Superdiffusion in Nonideal plasma

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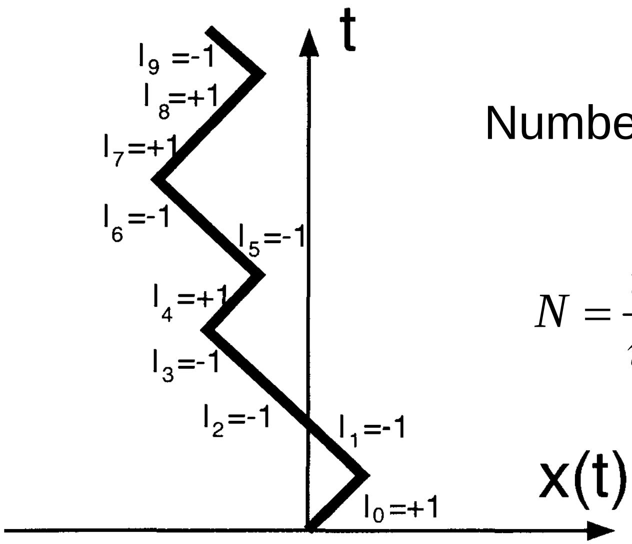
JIHT

Classical diffusion

Sornette, 2000



Number of steps

$$N = \frac{t}{\tau}$$

Langevin Equation

$$x(t + \tau) = x(t) + l(t)$$

$$x(t) = l(t-\tau) + l(t-2\tau) + \ldots + l(\tau) + l(0)$$

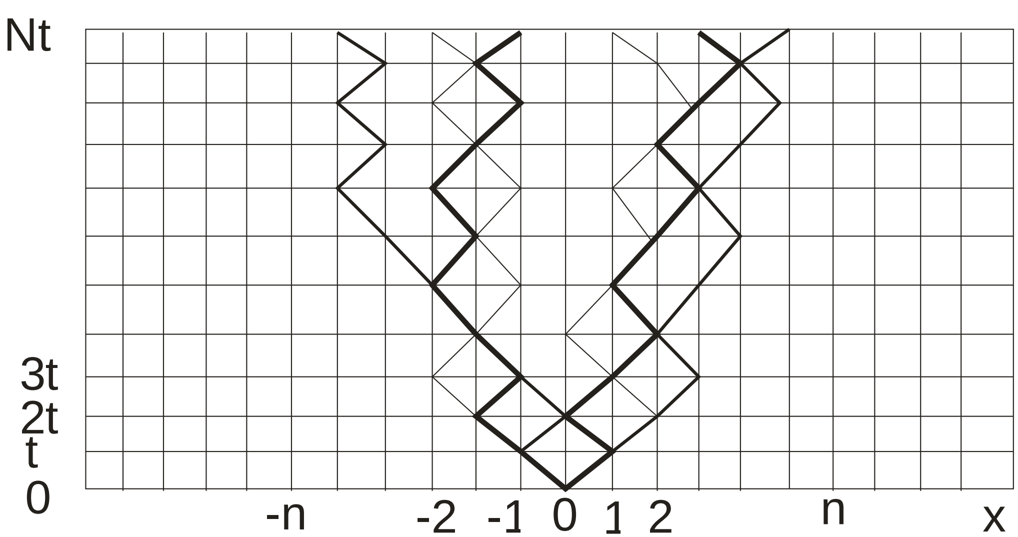
l(t) – random step of length l

Wiener process (i.i.d.)
Identically isotropic distributed

$$\langle l(t) l(t') \rangle = \delta_{tt'} \langle l^2(t) \rangle$$

Sum of random variables

$$x(t) = \sum_{i=1}^{N} l(i)$$



Sum of random variables

$$x(t) = \sum_{i=1}^{N} l(i)$$

Drift

$$< x(t) >= \sum_{i=1}^{N} < l(i) >= N < l >= \frac{< l > t}{\tau} = ut$$

Diffusion

$$< x^2(t) > - < x(t) >^2 = \sum_{i=1}^{N} \sum_{i=1}^{N} \left(< l_i l_j > - < l_i >< l_i > \right) = \sum_{i=1}^{N} \sum_{i=1}^{N} C_{ij}$$

Correlation function

$$C_{ij} =< l_i l_j > - < l_i >< l_i >$$

Uncorrelated $\{l_i\}$

$$C_{ij} = \left(< l^2 > - < l >^2 \right) \delta_{ij}$$

$$< x^2(t) > - < x(t) >^2 = N \left[< l^2 > - < l >^2 \right] = N\sigma^2 = \frac{t}{\tau} \sigma^2 = 2Dt$$

Diffusion coefficient

$$D = \frac{\sigma^2}{2\tau} \qquad \sigma^2 =< l^2 > - < l >^2$$

Diffusion in d - dimension space

$$< x^2(t) > - < x(t) >^2 = 2dDt$$

Long-range correlations

$$C(n) = \langle l_i l_{i+n} \rangle - \langle l_i \rangle \langle l_{i+n} \rangle,$$

$$C(n) = \frac{1}{N-n} \sum_{i=1}^{N-n} l_i l_{i+n} - \left(\frac{1}{N} \sum_{i=1}^{N} l_i \right)^2 .$$

$$x(t) = l(t-\tau) + l(t-2\tau) + ... + l(\tau) + l(0)$$

$$\langle [x(t)]^2 \rangle = tC(0) + 2\sum_{k=1}^{t} (t-k) C(k)$$

$$= t\left[C(0) + 2\sum_{k=1}^{t} C(k) \right] - 2\sum_{k=1}^{t} kC(k)$$

If sum converges, we have diffusion, but not D

$$\sum_{k=1}^{t} C(k)$$

Superdiffusion

$$< x^2(t) >= tC(0) + 2\sum_{k=1}^{t}(t-k)C(k)$$

Classical diffusion

$$l = const, \quad w^{\pm} = 1/2$$

$$\sum_{k=1}^{t}(t-k)C(k) << tC(0)$$

$$< x^2(t) >= 2Dt$$

$$C(k) \sim \frac{1}{k^{\nu}} \qquad \sum_{k=1}^{t} C(k) = \int_1^t C(k)dk$$

Superdiffusion

$$l \neq const, \quad w^{\pm} = w(i, f) \neq 1/2$$

$$\sum_{k=1}^{t}(t-k)C(k) > tC(0)$$

$$< x^2(t) >\sim t^{2-\nu}, \quad \nu < 1$$

$$< x^2(t) >\sim t \ln t, \quad \nu = 1$$

Superdiffusion – general case in a plasma

Classical diffusion in a nonideal plasma does not exist

Superdiffusion in plasma

Superdiffusion parameter in a gas

$$\tau_{coll} = \frac{a}{u_{therm}}, \quad \tau_{flight} = \frac{\lambda}{u_{therm}}, \quad \lambda = \frac{1}{na^2}, \quad S_{sd}^{gas} = \frac{\tau_{coll}}{\tau_{flight}} = \frac{a}{\lambda} = na^3 << 1$$

Superdiffusion parameter in a plasma

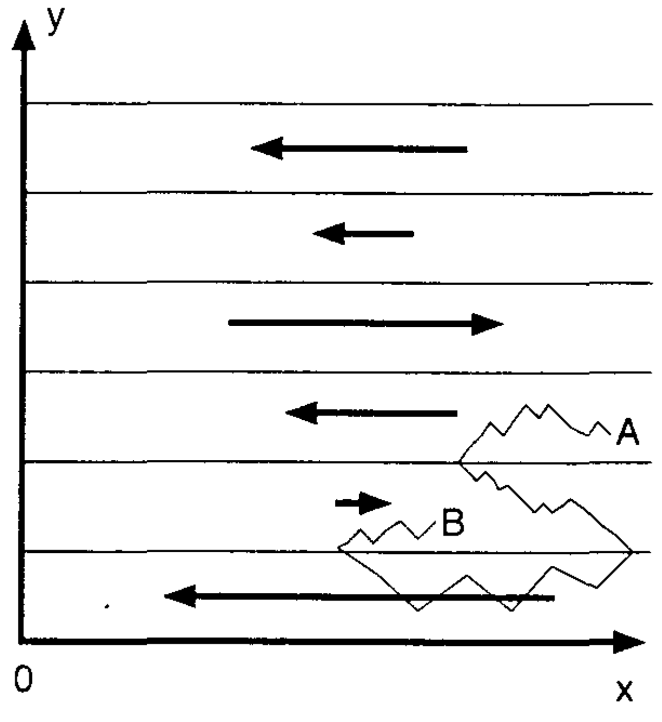
$$S_{sd}^{plasma} = \frac{\tau_{coll}}{\tau_{flight}} = \frac{a}{\lambda} = nr_D^3 = \Gamma$$

Theory of superdiffusion in plasma is based on 2-point time – correlation Green function

$$G(t',\mathbf{r}',\mathbf{p}';t,\mathbf{r},\mathbf{p}) = G^{RPA}(t',\mathbf{r}',\mathbf{p}';t,\mathbf{r},\mathbf{p}),\ G^{LD}(t',\mathbf{r}',\mathbf{p}';t,\mathbf{r},\mathbf{p}),..$$

Dyson eqn for propagator G2, etc.

Superviscosity



Vertical movement - diffusive

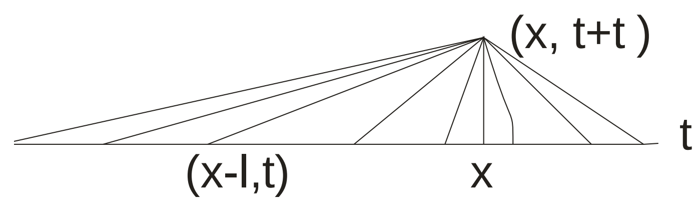
Horisontal movement – sum of varables

$$x_{\|}(t) \sim D^{-1/2}\ t^{\frac{3}{4}}$$

Probability density function (pdf)

Master equation for pdf

$$P(x,t+\tau)=\int \Pi(l)P(x-l,t)dl$$



О методе ФПВ в ламинарных и турбулентных потоках

Э.Е.Сон

15 июля 2008 г.

Содержание

Fokker-Plank equation (order l2)

$$j(x,t)=vP(x,t)-D\frac{\partial P(x,t)}{\partial x}$$

$$\frac{\partial P(x,t)}{\partial t}=-\frac{\partial j(x,t)}{\partial x}=-v\frac{\partial P(x,t)}{\partial x}+D\frac{\partial^2 P(x,t)}{\partial x^2}$$

Pdf in diffusion

1D-diffusion (0,L)

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} \qquad \left.\frac{\partial(x,t)}{\partial x}\right|_0 = \left.\frac{\partial(x,t)}{\partial x}\right|_L = 0.$$

Solution

$$c(x,t) = \int_0^L c_0(x')G(x-x',t)dx'$$

Green Function

$$G(x,t) = \frac{1}{\sqrt{2\pi Dt}}e^{-\frac{x^2}{2Dt}}$$

Probability function

$$dP(c) = \frac{dx(c)}{\sum_{c'} dx(c')},$$

Pdf

$$F(c) = \frac{dx(c)/dc}{\int_0^{c_0}(dx(c')/dc')dc'}.$$

Equation for pdf

$$\frac{\partial F}{\partial t} = DA^2F\frac{\partial}{\partial c}\left(\frac{1}{F}\frac{\partial F}{\partial c}\right)$$

$$F(t,c) = \int_0^L f(t,x,c)dx.$$

Pdf equations

Diffusion

$$\frac{\partial f(t,x,c)}{\partial t} = -\frac{\partial}{\partial c}\left(f(t,x,c)D\frac{\partial^2 \overline{c}(x,t)}{\partial x^2}\right)$$

$$\overline{F} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \nu\frac{\partial^2 \overline{v}}{\partial x^2},$$

$$\frac{\partial f(t,x,c)}{\partial t} + \overline{v}\frac{\partial f(t,x,c)}{\partial x} + \overline{\Pi}\frac{\partial f(t,x,c)}{\partial c} = 0,$$

$$\overline{\Pi} = D\frac{\partial^2 \overline{c}}{\partial x^2}.$$

Diffusion in the flow

$$\frac{\partial f}{\partial t} + \overline{v}\frac{\partial f}{\partial x} + \overline{F}\frac{\partial f}{\partial v} + \overline{\Pi}\frac{\partial f}{\partial c} = 0,$$

2- and N-point pdf

Definition 2- point pdf

$$dP_2(t,\vec{x}^{(1)},\vec{x}^{(2)},\vec{v}^{(1)},\vec{v}^{(2)},c^{(1)},c^{(2)}) = f_2(t,\vec{x}^{(1)},\vec{x}^{(2)},\vec{v}^{(1)},\vec{v}^{(2)},c^{(1)},c^{(2)})d^3v^{(1)}d^3v^{(2)}dc^{(1)}dc^{(2)}.$$

Equation for 2- point pdf

$$\frac{\partial f_2}{\partial t} + \overline{v_k^{(1)}}\frac{\partial f_2}{\partial x_k^{(1)}} + \overline{v_k^{(2)}}\frac{\partial f_2}{\partial x_k^{(2)}} + \overline{F_k^{(1)}}\frac{\partial f_2}{\partial v_k^{(1)}} + \overline{F_k^{(2)}}\frac{\partial f_2}{\partial v_k^{(2)}} + \overline{\Pi_k^{(1)}}\frac{\partial f_2}{\partial c_k^{(1)}} + \overline{\Pi_k^{(2)}}\frac{\partial f_2}{\partial c_k^{(2)}} = 0,$$

N- point pdf

$$\frac{\partial f_N}{\partial t} + \sum_{n=1}^{N}\overline{v_k^{(n)}}\frac{\partial f_N}{\partial x_k^{(n)}} + \sum_{n=1}^{N}\overline{F_k^{(n)}}\frac{\partial f_N}{\partial v_k^{(n)}} + \sum_{n=1}^{N}\overline{\Pi^{(n)}}\frac{\partial f_N}{\partial c^{(n)}} = 0.$$

Pdf equation for correlated system (SCCP, Turbulence)

$$\frac{\partial < f >}{\partial t} + v_k\frac{\partial < f >}{\partial x_k} = -\frac{\partial}{\partial v_k}\left(< f > \overline{F_k}\Big|_{\overline{\vec{v}}=\vec{v}}\right) - \frac{\partial}{\partial c}\left(< f > \overline{\Pi}\Big|_{\overline{c}=c}\right).$$

Fractional derivatives and superdiffusion

$$f(z) = \frac{1}{2\pi i} \oint \frac{f(\zeta)}{\zeta - z} d\zeta$$

Integral Caushy

Riemann-Liouville Fractional integral

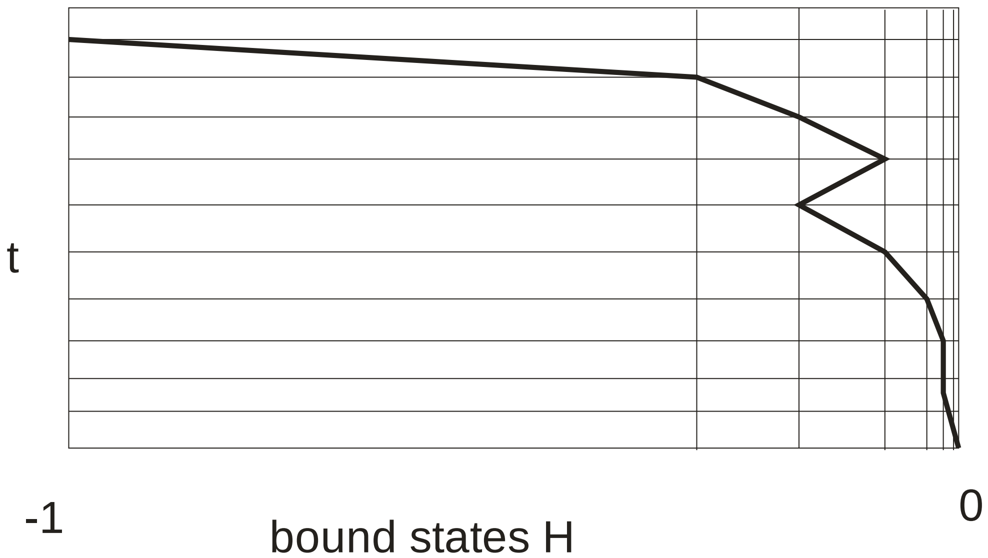
$$(I_{a+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-y)^{\alpha-1} f(y)\, \mathrm{d}y$$

$$(\partial_+^{\alpha} f)(x) = \frac{\mathrm{d}}{\mathrm{d}x} (I_+^{1-\alpha} f)(x)$$

$$\partial_+^{\alpha} \mu(t) = 0$$

$$\mu(t) = C_0\ t^{\alpha-1}$$

Diffusion in the energy space



Nonclassical diffusion

$$\varepsilon_n = -\frac{I}{n^2},\ x_n = -\frac{1}{n^2}$$

X(0)=0, flux down - recombination

X(0)=-1, flux up - ionization

$$\Delta x_n = x_{n+1} - x_n$$

Plasma problems (tbs)

- Superdiffusion in x-space
- Superdiffusion in energy space
  - Cold plasma
  - Thermal plasma
- Quantum Superdiffusion
- Temperature Green Function approach
- Superthermal conductivity
- Supertransport in a plasma
- Pdf equation in a plasma
- Difference – Differential equations !!!
- Difference equations in the nature of quantum physics