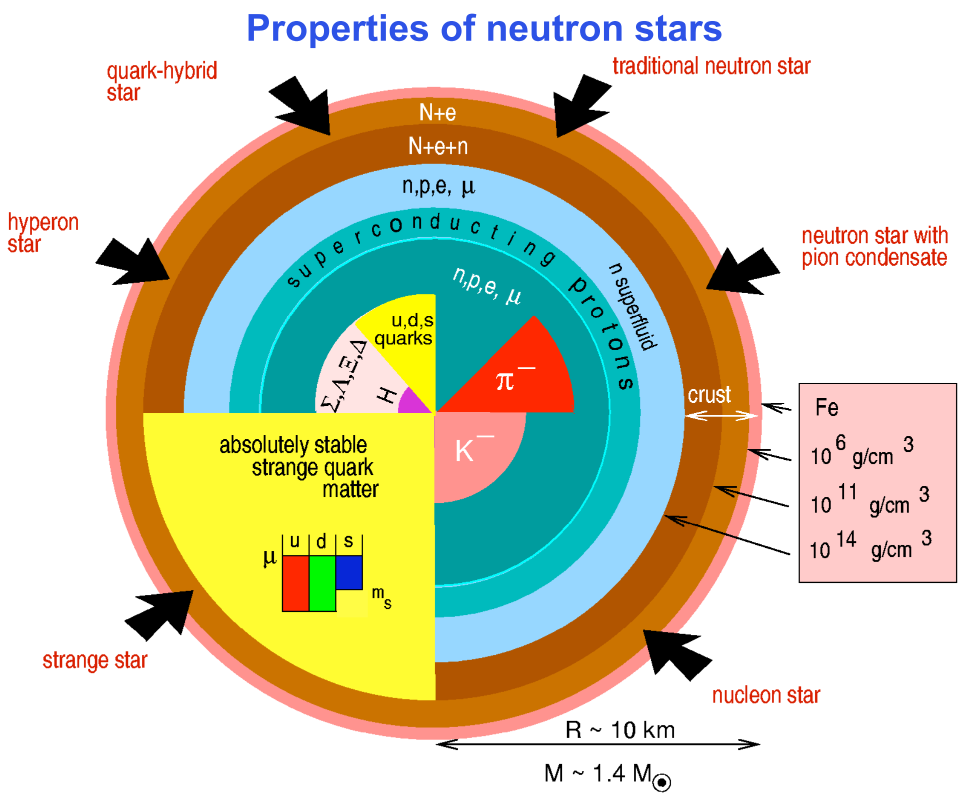


Astro & Cosmic QGP Searches Programs

Can we determine the QGP surface tension in a model independent way?

These findings initiated the research on QGP surface tension.

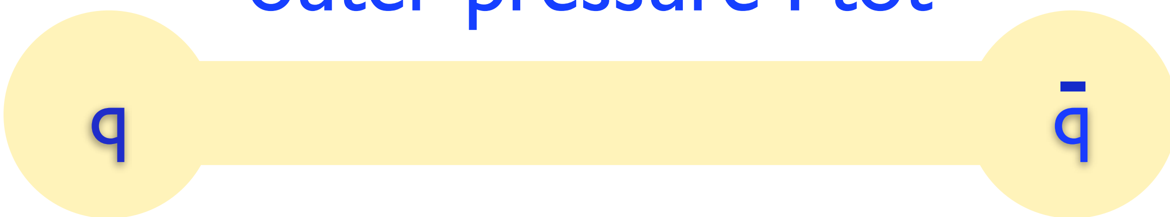
Unfortunately, the QGP surface tension is UNKNOWN despite many efforts and model results!



Confining String = Color Tube

Consider confining string between static q & anti q of length L and radius R<<L

outer pressure Ptot



color

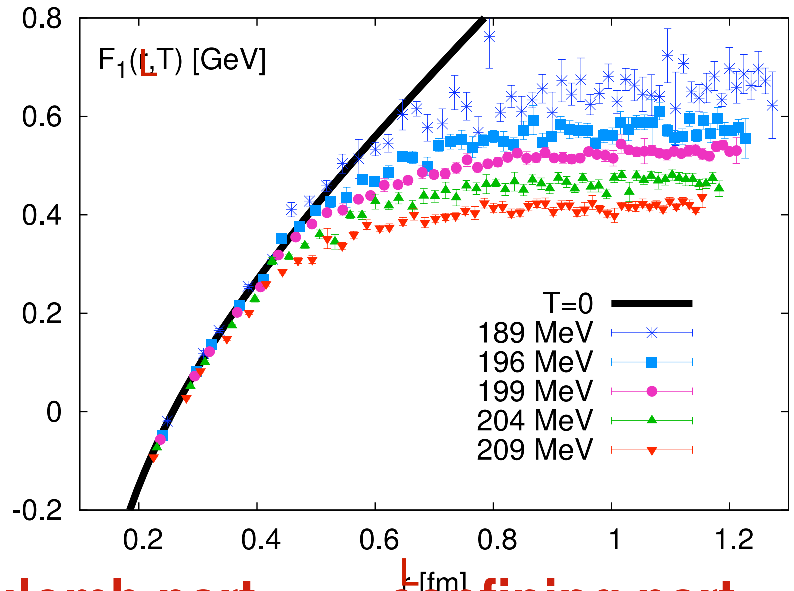
anticolor

Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$

Confinement means infinite free energy for infinite L

Deconfinement means that string tension vanishes

Can be rigorously found by Lattice QCD



Coulomb part **confining part**

String Tension vs Surface Tension

K.A.B., G.M.Zinovjev, Nucl. Phys. A848 (2010)

Consider now this tube as the cylindrical bag of length L and radius R<<L

Neglect effects of color sources and get cylinder FREE ENERGY as:

$$F_{cyl}(T,L,R) \equiv -\underbrace{p_v(T)\pi R^2 L}_{thermal} + \underbrace{\sigma_{surf}(T)2\pi RL}_{surface} + \underbrace{T\tau \ln\frac{V}{V_0}}_{small}$$

Equating the cylinder FREE ENERGY to string free energy

$$\boxed{\sigma_{str}(T) = \sigma_{surf}(T)\, 2\pi R \;-\; p_v(T)\pi R^2} + \frac{T\tau}{L}\ln\left[\frac{\pi R^2 L}{V_0}\right]$$

We got a new possibility to determine QGP bag surface tension directly from LQCD!

From bag model pressure $p_v(T=0) = -(0.25)^4 \ \mathrm{GeV}^4$, $R = 0.5$ fm and $\sigma_{str}(T=0) = (0.42)^2\ \mathrm{GeV}^2 \quad \Rightarrow$

$$\sigma_{surf}(T=0) = (0.2229\ \mathrm{GeV})^3 + 0.5\, p_v\, R \approx \boxed{(0.183\ \mathrm{GeV})^3} \approx 157.4\ \mathrm{MeV\ fm}^{-2}.$$

String Tension vs Surface Tension

K.A.B., G.M.Zinovjev, Nucl. Phys. A848 (2010)

QGP surface tension at T=0 is above the critical value!
Can quark matter survive the boiling away?

Equating the cylinder FREE ENERGY to string free energy

$$\sigma_{str}(T) = \sigma_{surf}(T)\, 2\pi R \;-\; p_v(T)\pi R^2 + \frac{T\tau}{L}\ln\left[\frac{\pi R^2 L}{V_0}\right]$$

We got a new possibility to determine QGP bag surface tension directly from LQCD!

From bag model pressure $p_v(T=0) = -(0.25)^4\ \mathrm{GeV}^4$, $R = 0.5$ fm and $\sigma_{str}(T=0) = (0.42)^2\ \mathrm{GeV}^2 \quad \Rightarrow$

$$\sigma_{surf}(T=0) = (0.2229\ \mathrm{GeV})^3 + 0.5\, p_v\, R \approx \boxed{(0.183\ \mathrm{GeV})^3} \approx 157.4\ \mathrm{MeV\ fm}^{-2}.$$

Surface Tension at Cross-over

For vanishing σ_{str} one has $\sigma_{str}^{LQCD} \approx \frac{\ln(L/L_0)}{R^2} C$

This is due to increase of surface fluctuations $\Rightarrow$ in general

$$\sigma_{str}(T)\, R^k \rightarrow \omega_k > 0 \quad \text{for} \quad k > 0$$

Parametrize $\sigma_{str} = \sigma_{str}^0\, t^\nu$, where $t \equiv \frac{T_{tr}(\mu) - T}{T_{tr}(\mu)} \rightarrow +0$

and find total pressure and total entropy density

for $\mu = 0$ (baryonic chemical potential)

$$p_{tot} = p_v(T) - \frac{\sigma_{surf}(T)}{R} \equiv \frac{\sigma_{surf}(T)}{R} - \frac{\sigma_{str}}{\pi R^2} \rightarrow \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \left[\sigma_{surf} - \frac{\omega_k}{\pi}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{k+1}{k}}\right]$$

$$s_{tot} = \left(\frac{\partial\, p_{tot}}{\partial\, T}\right)_\mu \rightarrow \underbrace{\frac{1}{k\,\sigma_{str}}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial\, \sigma_{str}}{\partial\, T}\, \sigma_{surf}}_{dominant \quad since \quad \sigma_{str} \rightarrow\, 0} + \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial\ \sigma_{surf}}{\partial\ T} - \frac{k+2}{\pi\, k}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{2}{k}} \frac{\partial\ \sigma_{str}}{\partial\ T}$$

For finite σ_{surf} and $\frac{\partial\ \sigma_{str}}{\partial\ T} < 0$ $\Rightarrow$ $\sigma_{surf} < 0$ since $s_{tot} > 0$

Comparison with LQCD

$\Rightarrow$ Assume: we can apply our results to LQCD data with $L \gg R$

For $\sigma_{str} \rightarrow 0 \quad \Rightarrow \quad R \rightarrow \frac{2\,\sigma_{surf}}{p_v}$ and lattice entropy is

$$\frac{S_{lat}}{L} = -\frac{1}{L}\,\frac{\partial F_{lat}}{\partial T} \rightarrow - \frac{s_{tot}\,k\,\sigma_{str} R}{\sigma_{surf}} = - \frac{s_{tot}\,k\,\omega_k}{\sigma_{surf} R^{k-1}} \rightarrow t^{\nu-1}$$

$\Rightarrow$ again $\sigma_{surf} < 0$

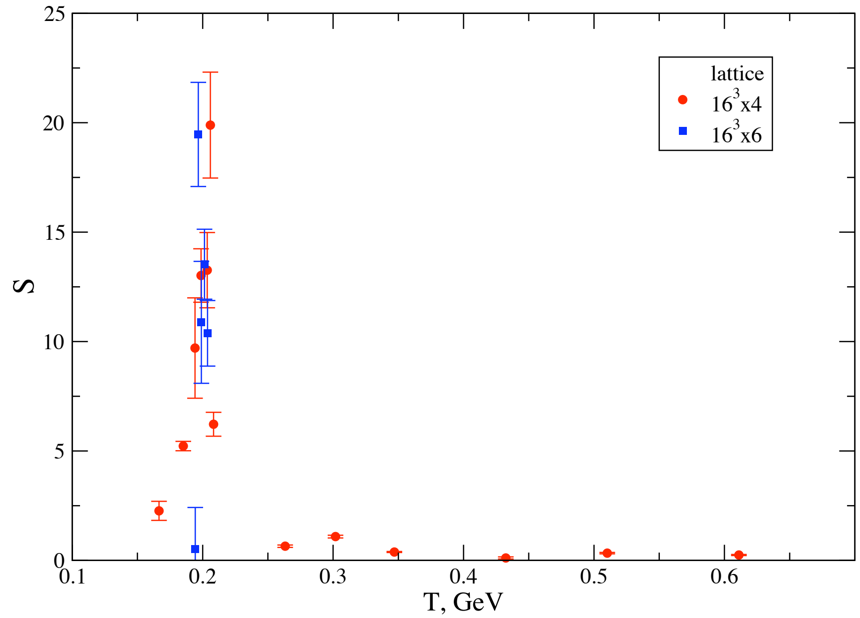
$\Rightarrow$ S_{lat} diverges for $\nu < 1$ and $R \rightarrow \infty$

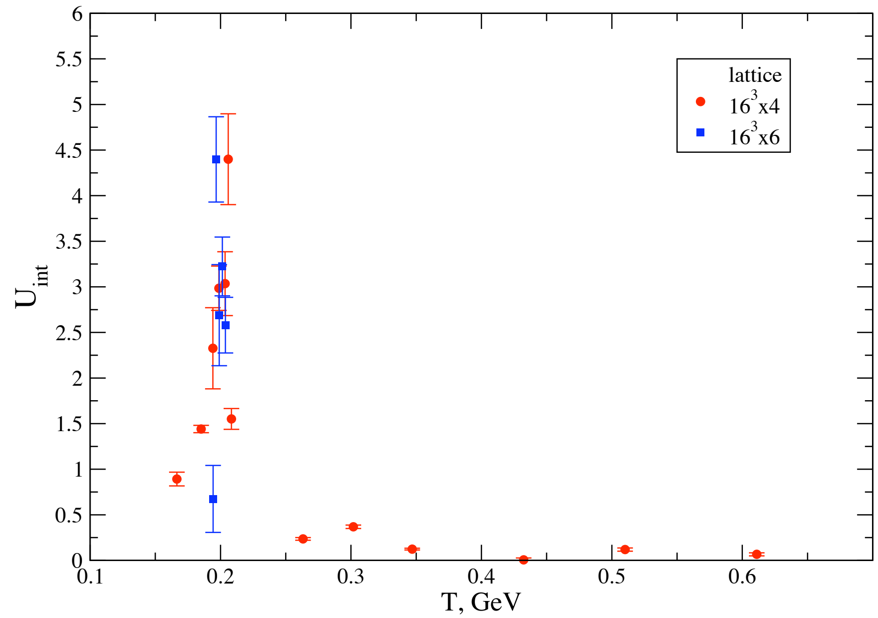
$\Rightarrow$ S_{lat} has a sharp inclease for $\nu < 1$ and $R \rightarrow R_{lat} < \infty$

Can we verify this result with LQCD data?

Mysterious Maximum

Entropy and Internal Energy





Similarly, consider the fall down of S_{lat} due to strong s_{tot} decrease

This explains 'a mysterious maximum in S_{lat}' (E. Shuryak)

Why Does the String Entropy Diverge at the Cross-over ?

$$\frac{S_{str}}{L} = \frac{\sigma^0_{str}\,\nu}{T_{tr}}\, t^{\nu-1} \rightarrow \frac{\nu}{T_{tr}} \left[\frac{\sigma^0_{str}}{\omega_k^{1-\nu}}\right]^{\frac{1}{\nu}} \boxed{R^{\frac{k(1-\nu)}{\nu}}}$$

String entropy diverges for $\nu < 1$ and $t \rightarrow +0$.

$\boxed{R \text{ power } \frac{k(1-\nu)}{\nu}}$ is FRACTAL for any $\nu \neq \frac{k}{k+n}$ where $n = 1, 2, 3, ...$

In LQCD the fractal structures are well known.

In this model the fractals appear at $t \rightarrow +0$ as surface deformations due to zero total pressure inside the color tube $\Rightarrow$ at NO ENERGY costs!

=> At the cross-over temperature there exist FRACTALS!

Conclusions

The relation between the string tension and the surface tension of QGP bags is found! It allows us to determine the surface tension of QGP bags directly from Lattice QCD.

The surface tension of QGP bags at T = 0 is larger than the critical one, but at the cross-over T ~ 170 MeV the surface tension is negative!

At the cross-over T ~ 170 MeV there exist fractals => fractal surfaces!

Negative surface tension of QGP bags at the cross-over does not allow the quark matter to survive till present time.

Thanks for your attention!

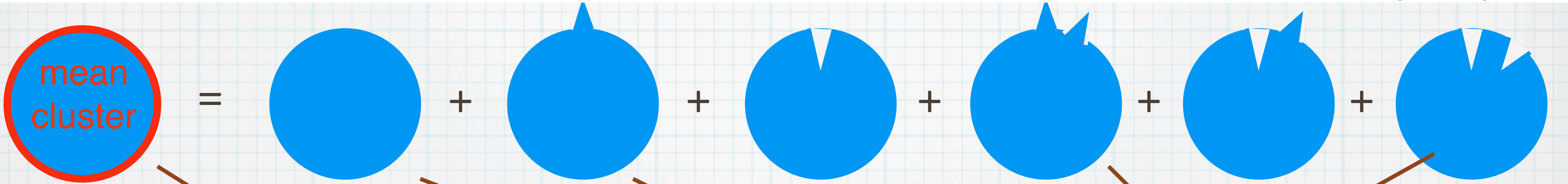
Backup slides

Surface Free Energy: F = E -TS

To find surface F one has to count for ALL surface deformations together with energy costs

Can be exactly done within Hills and Dales Model for v-volume cluster:

K.A.B. et al, PRE 72 (2005)



$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part} \underbrace{\exp(S)}_{Entropy\ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ Energy} \times \left\{1 + \left(\underbrace{w_H N_H}_{1\ Hill} + \underbrace{w_D N_D}_{1\ Dale}\right)\exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part} \underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

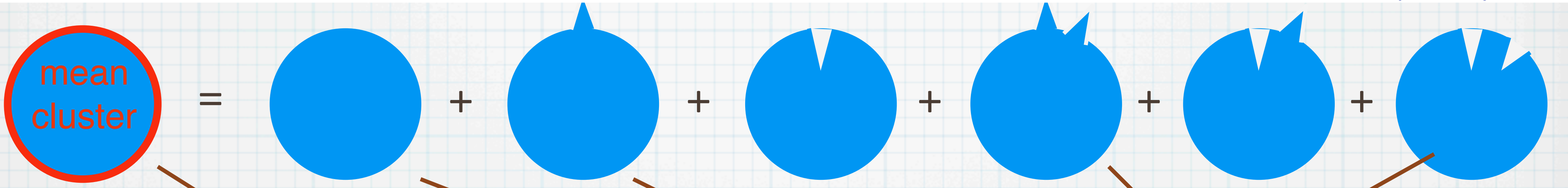
$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \quad \geq \quad F \quad \geq \quad \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}\,, \qquad \lambda_L \approx 0.28\, T_c^{-1}, \qquad \lambda_U \approx 1.06\, T_c^{-1}$$

K.A.B. & Elliott, UJP 52 (2007)

Surface Free Energy: F = E -TS

To find surface F one has to count for ALL surface deformations together with energy costs
Can be exactly done within Hills and Dales Model for v-volume cluster:

K.A.B. et al, PRE 72 (2005)



$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp(S)}_{Entropy\ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ Energy} \times \left\{1 + \Big(\underbrace{w_H N_H}_{1\ Hill} + \underbrace{w_D N_D}_{1\ Dale}\Big) \exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

$$\sigma_0 (1 - \lambda_L T)\, v^{\frac{2}{3}} \quad \geq \quad F \quad \geq \quad \sigma_0 (1 - \lambda_U T)\, v^{\frac{2}{3}}, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

K.A.B. & Elliott, UJP 52 (2007)

Thus, there is NOTHING wrong, if surface F < 0 above critical T!
This means only that entropy dominates!