GENERALIZED BETH-UHLENBECK EOS FOR THE NONIDEAL QUARK PLASMA

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Punchline:

- Hagedorn: "A system of interacting elementary particles can be reformulated as a system of noninteracting resonances" $\longrightarrow$ Resonance Gas
- Bound states can be treated as a new species $\Longrightarrow$ chemical picture
- Physical picture: there are also scattering states!
  EoS with bound and scattering states: Beth-Uhlenbeck EoS (1936/37)
- Generalized Beth-Uhlenbeck EoS includes the Mott transition:
  bound states $\Longrightarrow$ resonances in the scattering continuum

Seminar on Nonideal Plasma Physics; Moscow, 23.-24.11.2011

PARTITION FUNCTION FOR QUANTUM CHROMODYNAMICS (QCD)

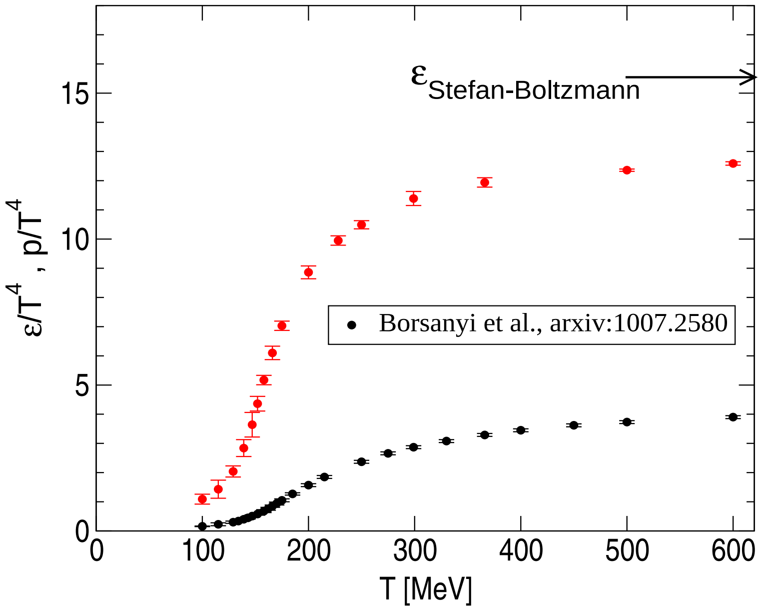
- Partition function as a Path Integral (imaginary time $\tau = i\,t$, $0 \le \tau \le \beta = 1/T$) $\Rightarrow$ PS I

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \exp\left\{-\int_0^\beta d\tau \int_V d^3x\, \mathcal{L}_{QCD}(\psi,\bar{\psi},A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength: $F^a_{\mu\nu}(A) = \partial_\mu A^a\nu - \partial_\nu A^a_\mu + g\, f^{abc}[A^b_\mu, A^c_\nu]$

$$\mathcal{L}_{QCD}(\psi,\bar{\psi},A) = \bar{\psi}[i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu]\psi - \frac{1}{4}F^a_{\mu\nu}(A)F^{a,\mu\nu}(A)$$

- Numerical evaluation: Lattice gauge theory simulations (hotQCD, Wuppertal-Budapest)



- Equation of state: $\varepsilon(T) = -\partial \ln Z[T,V,\mu]/\partial\beta$
- Phase transition at $T_c = 155$ MeV
- Problem: Interpretation ?

$\varepsilon/T^4 = \frac{\pi^2}{30}N_\pi \sim 1$ (ideal pion gas)

$\varepsilon/T^4 = \frac{\pi^2}{30}(N_G + \frac{7}{8}N_Q) \sim 15.6$ (quarks and gluons)

- Hadron resonance gas

Partition function for Quantum Chromodynamics (QCD)

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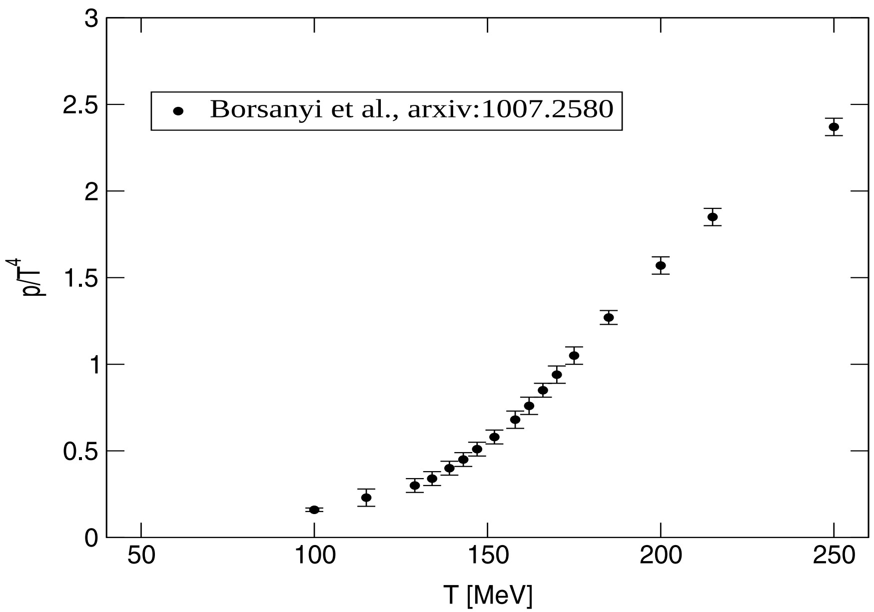
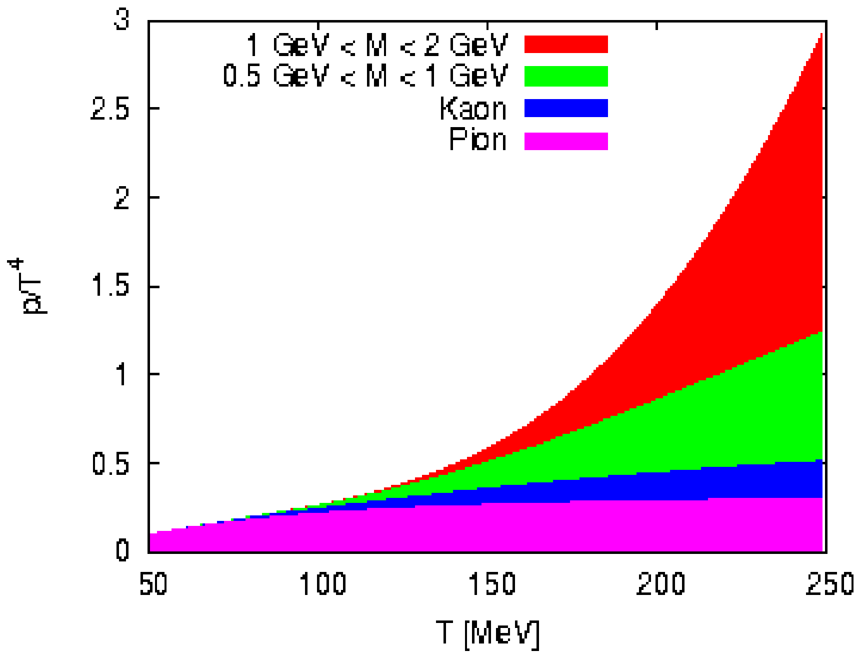
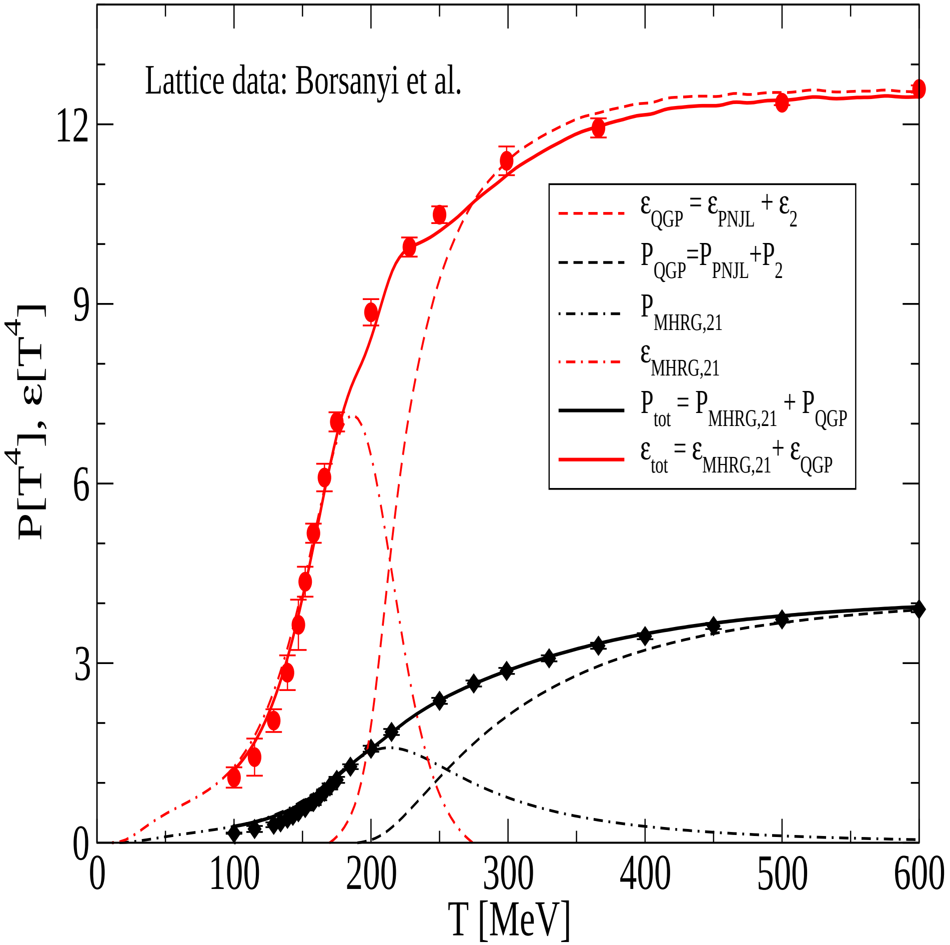




HYBRID APPOACH: PNJL & MOTT-HAGEDORN RESONANCE GAS

$$P_{\rm tot}(T,\{\mu_j\}) = P_{\rm PNJL}(T,\{\mu_i\}) + \sum_{r=M,B} \delta_r g_r \int ds\, A_r(s,m_r;T) \int \frac{d^3p}{(2\pi)^3} T \ln\left\{1+\delta_r \exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)\right\}$$



Spectral function for hadronic resonances:

$$A_r(s,m;T) = N_s \frac{m\Gamma_r(T)}{(s-m^2)^2 + m^2\Gamma_r^2(T)}$$

Ansatz motivated by chemical freeze-out model:

$$\Gamma_r(T) = \tau_r^{-1}(T) = \sum_h \lambda < r_r^2 >_T < r_h^2 >_T n_h(T)$$

Apparent phase transition at $T_c \sim 165$ MeV

Hadron resonances present up to $T_{\rm max} \sim 250$ MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)

Prog. Part. Nucl. Phys. 53, 197 (2004)

Blaschke, Prorok & Turko, in preparation

Hadronic states above T_c ! See also: Ratti, Bellwied et al., arXiv:1109.6243 [hep-ph]

Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\,t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}[i\gamma^\mu\partial_\mu - m - \gamma^0(\mu + \lambda_8\mu_8 + i\lambda_3\phi_3]\psi - \mathcal{L}_{\rm int} + U(\Phi)]\right\}$$

Polyakov loop: $\Phi = N_c^{-1}{\rm Tr}_c[\exp(i\beta\lambda_3\phi_3)]$ Order parameter for deconfinement

- Current-current interaction (4-Fermion coupling) and KMT determinant interaction

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\dots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2 - K[\det{}_f(\bar{q}(1+\gamma_5)q) + \det{}_f(\bar{q}(1-\gamma_5)q)]$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D\, {\rm e}^{-\sum_{M,D}\frac{M_M^2}{4G_M} - \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\},\Phi] + U(\Phi) + V_{\rm KMT}}$$

- Collective quark fields: Mesons (M_M) and Diquarks (Δ_D); Gluon mean field: Φ
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\ t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{-\int^{\beta} d\tau \int_V d^3x [\bar{q}(i\gamma^\mu\partial_\mu - m_0 - \gamma^0\mu)q + \sum_{M=\pi,\sigma} G_M(\bar{q}\Gamma_M q)^2]\right\}$$

- Couplings: $G_\pi = G_\sigma = G_S$ (chiral symmetry)
- Vertices: $\Gamma_\sigma = \mathbf{1}_D \otimes \mathbf{1}_f \otimes \mathbf{1}_c$; $\Gamma_\pi = i\gamma_5 \otimes \vec{\tau} \otimes \mathbf{1}_c$
- Bosonization (Hubbard-Stratonovich Transformation)

$$\exp\left[G_S(\bar{q}\Gamma_\sigma q)^2\right] = \text{const.} \int \mathcal{D}\sigma \exp\left[\frac{\sigma^2}{4G_S} + \bar{q}\Gamma_\sigma q\sigma\right]$$

- Integrate out quark fields $\longrightarrow$ bosonized partition function

$$Z[T,V,\mu] = \int \mathcal{D}\sigma\mathcal{D}\pi \exp\left\{-\frac{\sigma^2+\pi^2}{4G_S} + \frac{1}{2}\text{Tr}\ \ln S^{-1}[\sigma,\pi]\right\}$$

- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)

Mean Field plus (Gaussian) Fluctuations

- Separate the mean-field part of the quark determinant

$$\mathrm{Tr}\ \ln S^{-1}[\sigma,\pi] = \mathrm{Tr}\ \ln S^{-1}_{\mathrm{MF}}[m] + \mathrm{Tr}\ \ln[1+(\sigma+i\gamma_5\vec{\tau}\vec{\pi})S_{\mathrm{MF}}[m]]$$

- Mean-field quark propagator

$$S_{\mathrm{MF}}(\vec{p},i\omega_n;m) = \frac{\gamma_0(i\omega_n+\mu)-\vec{\gamma}\cdot\vec{p}+m}{(i\omega_n+\mu)^2-E_p^2}$$

- Expand the logarithm: $\ln(1+x) = -\sum_{n=1}^{\infty}(-1)^n x^n/n = x - x^2/2 + ...$
- Thermodynamic potential in Gaussian approximation

$$\begin{aligned}\Omega(T,\mu) &= -T\ln Z(T,\mu) = \Omega_{\mathrm{MF}}(T,\mu) + \Omega^{(2)}_{\mathrm{M}}(T,\mu) + \ldots \\ \Omega^{(2)}_{\mathrm{M}}(T,\mu) &= \frac{N_M}{2}\int\frac{d^2p}{(2\pi)^3}\frac{1}{\beta}\sum_n \mathrm{e}^{i\nu_n\eta}\ln\left[1-2G_S\Pi_M(\vec{p},i\nu_n)\right]\ ,\quad N_\sigma=1,\ N_\pi=3\end{aligned}$$

- Mesonic polarization loop

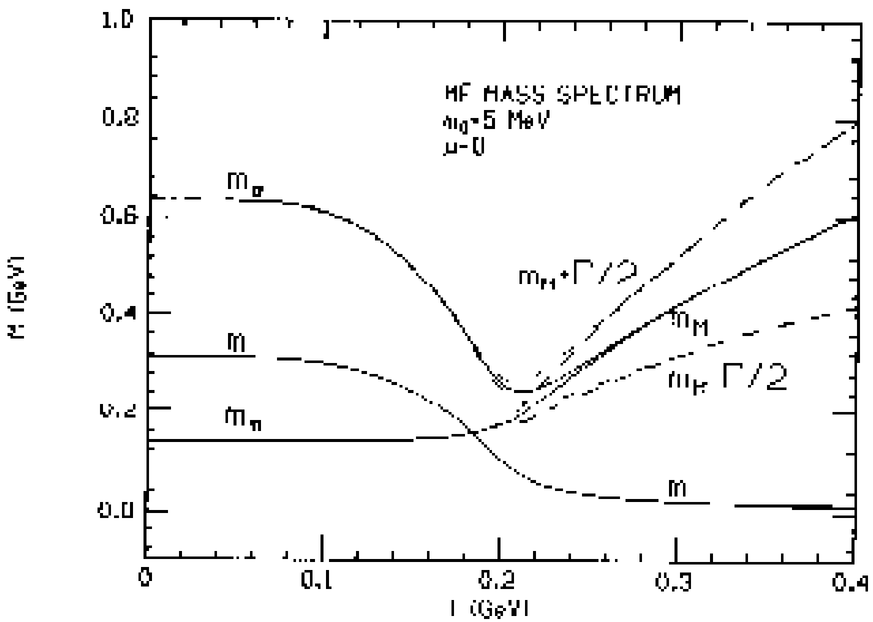
$$\Pi_M(\vec{p},i\nu_n) = -\frac{1}{\beta}\sum_{n'}\mathrm{e}^{i\nu_{n'}\eta}\int\frac{d^2k}{(2\pi)^3}\mathrm{Tr}\left[\Gamma_M S_{\mathrm{MF}}(-\vec{k},-i\omega_{n'})\Gamma_M S_{\mathrm{MF}}(\vec{k}+\vec{p},i\omega_{n'}+i\nu_n)\right]$$

GENERALIZED BETH-UHLENBECK EOS: NJL MODEL RESULTS

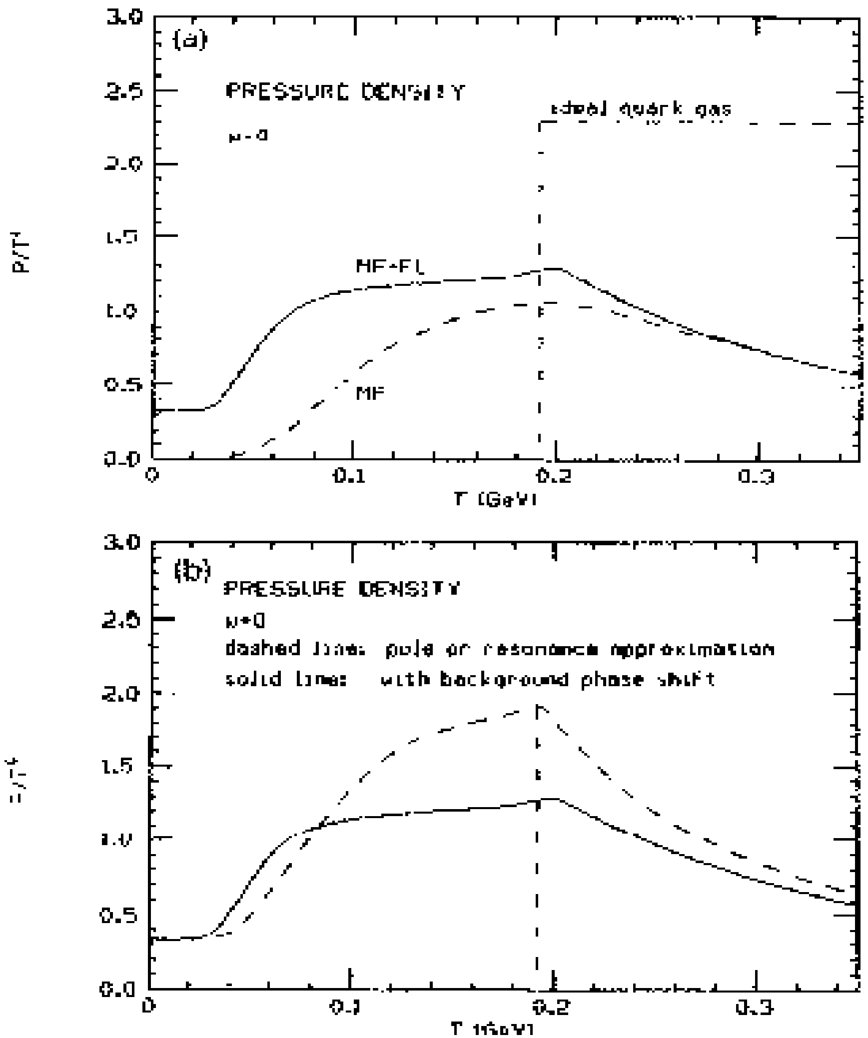
Generalized Beth-Uhlenbeck approach:

- Deuterons in nuclear matter
  Schmidt, Röpke, Schulz, Ann. Phys. (1990)
- Mesons in quark matter
  Hüfner, Klevansky, Zhuang, Voß, Ann. Phys. (1994)

P. Zhuang et al. / Nuclear Physics A 576 (1994) 525–552



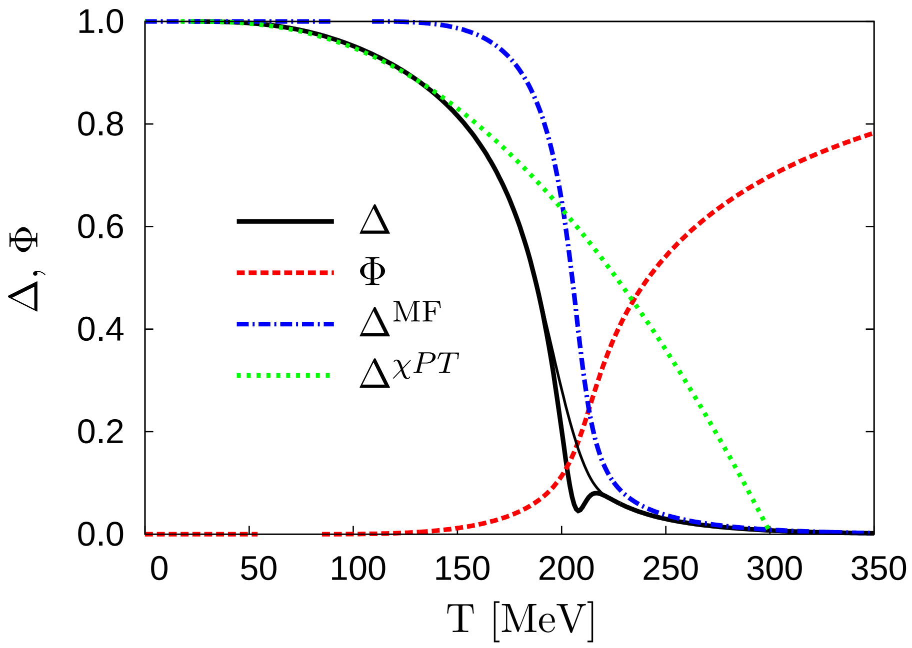
P. Zhuang et al. / Nuclear Physics A 576 (1994) 525–552

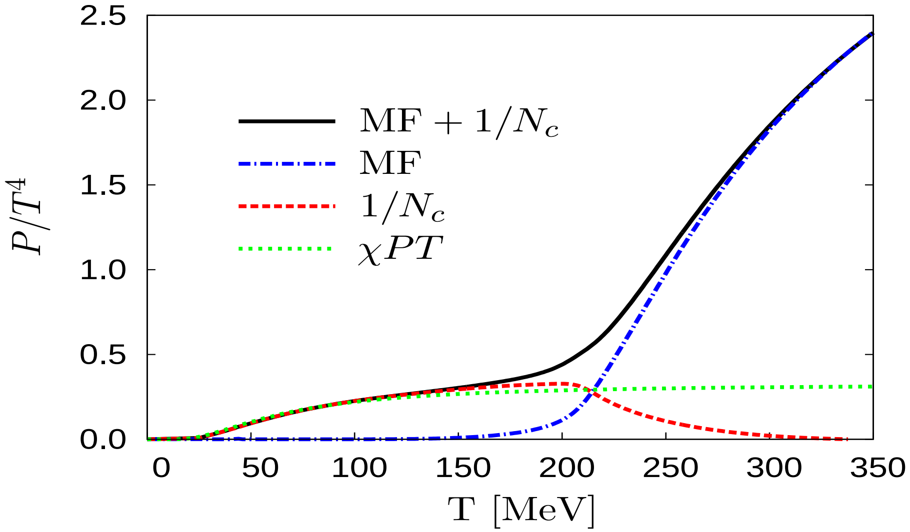


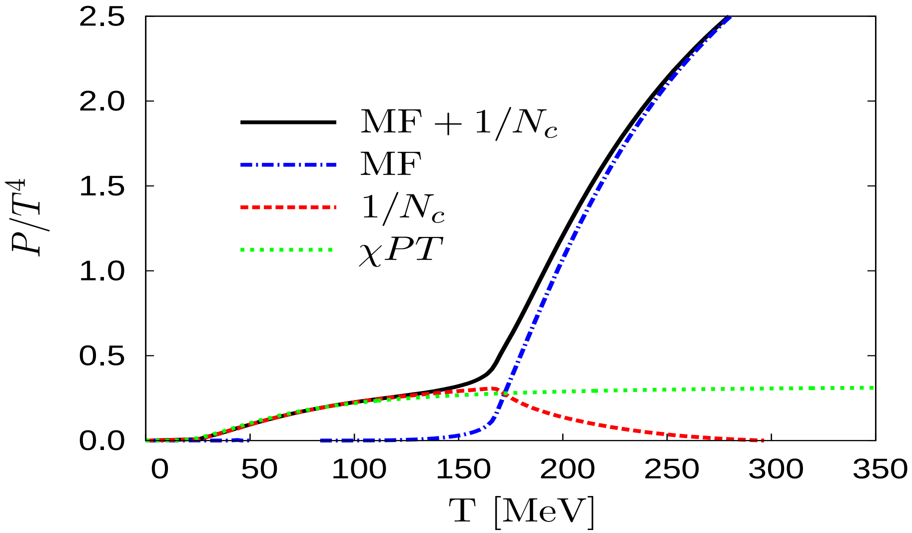
GEN. BETH-UHLENBECK EOS: NONLOCAL PNJL RESULTS

Nonlocal PNJL model beyond meanfield:

Blaschke et al., Yad. Fiz. 71 (2008)
Radzhabov et al., PRD 83 (2011) 116004







PL-Potential with $T_0 = 270$ MeV (upper panel),
and $T_0 = 208$ MeV (lower panel) $\Longrightarrow$

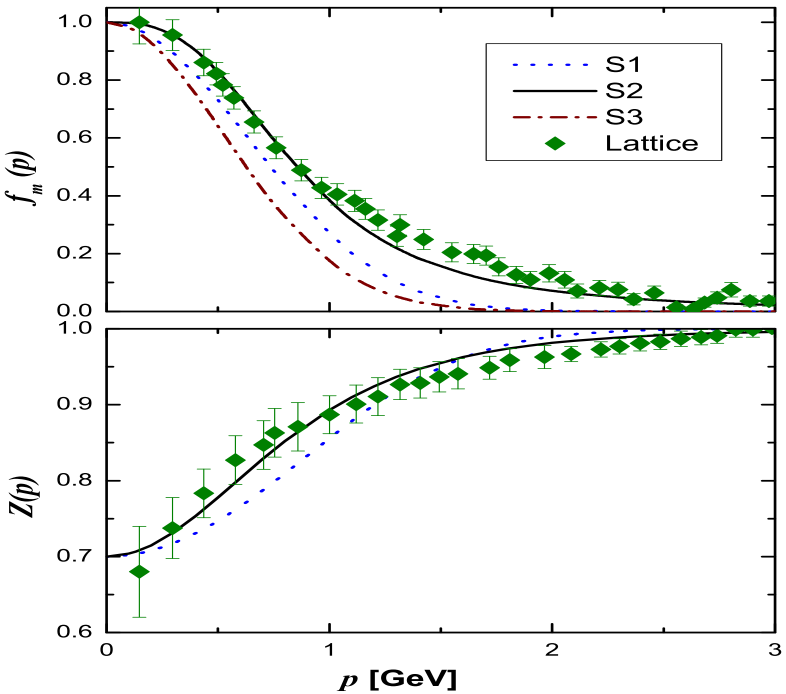
NONLOCAL PNJL MODEL VS. LATTICE QCD

$$S_E = \int d^4x \left\{ \bar{\psi}(x)\left(-i\gamma_\mu D_\mu + \hat{m}\right)\psi(x) - \frac{G_S}{2}[j_a(x)j_a(x) - j_P(x)j_P(x)] + \mathcal{U}\left(\Phi[A(x)]\right) \right\} ,$$

Nonlocal currents

$$j_a(x) = \int d^4z\, g(z)\, \bar{\psi}\left(x + \frac{z}{2}\right)\, \Gamma_a\, \psi\left(x - \frac{z}{2}\right)$$

$$j_P(x) = \int d^4z\, f(z)\, \bar{\psi}\left(x + \frac{z}{2}\right)\, \frac{i\overleftrightarrow{\partial\!\!\!/}}{2\,\kappa_p}\, \psi\left(x - \frac{z}{2}\right)$$



Formfactors fitted to Lattice results

$$g(q) = \frac{1+\alpha_z}{1+\alpha_z\; f_z(q)} \frac{\alpha_m\; f_m(q) - m\;\alpha_z f_z(q)}{\alpha_m - m\;\alpha_z}$$

$$f(q) = \frac{1+\alpha_z}{1+\alpha_z\; f_z(q)} f_z(q)$$

$$f_m(q) = \left[1 + \left(q^2/\Lambda_0^2\right)^{3/2}\right]^{-1}$$

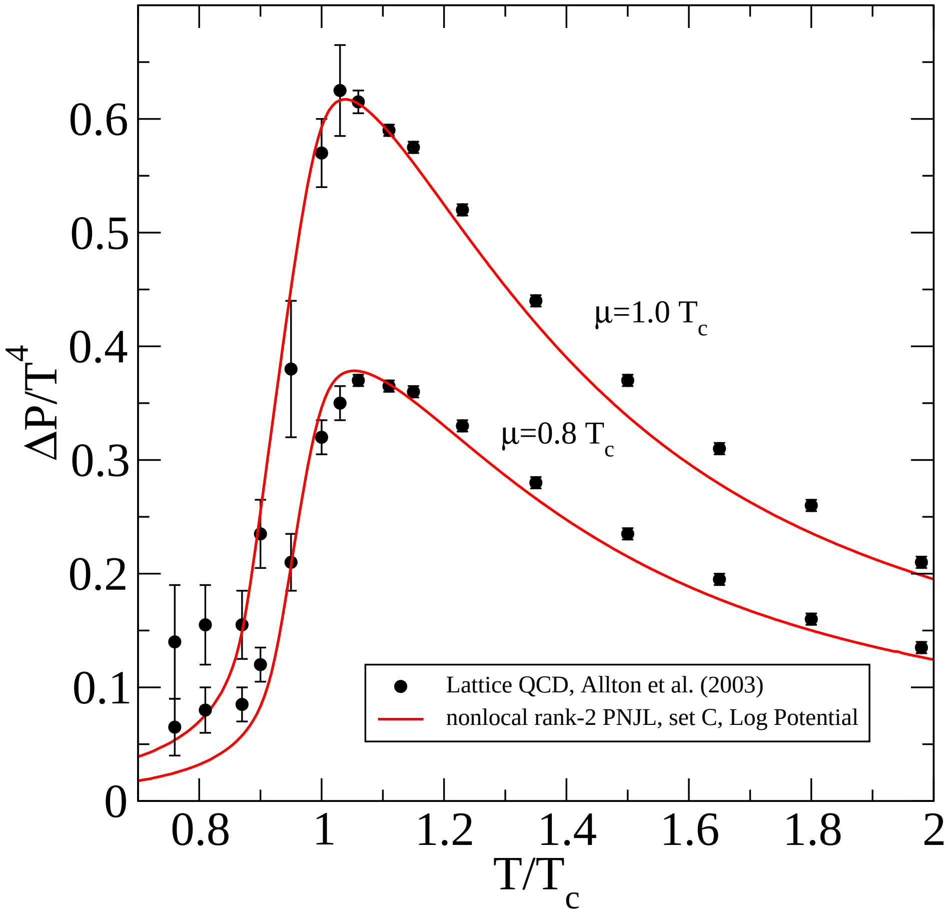
$$f_z(q) = \left[1 + \left(q^2/\Lambda_1^2\right)\right]^{-5/2} .$$

Parappilly et al., Phys. Rev. D

Noguera, Scoccola, PRD 78, 114002 (2008)

Nonlocal PNJL model vs. Lattice QCD (II)

$$\Omega^{\rm MFA} = -\frac{4T}{\pi^2}\sum_c \int_{p,n} \ln\left[\frac{(\rho^c_{n,\vec{p}})^2 + M^2(\rho^c_{n,\vec{p}})}{Z^2(\rho^c_{n,\vec{p}})}\right] + \frac{\sigma_1^2 + \kappa_p^2\ \sigma_2^2}{2G_S} + \mathcal{U}(\Phi,T)$$



Mass function and WF renormalization

$$M(p) = Z(p)\,[m + \sigma_1\ g(p)]$$

$$Z(p) = [1 - \sigma_2\ f(p)]^{-1}\,.$$

Finite T,μ formalism: Matsubara

$$(\rho^c_{n,\vec{p}})^2 = [(2n+1)\pi T - i\mu + \phi_c]^2 + \vec{p}^{\,2}\ ,$$

Polyakov-loop potential:

Dexheimer-Schramm, arXiv:0910.1312

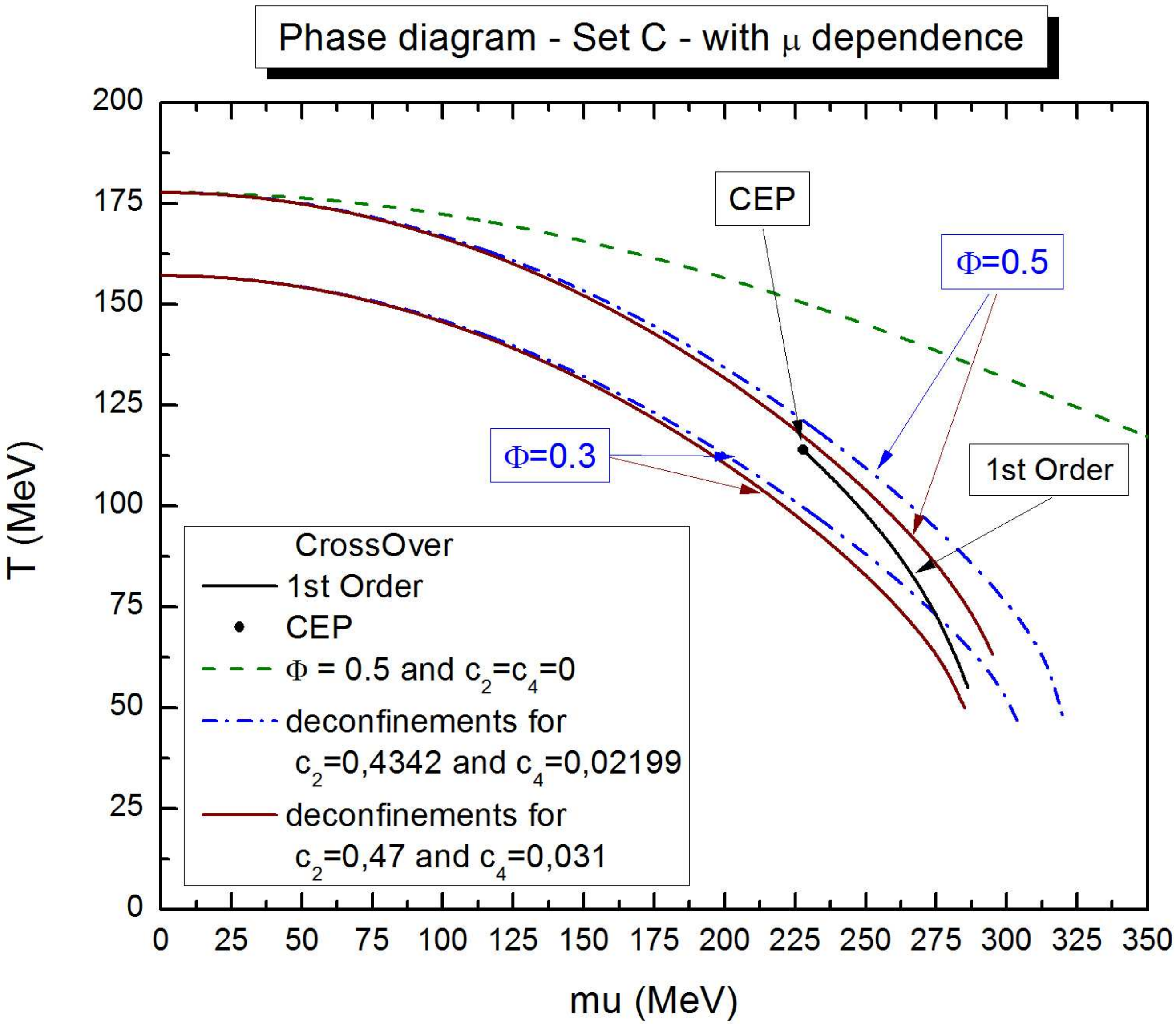
$$\mathcal{U}(\Phi,T,\mu) = (a_0 T^4 + a_1\mu^4 + a_2 T^2\mu^2)\Phi^2 + a_3 T_0^4 \ln\left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4\right)$$

Parameters:

$a_0 = 1.85$, $a_1 = 1.44\text{x}10^{-3}$, $a_2 = 0.08$, $a_3 = 0.40$

Contrera, D.B., in preparation

NONLOCAL PNJL MODEL: PHASE DIAGRAM AND CP



CEP Parameters:

$T_{\rm CP} = 128.6$ MeV
$\mu_{\rm CP} = 223.3$ MeV
$\mu/T\big|_{CEP} = 1.74$

Gustavo Contrera, D.B., in preparation (2011)

FROM DIQUARKS TO BARYONS (I)

The inverse diquark propagator is then obtained from

$$(S_D^A)^{-1}(k_0,k) = \frac{1}{4G_D} - \Pi_D^A(k_0,k) \;\;,\;\; \Pi_D^A(k_0,k) = \int \frac{d^4q}{(2\pi)^4} S_Q(q)\Sigma^A(k)S_Q(q-k)\Sigma^A(k)$$

Propagator can be expressed via the spectral density after analytic continuation

$$S_D^A(z,k) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\frac{\varrho_D^A(\omega,k)}{z-\omega} \;\;,\;\; \varrho_D^A(\omega,k) = \lim_{\varepsilon\to 0}\frac{8G_D^2\mathrm{Im}\Pi_D^A(\omega+i\varepsilon,k)}{[1-2G_D\mathrm{Re}\Pi_D^A(\omega+i\varepsilon,k)]^2+[2G_D\mathrm{Im}\Pi_D^A(\omega+i\varepsilon,k)]^2}$$

Similar, baryon propagator and spectral density

$$S_B^{-1}(P_0,P) = \frac{1}{G_B} - \Pi_B(P_0,P) \;\;,\;\; \Pi_B(P_0,P) = \sum_{A=2,5,7}\int \frac{\mathrm{d}k^4}{(2\pi)^4} S_Q^{11,A}(P-k)S_D^A(k)$$

Further details:

Wang, Wang, Rischke, PLB (2011); arXiv:1008.4029 [nucl-th]

Zablocki, Blaschke, Buballa, in preparation (2011)

THERMODYNAMIC POTENTIAL IN TERMS OF PHASE SHIFT

Integrating the spectral density over the coupling constant leads to

$$\int_0^G \frac{dg}{g^2}\rho_{\mathrm{B}}^g(\omega,\mathbf{P}) = \frac{i}{2}\log\left(\frac{\frac{1}{G}-\Pi_{\mathrm{B}}(\omega+i\delta,\mathbf{P})}{\frac{1}{G}-\Pi_{\mathrm{B}}(\omega-i\delta,\mathbf{P})}\right) \equiv \delta_{\mathrm{B}}(\omega,\mathbf{P}).$$

The in-medium phase shift $\delta_{\mu,T}(\omega,\mathbf{P})$ is the argument of the dynamic pair susceptibility

$$\frac{\frac{1}{G}-\Pi_{\mathrm{B}}(\omega\pm i\delta,\mathbf{P})}{|\frac{1}{G}-\Pi_{\mathrm{B}}(\omega,\mathbf{P})|} = e^{\mp i\delta_{\mathrm{B}}(\omega,\mathbf{P})}.$$

Thermodynamical potential in Beth-Uhlenbeck type form

$$\begin{aligned}
\Omega_{\mathrm{B}}^{(2)}(T,\mu) &= -d_B\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{d\mathbf{P}}{(2\pi)^3}\tilde{f}_B(\omega)\delta_{\mathrm{B}}(\omega,\mathbf{P}),\\
&= d_B\int_0^{\infty}\frac{d\omega}{\pi}\frac{d\mathbf{P}}{(2\pi)^3}\left[1-f_{\mathrm{B}}(\omega)-f_{\bar{\mathrm{B}}}(\omega)\right]\delta_{\mathrm{B}}(\omega,\mathbf{P}),\\
&= d_B\int_0^{\infty}\frac{d\omega}{\pi}\frac{d\mathbf{P}}{(2\pi)^3}\left\{\omega+T\ln[1+\mathrm{e}^{-(\omega-3\mu)/T}]+T\ln[1+\mathrm{e}^{-(\omega+3\mu)/T}]\right\}\frac{\partial\delta_{\mathrm{B}}(\omega,\mathbf{P})}{\partial\omega},
\end{aligned}$$

Pole approximation $\rightarrow$ Walecka model: $\Omega_{\mathrm{tot}} = \Omega_{\mathrm{MF}} + \Omega_{\mathrm{B}}^{(2)}$

See also Abuki, NPA **791** (2007) 117 and Zablocki, D.B., Buballa, in prep. (2011)

CompOSE - CompStar Online Supernova EoS

Reference manual
version 1.0

CompOSE

CompStar **O**nline **S**upernovæ **E**quations of State

fertilising the fields of nuclear physics and astrophysics

www.compstar-esf.org/compose*
compose@compstar-esf.org†

European Science Foundation
Research Networking Program
CompStar

November 22, 2010

General Requirements:

- Densities: $10^{-8} \leq n/n_0 \leq 10$
- Temperatures: $0 \leq T \leq 200$ MeV
- Proton fractions: $0 \leq Y_p \leq 0.6$; $\beta = 1 - 2Y_p$

New Developments:

- Dissolution of clusters due to Pauli blocking
- Realistic high-density modeling: DD-RMF/3FSC PNJL
- Thermodynamics of 1^{st} order PT; pasta phases

I. For Contributors:

- How to prepare EoS tables
- How to submit EoS tables
- Extending CompOSE

II. For Users:

- Hadronic EoS: Statistical, Skyrme, DBHF, ...
- Quark Matter EoS: Bag, PNJL, ...
- Phase transition: Maxwell, Gibbs, Pasta, ...

INVITATION: CONTRIBUTE TO THE NICA WHITE PAPER

NICA

Draft v 3.03

June 20, 2010

SEARCHING for a QCD MIXED PHASE at the NUCLOTRON-BASED ION COLLIDER FACILITY (NICA White Paper)

http://theor.jinr.ru/twiki-cgi/view/NICA/WebHome

http://theor.jinr.ru

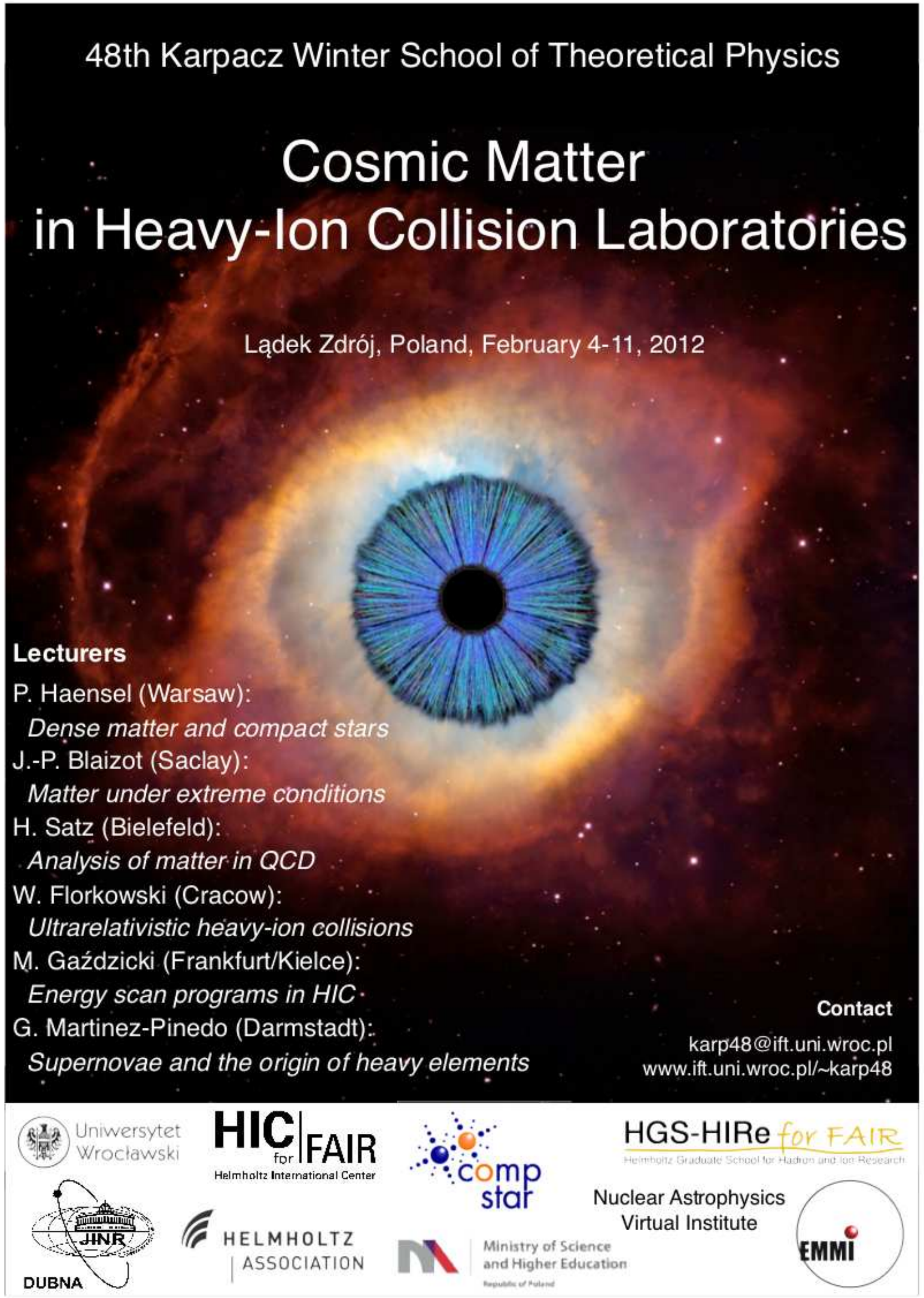
Contents

SUMMARY

- Generalized Beth-Uhlenbeck approach as microphysical basis to account for hadron dissociation (Mott effect) at extreme temperatures and densities
- Benchmark: pion and sigma Mott effect within NJL model, revised within nonlocal PNJL model
- Nonlocal PNJL model calibrated with lattice quark propagator data, EoS at finite T and μ, Phase diagram with critical point
- Application of GBU to interprete chemical freeze-out as Mott-Anderson localization
- Effective GBU model description: Mott-Hagedorn resonance gas + PNJL model describes Lattice QCD thermodynamics

OUTLOOK: NEXT STEPS ...

- Walecka model as limit of PNJL model: chiral transition effects in nuclear EoS
- Prospects for HIC (CBM & NICA) and Supernovae: color superconducting (quarkyonic) phases accessible!



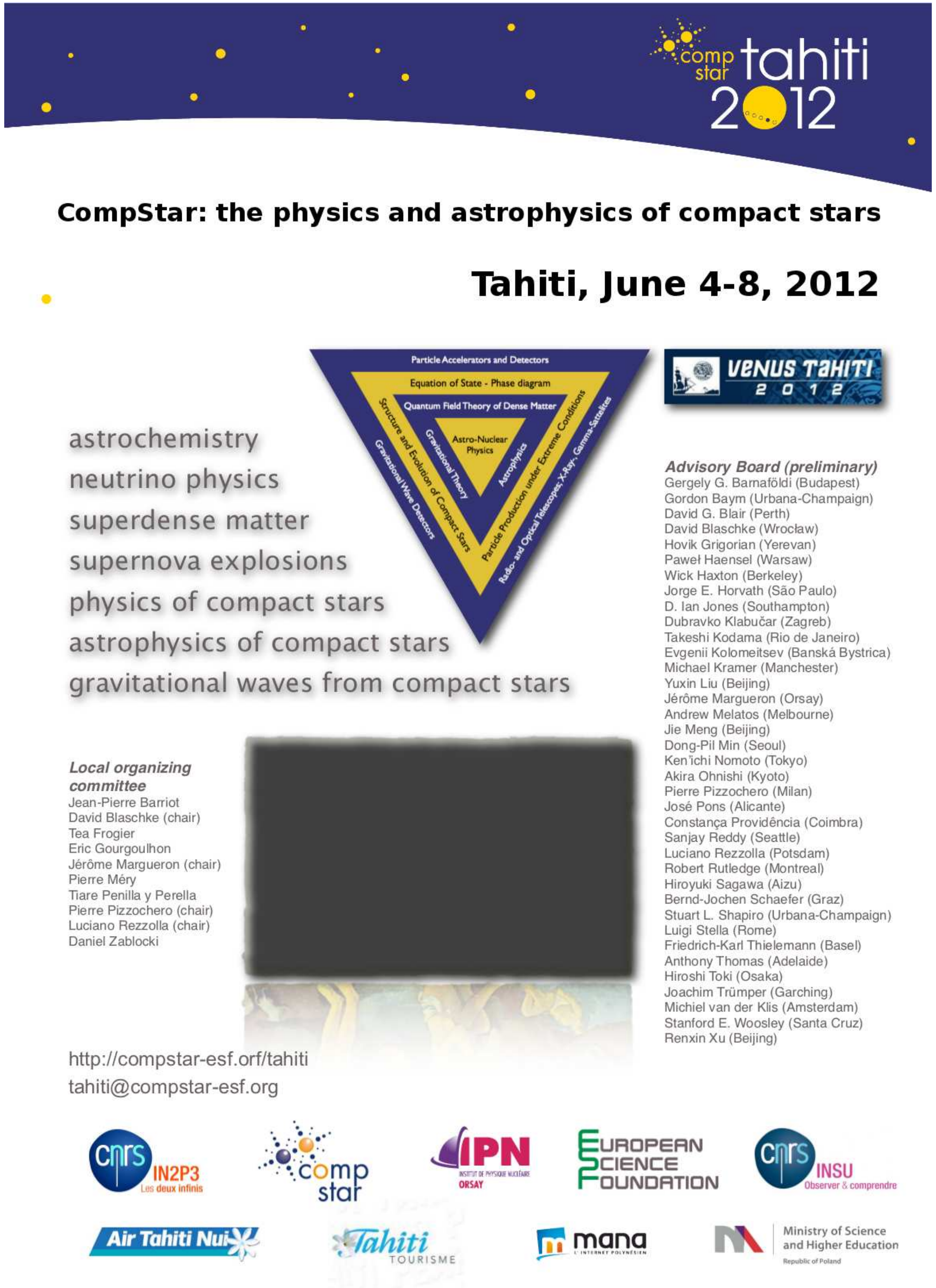
Invitations:

Karpacz Winter School on Theoretical Physics
"Cosmic Matter in Heavy-Ion Collision Laboratories"
Lądek Zdròj, Poland, February 4-11, 2012
http://www.ift.uni.wroc.pl/∼karp48

International Conference
"CompStar:
the physics and astrophysics of compact stars"
Tahiti, June 4-8, 2012
http://compstar-esf.org

Helmholtz International Summer School
"Dense Matter in HIC & Astrophysics"
Dubna, Russia, 2012
http://theor.jinr.ru/meetings

CompStar School & Workshop
"EoS in Compact Star Astrophysics & HIC"
Zadar, Croatia, September 2012
http://compstar-esf.org



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IT'S FUN TO INVESTIGATE HOT & DENSE STATES OF MATTER !!