Quantum Monte Carlo simulations of the strongly coupled quark-gluon plasma.

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OUTLINE

- Difficulties of theoretical treatment of strongly correlated quantum Coulomb systems of particles
- Conclusions from Lattice QCD simulations
- Simulation of quantum many-particle systems
- Applications to the quark-gluon plasma

Interaction and quantum effects in dense 3D and 2D plasma media with different mass ratio of charges.

Classical one-component plasma - COCP

Quantum one-component plasma - QOCP

Classical two-component plasma - CTCP

Quantum two-component plasma - QTCP

Coulomb interaction: $U_{ab}(r) = e_a e_b / r$

$T \sim 10^{13} K, n \sim 10^{45} cm^{-3}$

— Nonideality boundary:

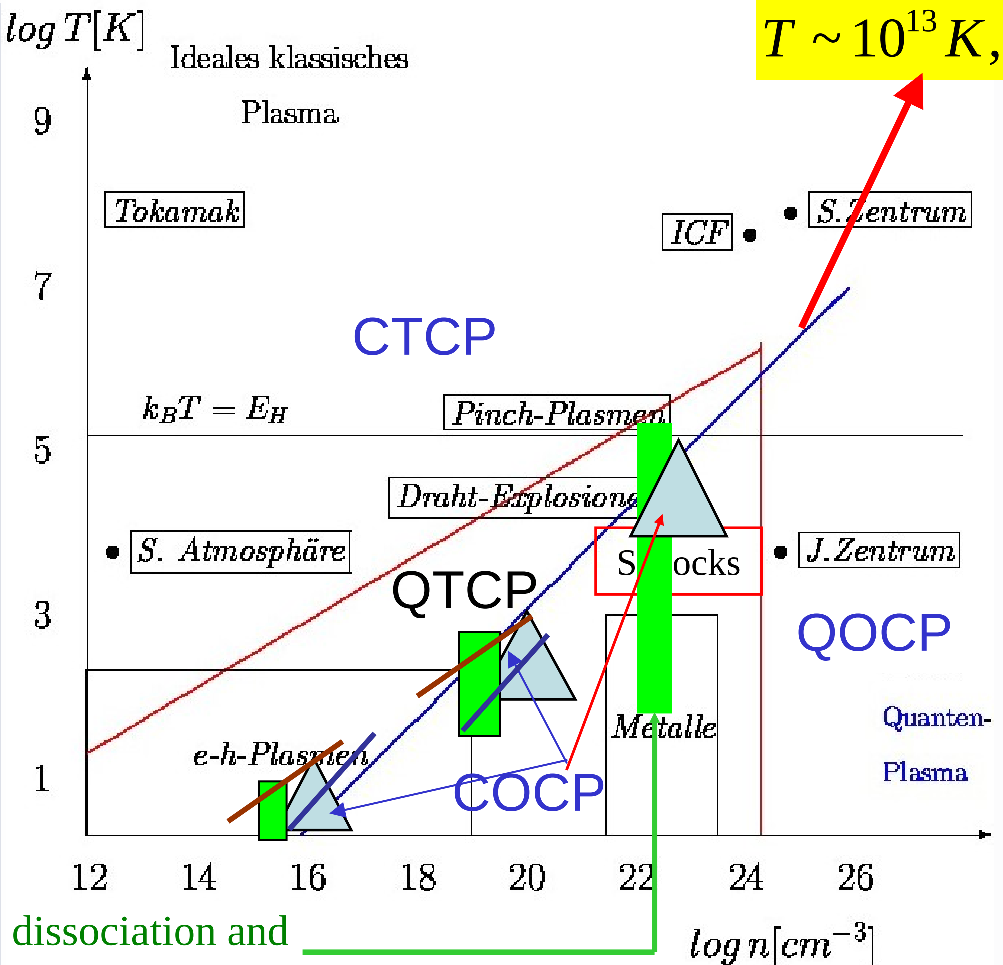
$$< U_{Coul} > = < E_{Kin} >$$

Inside: Strong Coulomb interaction,
Many-body effects
atomes, molecules, clusters

Degeneracy boundary

$$\lambda_e = \bar{r}$$

Below: overlapping electron
Wave functions,
Quantum and spin effects

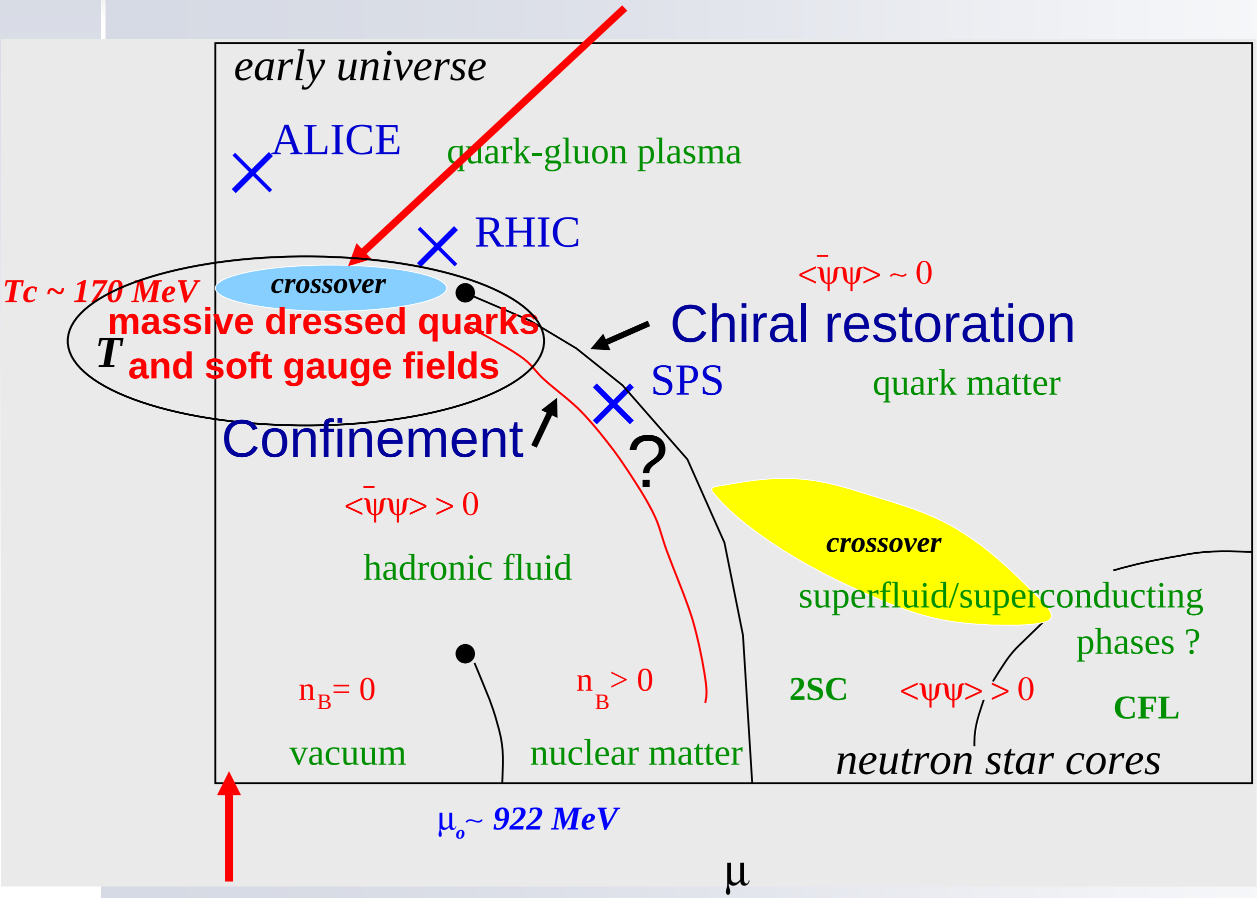


Pressure dissociation and ionization, Mott effect

Quasiparticle approximation for non-Abelian plasmas

In restricted part of phase diagram results of resummation technique and lattice simulations allow for consideration of quark-gluon plasma as system of dressed quarks, antiquarks and gluons which can be presented by massive color Coulomb quasiparticles with T-dependent dispersion curves and width (at least at μ=0 at T~T_d or above T_d and below T_c if T_d<T_c)

Feinberg, Litim, Manuel, Stoecker,Bleicher,, Richardson, Bonasera,Maruyama, Hatsuda, Shuryak,….



Phase diagram
(F.Karsch)

Basic asumptions of the quasiparticle model of quark – gluon plasma

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width.

(Shuryak , Phys.Lett.B478,161(2000), Phys. Rev. C, **74**, 044909, (2006))

- We consider color quasiparticles representing gluons and the most stable quarks of three flavors (up, down and strange) with m~T.
- Interparticle interaction is domonated by a color Coulomb potential with distance dependent coupling constant. Magnetic effects are neglected.
- The color operators are substituted by their average values
  - classical color vectors Q in SU(3) (8D vectors with 2 Casimirs conditions.).

•The model input requires :

- The temperature dependence of the quasiparticle masses.
- The temperature dependence of the coupling constant.

All input quantities should be deduced from lattice QCD calculations or experimental data and substitued in quantum Hamiltonian.

Thermodynamics of quark - gluon plasma in grand canonical ensemble within Feynman formulation of quantum mechanics

$$H_\beta = K_\beta + U_C = \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + U_C =$$

$$= \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

Grand canonical partition function

$$\Omega(\mu, \mu_g = 0, V, \beta) = \sum_{N_u, N_d, N_s, N_{\bar{u}}, N_{\bar{d}}, N_{\bar{s}}, N_g} \exp(\beta\mu_B(N_q - N_{\bar{q}}))\exp(\beta\mu_S(N_s - N_{\bar{s}}))\times$$

$$\times Z(N_q, N_{\bar{q}}, N_g, \beta) / N_u! N_d! N_s! N_{\bar{u}}! N_{\bar{d}}! N_{\bar{s}}! N_g!$$

$$N_q = N_u + N_d + N_s; N_{\bar{q}} = N_{\bar{u}} + N_{\bar{d}} + N_{\bar{s}}; N_a = N_b / 3, N_{\bar{a}} = N_{\bar{b}} / 3$$

$$Z(N_q, N_{\bar{q}}, N_g, \beta) = \sum_\sigma \int_V dr d\vec{Q} \rho(r, \vec{Q}, \sigma; \beta)$$

$$\rho = \exp(-\beta H(\beta)) = \underbrace{\exp(-\Delta\beta H(\beta)) \times \ldots \times \exp(-\Delta\beta H(\beta))}_{n+1}$$

$$\Delta\beta = \beta / (n+1)$$

For case

$$\beta = 1/kT, \mu_B = 0, \mu_s = 0$$

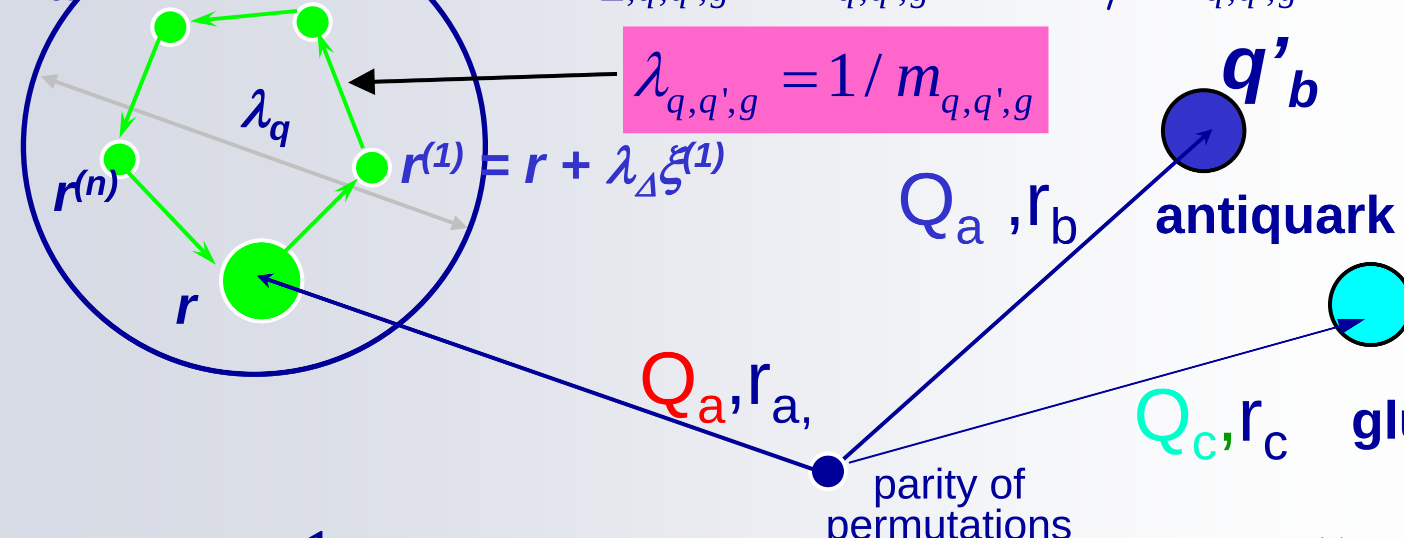
PATH INTEGRAL MONTE-CARLO METHOD

quark, antiquark, gluon

$$\lambda^3_{\Delta,q,q',g} = \lambda^3_{q,q',g}\sqrt{\pi/2}\Big/\left(m_{q,q',g}/(n+1)T\right)^{5/2}$$

$r^{(n+1)} \equiv r$

$\sigma' \equiv \sigma$



$$\rho\left(r,\vec{Q},\sigma;\beta\right) = \frac{1}{\lambda_{\Delta q}^{3N_q}\lambda_{\Delta\bar{q}}^{3N_{\bar{q}}}\lambda_{\Delta g}^{3N_g}} \sum_{P=P_q,P_{\bar{q}},P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)}\ldots dr^{(n)} d\vec{Q}^{(1)}\ldots d\vec{Q}^{(n)} \times$$

$$\rho\left(r,\vec{Q};r^{(1)},\vec{Q}^{(2)};\Delta\beta\right)\ldots\rho\left(r^{(n)},\vec{Q}^{(n)};\hat{P}r^{(n+1)},\hat{P}\vec{Q}^{(n+1)};\Delta\beta\right) S\left(\sigma,\hat{P}\sigma'\right)$$

spin matrix

$$\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)}) \approx \delta(Q^{(l)}-Q^{(l+1)})\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l)})$$

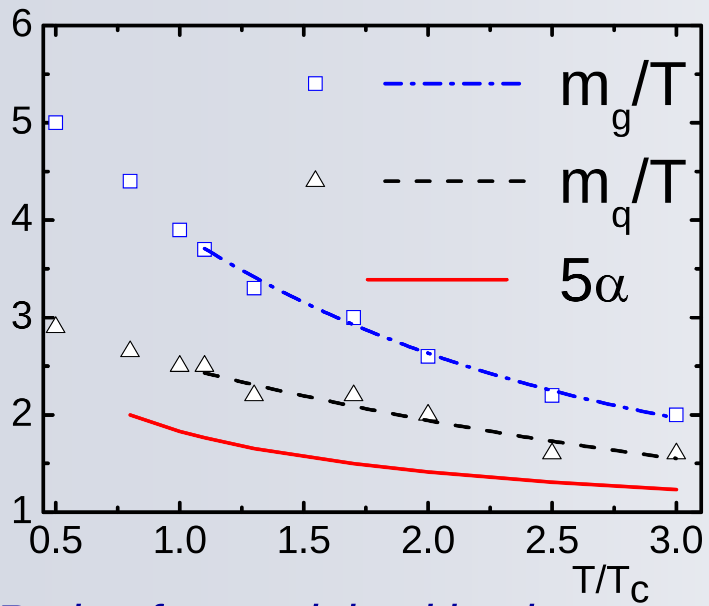
Input quantities

1) Coupling constant

$$\alpha(T) = g^2(T)/4\pi < 1$$

G.M.Prosperi, M.Raciti, C.Simolo, Prog. Part. Nucl.Phys. **58**, 387, 2010

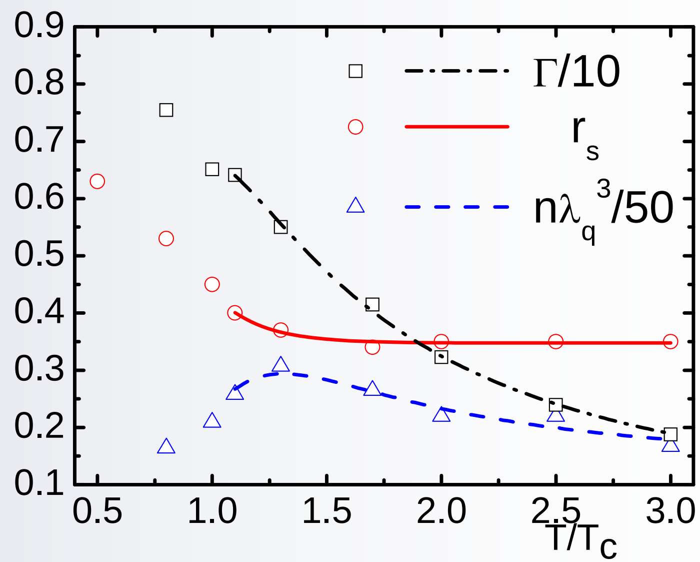
2) Quasiparticle masses: $m_q = m_{q'} < m_g$



Ratio of potential to kinetic energy per quasiparticle

$$\Gamma(T) \sim U/K \sim 5$$

$$T_c = 175 MeV$$

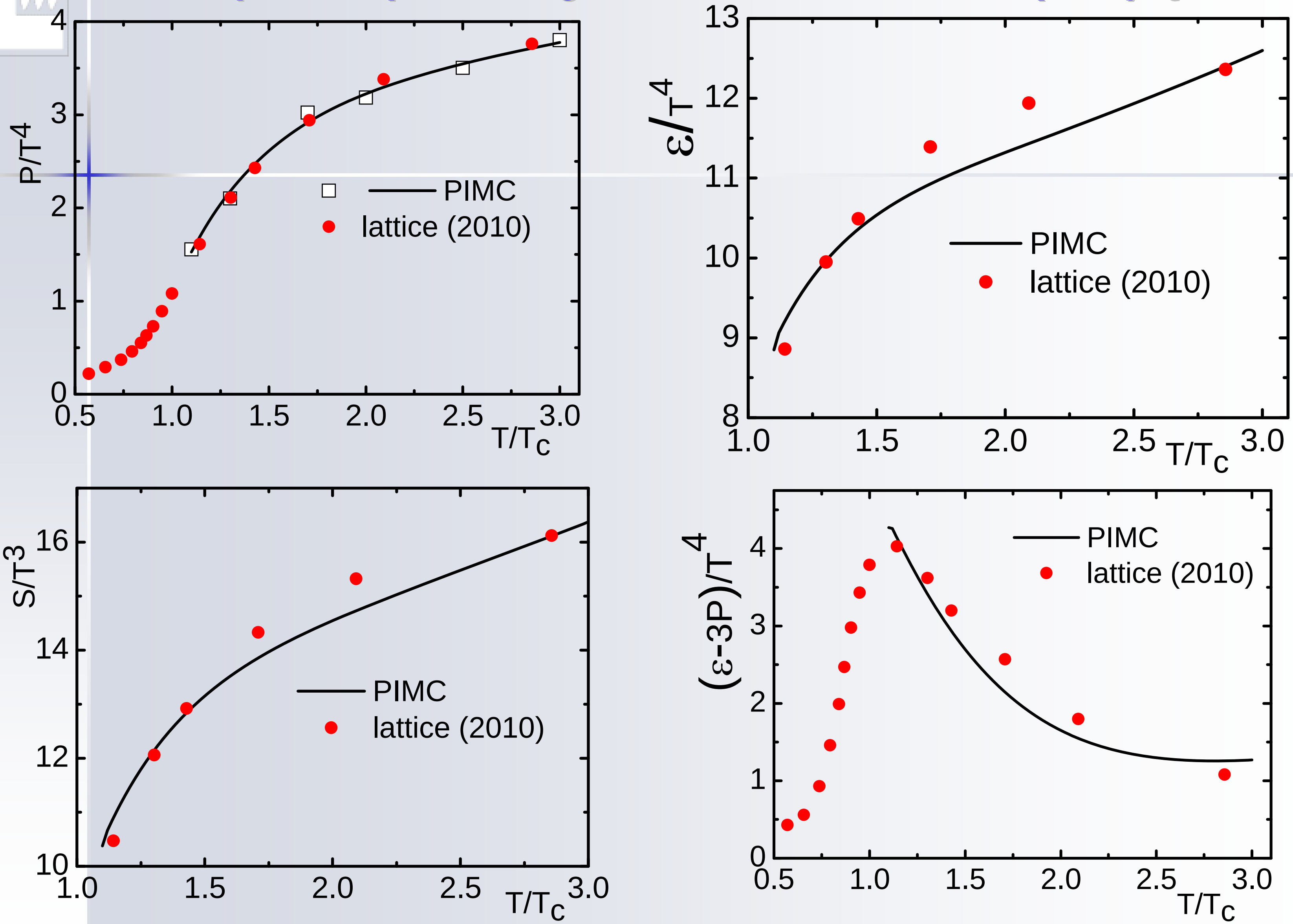


Density from grand canonical ensemble

r_s - Wigner-Seitz radius,

$$4\pi r_s^3 n = 1$$

Equation of State. The entropy density. The trace anomaly.
Comparison path integral results with lattice (2+1) QCD



The QCD equation of state with dynamical quarks
S.Borsanyi, G.Endrodi, Z.Fodor, A.Jakovac, S.D.Katz, S.Krieg, C.Ratti, K.K.Szabo, JHEP 11, 077, 1011, (2010)

Kinetic properties of quark – gluon plasma in canonical ensemble

$$C_{FA}(t) = Z^{-1} Tr\{F \exp(i\frac{Ht_c}{h}) A \exp(-i\frac{Ht_c}{h})\};$$

$$H = K + V(q,Q), t_c = t - i\frac{\beta h}{2}, \beta = \frac{1}{kT},$$

$$Z = Tr\{\exp(-\beta H)\}$$

$$C_{FA}(t) = \frac{1}{(2\pi h)^{2\nu}} \iint dQ_1 dp_1 dq_1 dp_2 dq_2 F(p_1,q_1) A(p_2,q_2) \times$$

$$W(p_1,q_1,Q_1; p_2,q_2,Q_1; t; i\beta h),$$

$$A(p,q) = \iint d\xi \exp(-i\frac{p\xi}{h}) < q - \frac{\xi}{2} | A | q + \frac{\xi}{2} >$$

$$W(p_1,q_1,Q_1; p_2,q_2,Q_1; t; i\beta h) = Z^{-1} \iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h}) \exp(i\frac{p_2\xi_2}{h}) \times$$

$$< q_1 + \frac{\xi_1}{2} | \exp(i\frac{Ht_c}{h}) | q_2 - \frac{\xi_2}{2} >< q_2 + \frac{\xi_2}{2} | \exp(-i\frac{Ht_c}{h}) | q_1 - \frac{\xi_1}{2} >$$

Integral color Wigner – Liouville equation

$$W(p_1,q_1,Q_1;p_2,q_2,Q_1;t;i\beta h)=\overline{W}(p^0{}_1,q^0{}_1,Q_1^0;p^0{}_2,q^0{}_2,Q_2^0;0;i\beta h)+$$

$$+\int_0^t d\tau \iint ds \iint d\eta W(p_1^\tau - s,q_1^\tau,Q_1^\tau;p_2^\tau-\eta,q_2^\tau,Q_2^\tau;\tau;i\beta h)\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau),$$

$$\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau)=\frac{1}{2}\{\omega(s,q_1^\tau,Q_1^\tau)\delta(\eta)-\omega(\eta,q_2^\tau,Q_2^\tau)\delta(s)\},F(q,Q)=-\nabla_q V(q,Q)$$

$$\omega(\eta,q,Q)=\frac{4}{(2\pi h)^\nu h}\iint dq'V(q-q',Q)Sin(\frac{2sq'}{h})+F(q,Q)\bullet\frac{d\delta(s)}{ds}$$

$$\frac{dq_1^t}{dt}=\frac{1}{2}\frac{p_1^t}{\sqrt{m^2+(p_1^t)^2}},\frac{dp_1^t}{dt}=\frac{1}{2}F(q_1^t,Q_1^t),$$

$$\frac{dQ_{1,i}^{t,a}}{dt}=\frac{1}{2}\sum_{b,c}f^{abc}Q_{1,i}^b\nabla_{Q_{1,i}^c}V(q_1^t,Q_1^t),$$

$$p_1^t(t,p_1,q_1,Q_1)=p_1,q_1^t(t,p_1,q_1,Q_1)=q_1,Q_1^t(t,p_1,q_1,Q_1)=Q_1$$

$$\frac{dq_2^t}{dt}=-\frac{1}{2}\frac{p_2^t}{\sqrt{m^2+(p_2^t)^2}},\frac{dp_2^t}{dt}=-\frac{1}{2}F(q_2^t,Q_2^t),$$

$$\frac{dQ_{2,i}^{t,a}}{dt}=-\frac{1}{2}\sum_{b,c}f^{abc}Q_{2,i}^b\nabla_{Q_{2,i}^c}V(q_2^t,Q_2^t),$$

$$p_2^t(t,p_2,q_2,Q_1)=p_2,q_2^t(t,p_2,q_2,Q_1)=q_2,Q_2^t(t,p_2,q_2,Q_1)=Q_1$$

Positive time direction

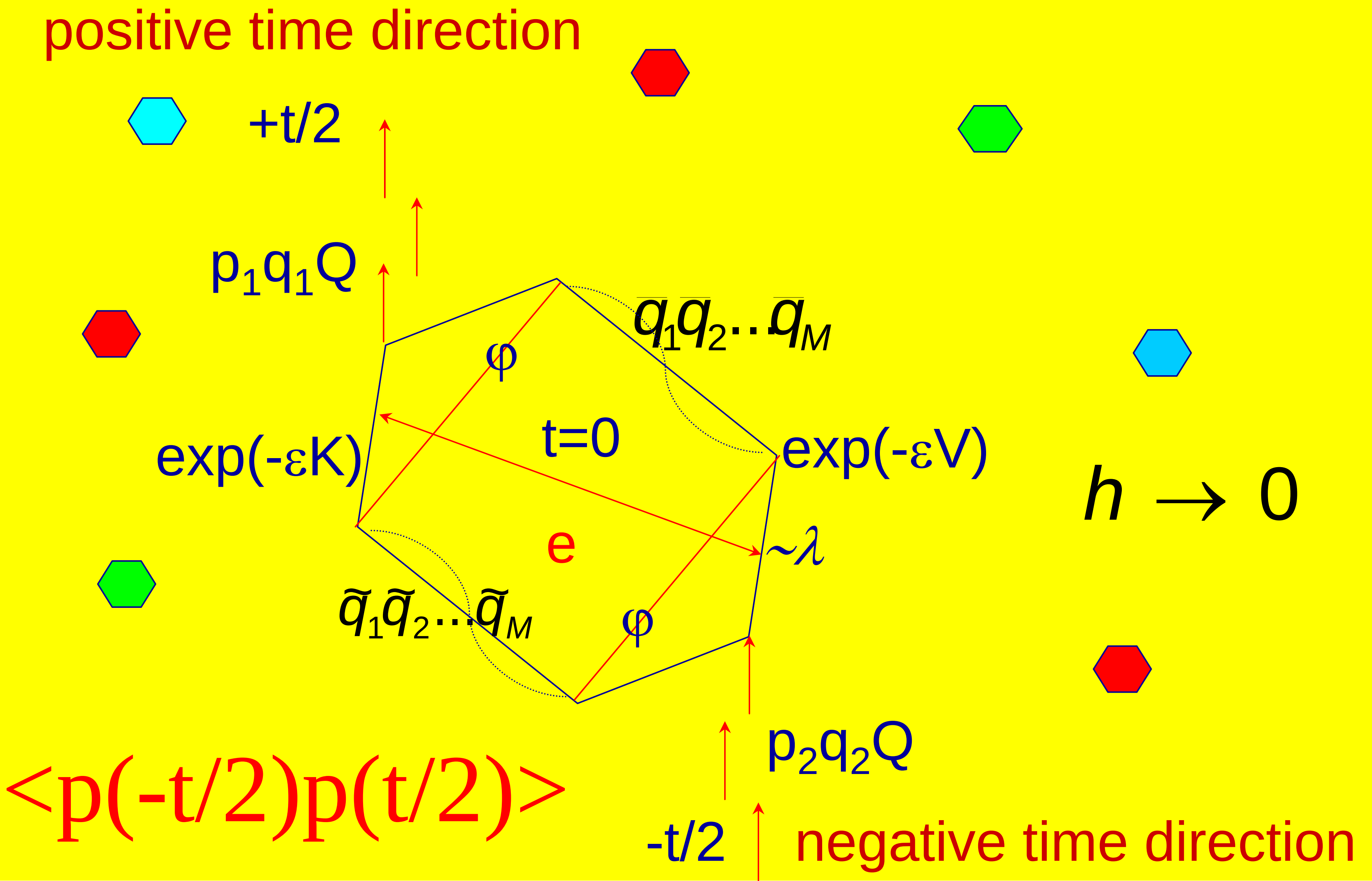
Color dynamics in SU(3)

Initial conditions

Hamiltonian eqations

Negative time direction

Schematic snapshot for color phase space dynamics

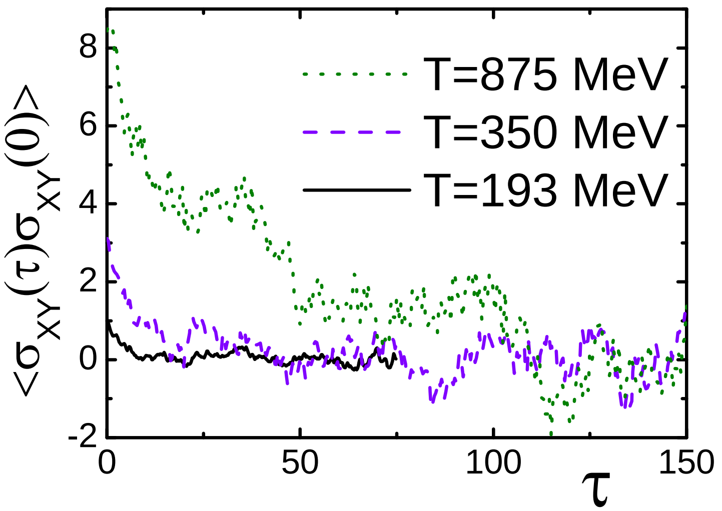


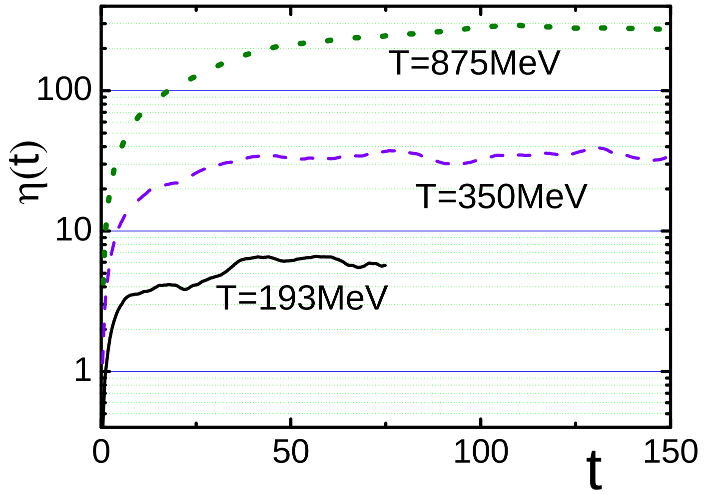
Time autocorrelation function of the stress energy tensor and shear viscosity of quark –gluon plasma

$$\eta(\tau) = \frac{n}{3k_B T}\left\langle \sum_{X<Y} \sigma_{XY}(\tau/2)\sigma_{XY}(-\tau/2) \right\rangle$$

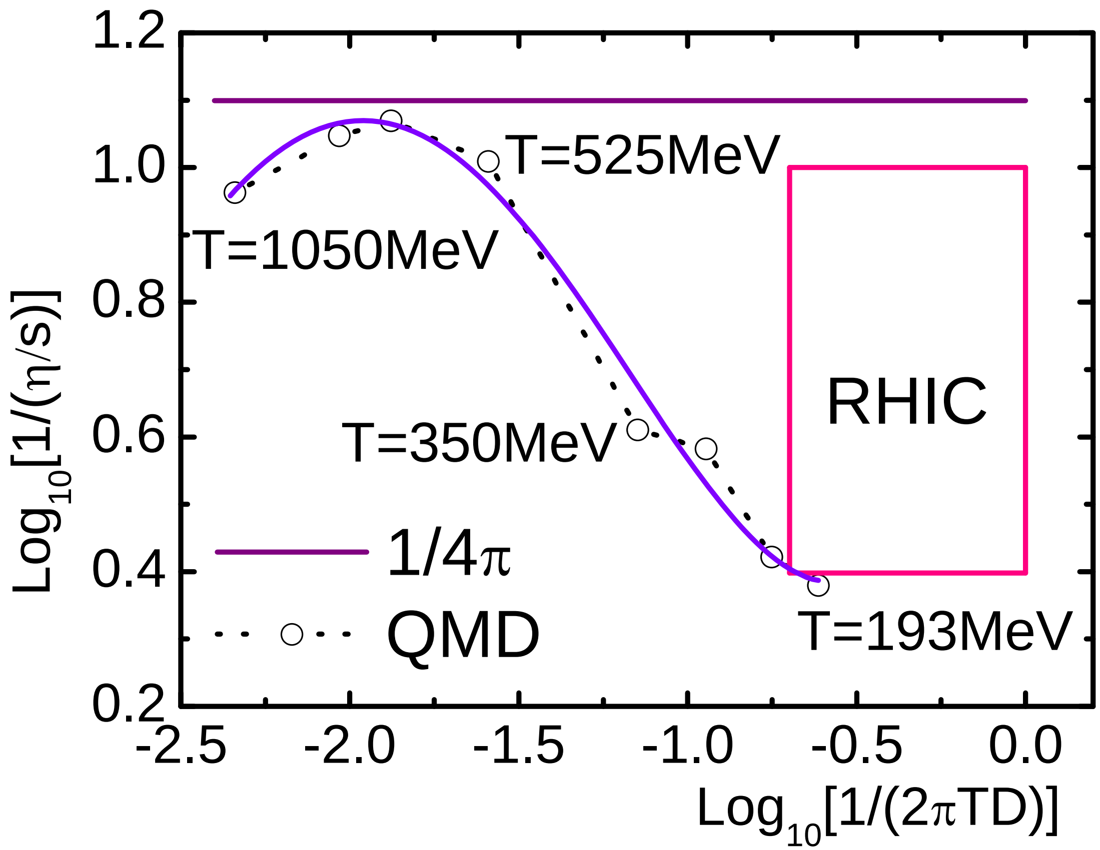
$$\sigma_{XY}(\tau) = \frac{1}{N}\left(\sum_{i=1}^{N} m_i v_{ix} v_{iy} + \frac{1}{2}\sum_{i\neq j} r_{ij,x} F_{ij,y} \right)$$

$$\eta = \lim_{t\to\infty} \int_0^t \eta(\tau) d\tau$$





Diffusion coefficient and shear viscosity



CONCLUSIONS

• Path integral Monte Carlo method and Wigner-Liouville dynamics are a reliable and fast method of simulation thermodynamic and kinetic properties in a wide range of plasma parameters

•The developed numerical approach can be applied to consideration of QG plasmas.

•Results of simulations agree with available theoretical and experimental data.

Thank you for attention.

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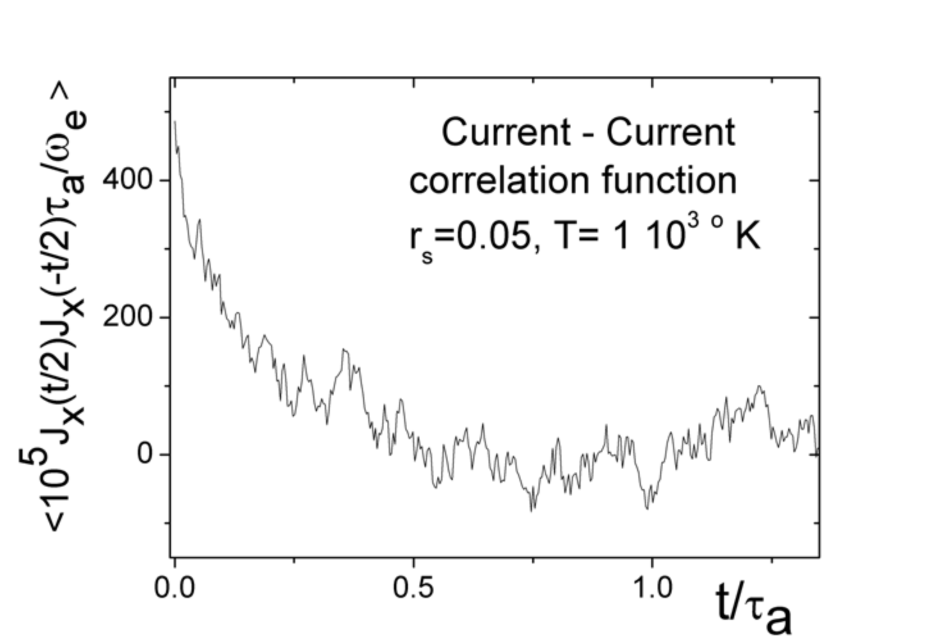
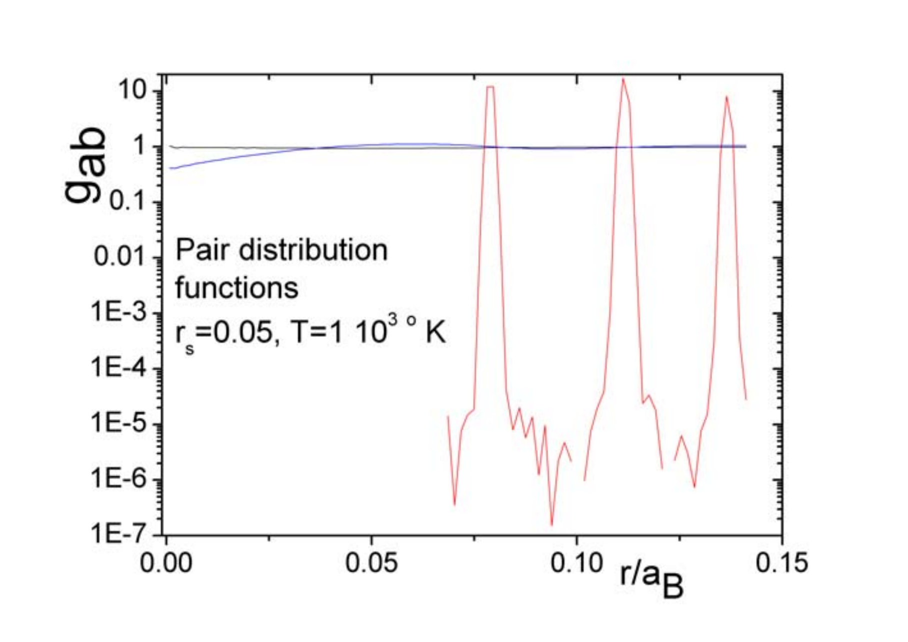
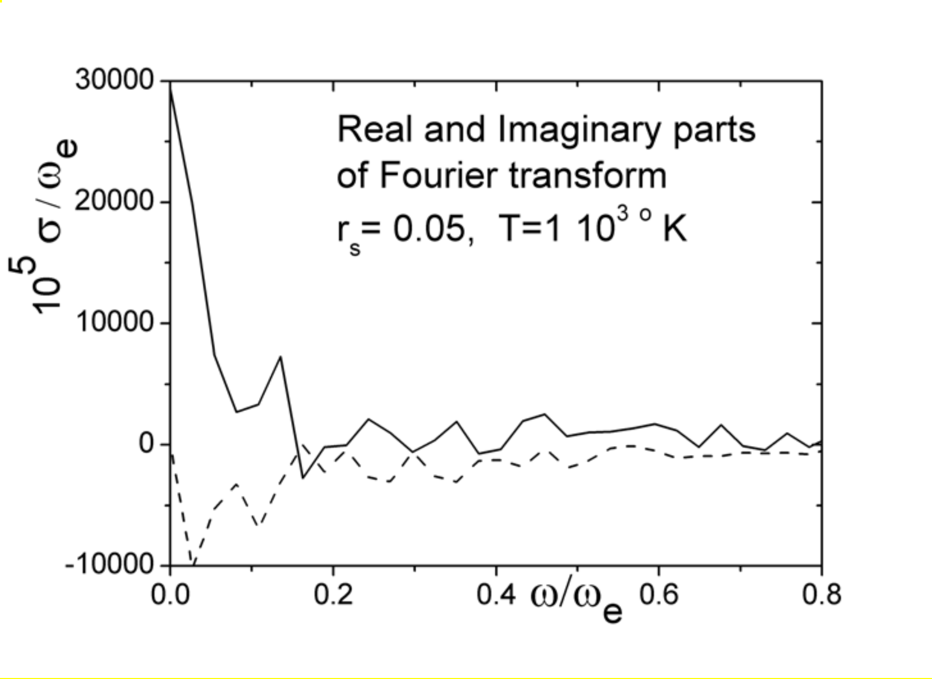
Correlation functions and transport coefficients EMP Hydrogen plasma $T = 10\ 000^o$ K







Integral equation

$$W(p_1,q_1,Q_1;p_2,q_2,Q_1;t;i\beta h)=\bar{W}(p^0{}_1,q^0{}_1,Q_1^0;p^0{}_2,q^0{}_2,Q_2^0;0;i\beta h)+$$

$$+\int_0^t d\tau\iint ds\iint d\eta W(p_1^\tau-s,q_1^\tau,Q_1^\tau;p_2^\tau-\eta,q_2^\tau,Q_2^\tau;\tau;i\beta h)\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau),$$

$$\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau)=\frac{1}{2}\{\omega(s,q_1^\tau,Q_1^\tau)\delta(\eta)-\omega(\eta,q_2^\tau,Q_2^\tau)\delta(s)\},F(q,Q)=-\nabla_q V(q,Q)$$

$$\omega(\eta,q,Q)=\frac{4}{(2\pi h)^\nu h}\iint dq'V(q-q',Q)Sin(\frac{2sq'}{h})+F(q,Q)\bullet\frac{d\delta(s)}{ds}$$

Positive time direction

$$\frac{dq_1^t}{dt}=\frac{1}{2m}p_1^t,\frac{dp_1^t}{dt}=\frac{1}{2}F(q_1^t,Q_1^t),$$

$$\frac{dQ_{1,i}^{t,a}}{dt}=\frac{1}{2}\sum_{b,c}f^{abc}Q_{1,i}^b\nabla_{Q_{1,i}^c}V(q_1^t,Q_1^t),$$

$$p_1^t(t,p_1,q_1,Q_1)=p_1,q_1^t(t,p_1,q_1,Q_1)=q_1,Q_1^t(t,p_1,q_1,Q_1)=Q_1$$

$$\frac{dq_2^t}{dt}=-\frac{1}{2m}p_2^t,\frac{dp_2^t}{dt}=-\frac{1}{2}F(q_2^t,Q_2^t),$$

$$\frac{dQ_{2,i}^{t,a}}{dt}=-\frac{1}{2}\sum_{b,c}f^{abc}Q_{2,i}^b\nabla_{Q_{2,i}^c}V(q_2^t,Q_2^t),$$

$$p_2^t(t,p_2,q_2,Q_1)=p_2,q_2^t(t,p_2,q_2,Q_1)=q_2,Q_2^t(t,p_2,q_2,Q_1)=Q_1$$

Color dynamics in SU(2) or SU(3)

Initial conditions

Hamiltonian eqations

Negative time direction

Initial conditions

$$\exp(-\frac{\beta}{2}H) = \exp(-\varepsilon H)\exp(-\varepsilon H)...\exp(-\varepsilon H), \varepsilon = \beta / 2M, t = 0$$

$$\exp(-\varepsilon H) = \exp(-\varepsilon K)\exp(-\varepsilon V)\exp(-\varepsilon^2[K,V]/2)...,$$

$$\bar{W}(p_1,q_1,Q_1;p_2,q_2,Q_1;0;i\beta h) \approx \iint d\bar{q}_1 d\bar{q}_2 ... d\bar{q}_M d\tilde{q}_1 d\tilde{q}_2 ... d\tilde{q}_M \times$$

$$\Psi\{p_1,q_1,Q_1;p_2,q_2,Q_1;\bar{q}_1,\bar{q}_2...\bar{q}_M;\tilde{q}_1,\tilde{q}_2...\tilde{q}_M;i\beta h\},$$

$$\Psi\{p_1,q_1,Q_1;p_2,q_2,Q_1;\bar{q}_1,\bar{q}_2...\bar{q}_M;\tilde{q}_1,\tilde{q}_2...\tilde{q}_M;i\beta h\} =$$

$$Z^{-1} < q_1 | \exp(-\varepsilon K) | \bar{q}_1 > \exp(-\varepsilon V(\bar{q}_1,\bar{Q}_1)) < \bar{q}_1 | \exp(-\varepsilon K) | \bar{q}_2 >$$

$$\exp(-\varepsilon V(\bar{q}_2,\bar{Q}_1))...\exp(-\varepsilon V(\bar{q}_M,\bar{Q}_1)) < \bar{q}_M | \exp(-\varepsilon K) | q_2 > \phi(p_2,\bar{q}_M,\tilde{q}_1)\times$$

$$< q_2 | \exp(-\varepsilon K) | \tilde{q}_1 > \exp(-\varepsilon V(\tilde{q}_1,\bar{Q}_1)) < \tilde{q}_1 | \exp(-\varepsilon K) | \tilde{q}_2 >$$

$$\exp(-\varepsilon V(\tilde{q}_2,\bar{Q}_1))...\exp(-\varepsilon V(\tilde{q}_M,\bar{Q}_1)) < \tilde{q}_M | \exp(-\varepsilon K) | q_1 > \phi(p_1,\tilde{q}_M,\bar{q}_1)$$

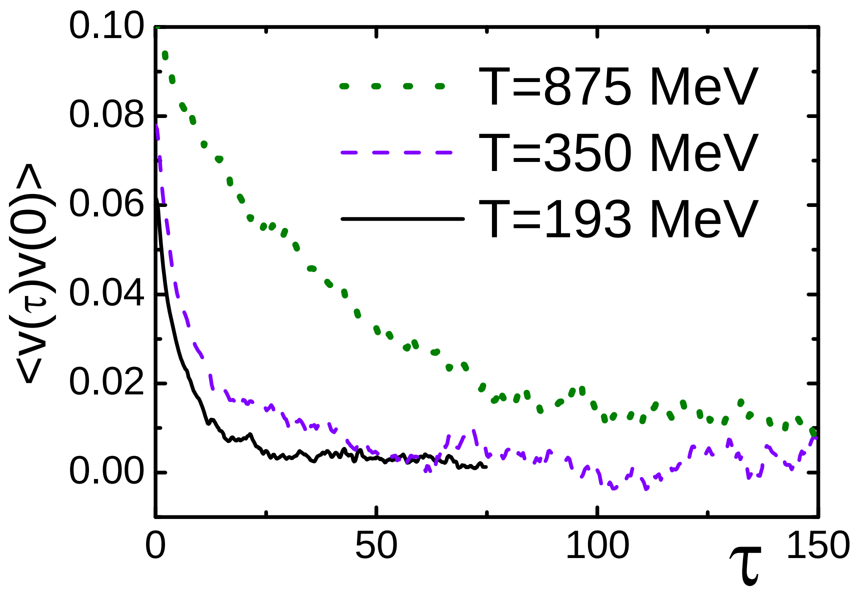
$$\phi(p,\bar{q},\tilde{q}) = \lambda^{\nu}\exp\left(\frac{< \frac{p\lambda}{h} + i\pi\frac{\bar{q}-\tilde{q}}{\lambda} | \frac{p\lambda}{h} + i\pi\frac{\bar{q}-\tilde{q}}{\lambda} >}{2\pi}\right), \lambda^2 = \frac{2\pi h^2\beta}{2Mm},$$

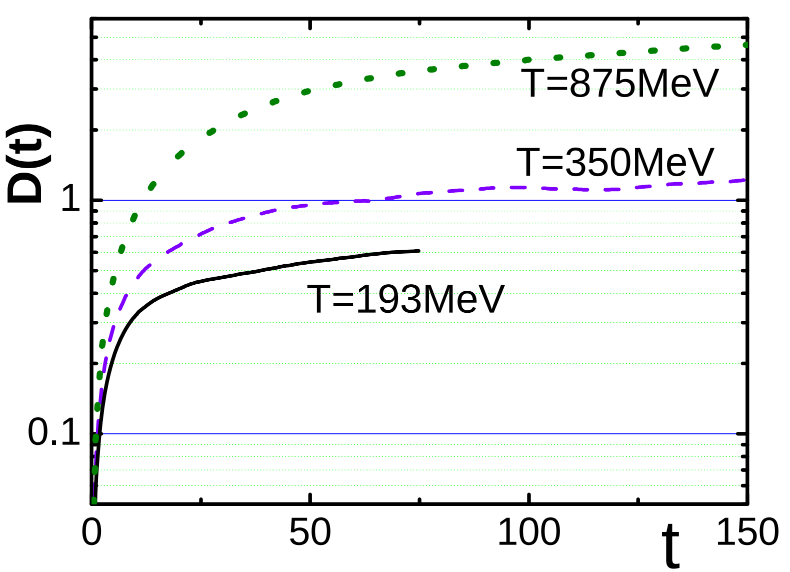
Velocity autocorrelation function and diffusion constant QGP

$$D(\tau) = < v(\tau/2) v(-\tau/2) > =$$

$$= \frac{1}{3N} < \sum_{i=1}^{N} \vec{v}_i(\tau/2) \bullet \vec{v}_i(-\tau/2) >$$

$$D = \lim_{t\to\infty} D(t) = \lim_{t\to\infty} \int_0^t d\tau D(\tau)$$

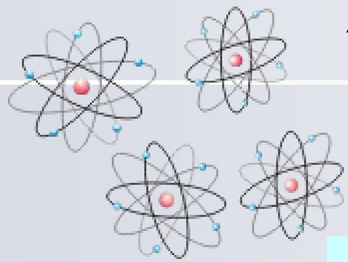


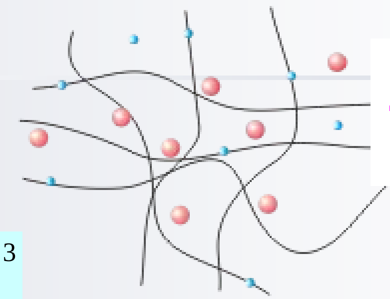


Matter transformation at high density and energy concentration

$\rho < 1 g/cm^3$

Atom

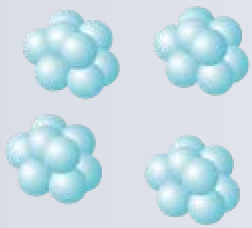


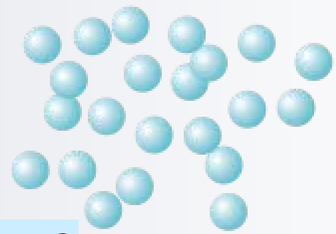


Electromagnetic plasma

$\rho = 10^{14} g/cm^3$

Atomic nucleus

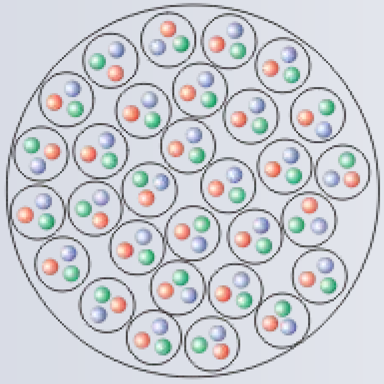


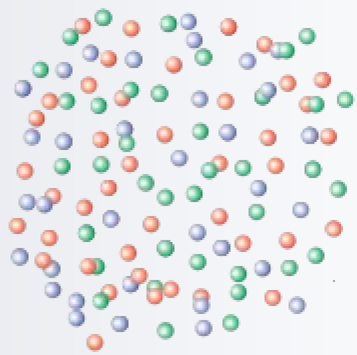


Nuclear Matter

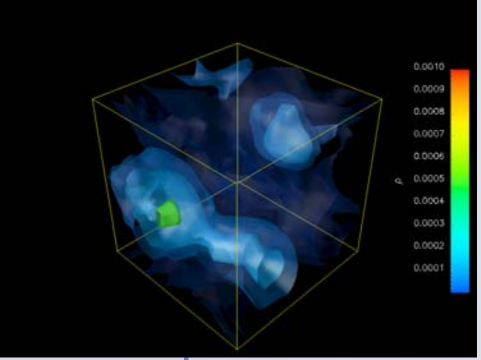
$\rho = 2.510^{15} g/cm^3$

Nucleon

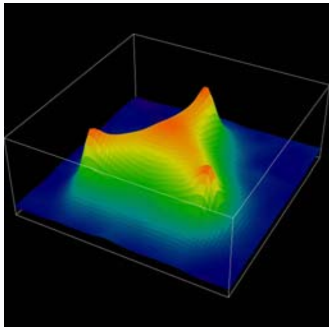




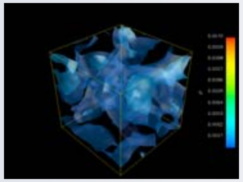
Quark-gluon plasma



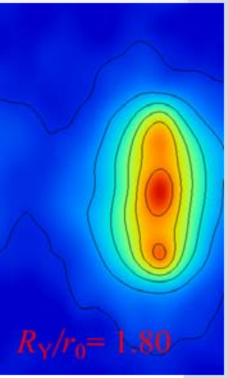
Conclusions from Lattice simulations

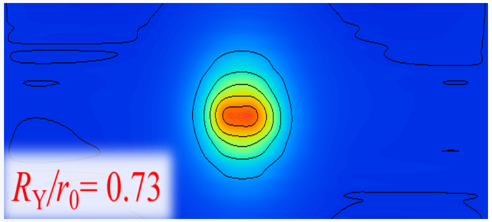


- Computer simulations a) reproduce well known hadron properties b) predict new phenomena c) help to create new theoretical ideas and models.



- Low dimensional objects (regions or quasiparticles - dressed quarks and gluons) are responsible for most interesting nonperturbative effects such as chiral symmetry breaking and confinement.





http://www.ihed.ras.ru/fortov/polikarpov.ppt

Density matrix

$$\sum_{\sigma}\rho\left(r,\vec{Q},\sigma;\beta\right)=\frac{1}{\lambda_{\Delta}^{3N_q}\lambda_{\Delta}^{3N_{\bar{q}}}\lambda_{\Delta}^{3N_g}}\sum_{\sigma}\rho\left(\left[r\vec{Q}\right],\beta\right)$$

$$\rho\left(\left[rQ\right],\beta\right)=\exp\left\{-\beta U\left(\left[rQ\right],\beta\right)\right\}\times$$

$$\times\prod_{l=1}^{n}\prod_{p=1}^{N_q}\varphi_{pp}^{l}\det\left|\psi_{ab}^{n,1}\right|_{N_q}\prod_{p=1}^{N_{\bar{q}}}\tilde{\varphi}_{pp}^{l}\det\left|\tilde{\psi}_{ab}^{n,1}\right|_{N_{\bar{q}}}\prod_{p=1}^{N_g}\tilde{\tilde{\varphi}}_{pp}^{l}\,per\left|\tilde{\tilde{\psi}}_{ab}^{n,1}\right|_{N_g}$$

$$U\left(\left[rQ\right],\beta\right)=\sum_{l=0}^{n}\frac{U_l\left(\left[r^{(l)}Q\right],\beta\right)}{n+1}$$

Pairwise sum of Kelbg potentials for each l=0,…,n

Exchange matrix

$$\left\|\psi_{ab}^{n,1}\right\|_{s}\equiv\left\|\delta_{\sigma_a,\sigma_b}\delta_{f_a,f_b}K_2\left\{\sqrt{(m_a/((n+1)T))^2+\left|\left(r_a^{(0)}-r_b^{(n)}\right)\right|^2\Big/\lambda_a^2}\right\}\right\|$$

Color Kelbg potential

Richardson, Gelman, Shuryak, Zahed, Harmann, Donko, Levai, Kalman (r=0 ?)

$$x_{ab} = |\mathbf{r}_{ab}| / \tilde{\lambda}_{ab} \qquad \tilde{\lambda}_{ab} = \hbar^2 \Delta\beta / 2\mu_{ab}$$

$$\Phi^{ab}(x_{ab}, \Delta\beta) = \frac{<\vec{Q}_a | \vec{Q}_b > g^2}{4\pi\tilde{\lambda}_{ab} x_{ab}} \left\{ 1 - e^{-x_{ab}^2} + \sqrt{\pi} x_{ab} \left[1 - erf(x_{ab}) \right] \right\}$$

$$|\mathbf{r}_{ab}| \to 0$$

$$\sim \frac{<Q_a | Q_b > g^2 \sqrt{\pi}}{4\pi\tilde{\lambda}_{ab}}$$

$$|\mathbf{r}_{ab}| >> \tilde{\lambda}_{ab}$$

$$\frac{<Q_a | Q_b > g^2}{4\pi\tilde{\lambda}_{ab} |x_{ab}|}$$

Objects Q are color coordinates of quarks and gluons

There is no divergence at small interparticle distances and it has a true asymptotics (T, x_{ab})

Ha -> $k_B T_c$, T_c =175 Mev,

$T_c < T$, $m_a \sim 5 k_B T_c / c^2$,

$L_o \sim hc/k_B T_c$, $r_s = <r>/L_o < 0.1$,

$L_o \sim 1.2\ 10^{-15}$ m, $x_{ab} \sim 1$

Pair distribution functions in canonical emsemble

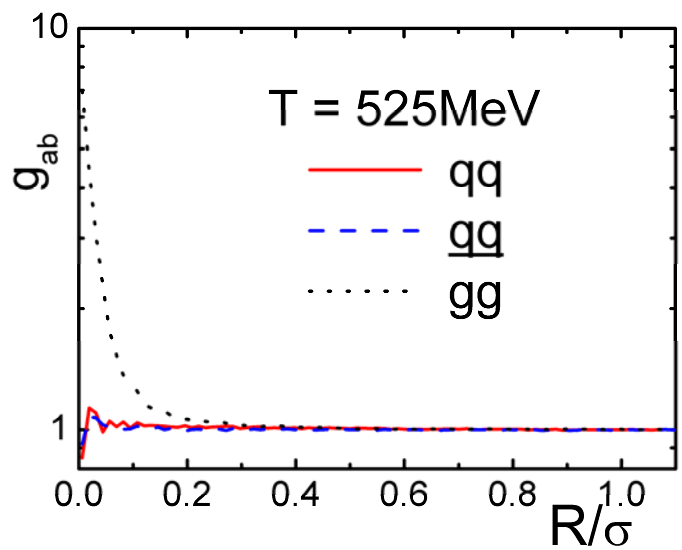
$$H_\beta = \sum_a \sqrt{m_a(\beta)^2 + p_\alpha{}^2} + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) C_{ab} < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

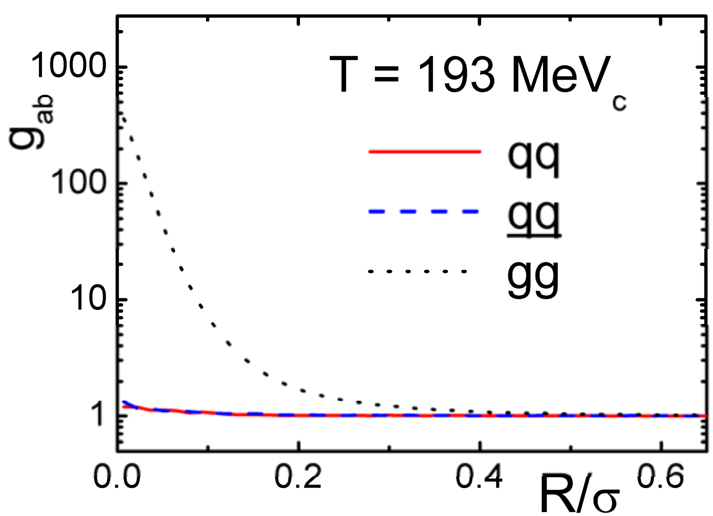
$$g_{ab}(|R_1 - R_2|) = g_{ab}(R_1, R_2) = \frac{1}{Z(N_q, N_{\bar{q}}, N_g)} \times$$

$$\sum_\sigma \int_V dr\, dQ\, \delta(R_1 - r^a{}_1)\delta(R_2 - r^b{}_2)\rho(r, Q, \sigma; \beta),$$

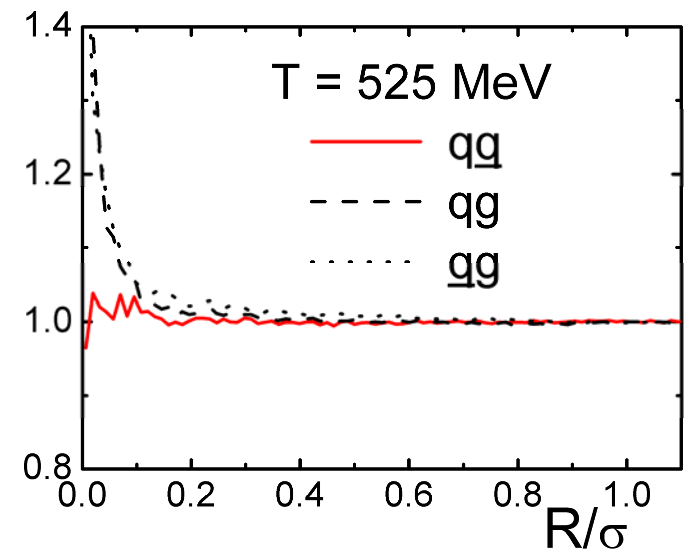
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

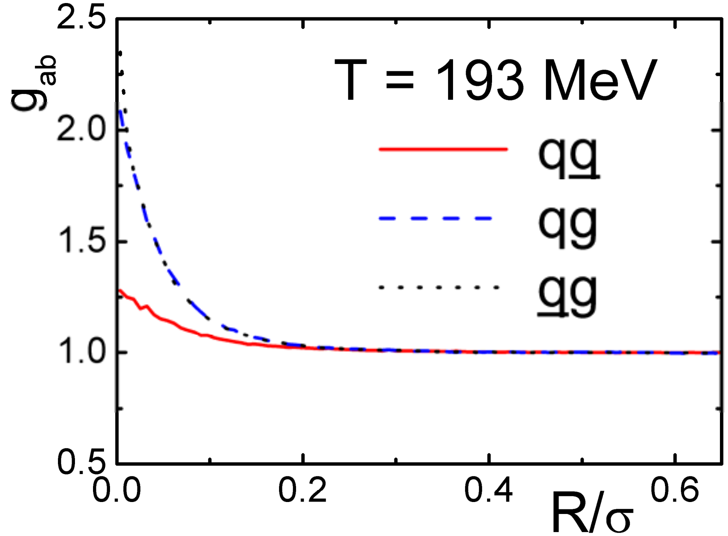
Similar quasiparticles





Different quasiparticles





CONCLUSIONS

- Path integral Monte Carlo is a reliable method of simulation in a wide range of parameters
- It doesn’t use any approximation, so it can predict new properties of matter
- Plasma phase transition and crystallization in Coulomb quantum plasma can be revealed in calculations. These regions should be investigated experimentally.
- Quantum dynamics method can be constructed on the basis of PIMC and Wigner formulation of quantum mechanics