

Moscow, 23.11.2011

Исследования неидеальной плазмы
**SPATIALLY RESOLVED COLLECTIVE EXCITATIONS
IN NANO-PLASMAS**

Thomas Raitza, Heidi Reinholz, Gerd Röpke,
Rostock University



Introduction

Nanoplasmonics

- „optical phenomena in the nanoscale vicinity of metal surfaces“
- absorption and scattering of light from small particles
 - resonant enhancement of specific frequencies



Lycurgus Cup (400 A.D.)

- wavelength \gg nanoscale system
(planar wave \rightarrow longwavelength limit)
- oscillator strength f
(sum rule of absorption cross section)
- dependence on size/shape, composition/geometry,
surrounding dielectric material

M. Stockman, Phys. today 64(2) 39 (2011), arXiv:0908.3559

S. Raza et al., arXiv:1106.2175

T. Sandu et al., arXiv:1104.5666

V.B. Gildenburg et al., Phys. Plasmas 18, 092101 (2011)

Equilibrium correlation functions and transport coefficients (fluctuation-dissipation theorem)

Generalized linear response

$$\epsilon(\vec{k}, \omega) = 1 + \frac{i}{\epsilon_0 \omega} \sigma(\vec{k}, \omega) = 1 - \frac{\omega_{\text{pl}}^2}{\omega(\omega - i\nu(\vec{k}, \omega))}$$

Reinholz, Ann. Phys. (Fr) 2005

- generalized statistical operator
- **Kubo** formula vs. generalized **Drude** formula

$$\sigma(\omega) \propto \langle \vec{j}; \vec{j} \rangle_{\omega+i\eta} \qquad \nu(\omega) \propto \langle \vec{F}; \vec{F} \rangle_{\omega+i\eta}$$

dynamical **conductivity**

current-current correlation function

dynamical **collision frequency**

force-force correlation function

Zubarev, Morozov, Röpke, *Stat. Mech. of Non-Equilibrium Processes* (1996)

MD simulations, homogeneous plasmas

- bilocal dynamical structure factor, Fourier transform
- wave vector k to characterize collective excitations

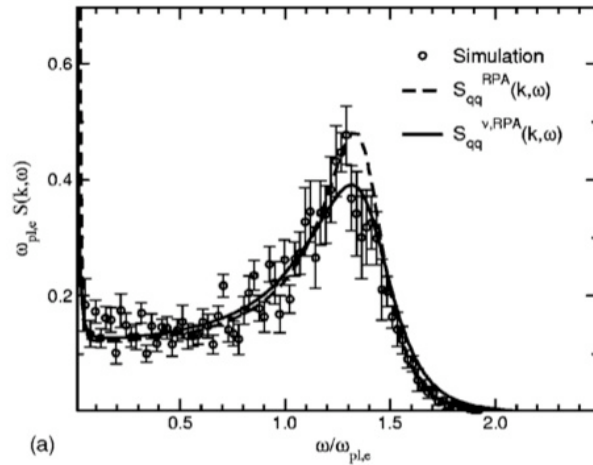
$$S(\vec{k}, \omega) = \frac{1}{\pi V(k)} \frac{1}{e^{-\beta\hbar\omega} - 1} \text{Im}\epsilon_l^{-1}(\vec{k}, \omega)$$

different auto-correlation functions (ACF):
current-current, force-force, density-density

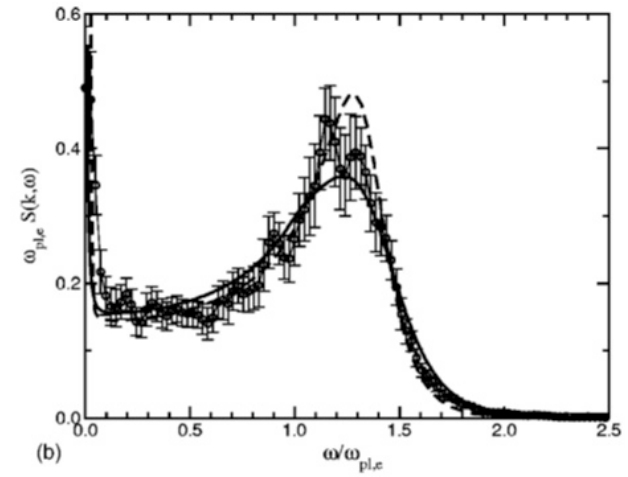
$$\epsilon_l^{-1}(k, \omega) \propto \frac{1}{\omega} \langle j_k^z; j_k^z \rangle_{\omega+i\eta} \propto \frac{\omega}{k^2} \langle \delta n_k; \delta n_k \rangle_{\omega+i\eta}$$

Dynamical structure factor for H plasmas

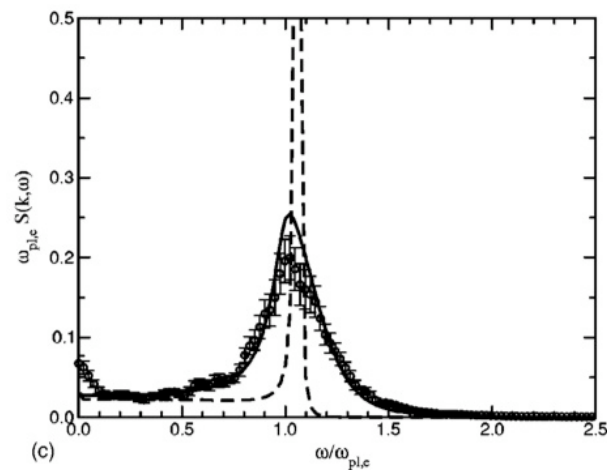
$\Gamma=0.5$
 $k/\kappa=0.51$



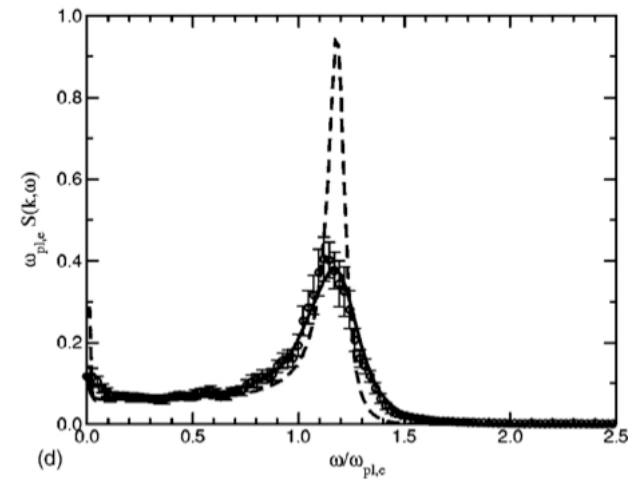
$\Gamma=1$
 $k/\kappa=0.51$



$\Gamma=2$
 $k/\kappa=0.25$



$\Gamma=1$
 $k/\kappa=0.36$

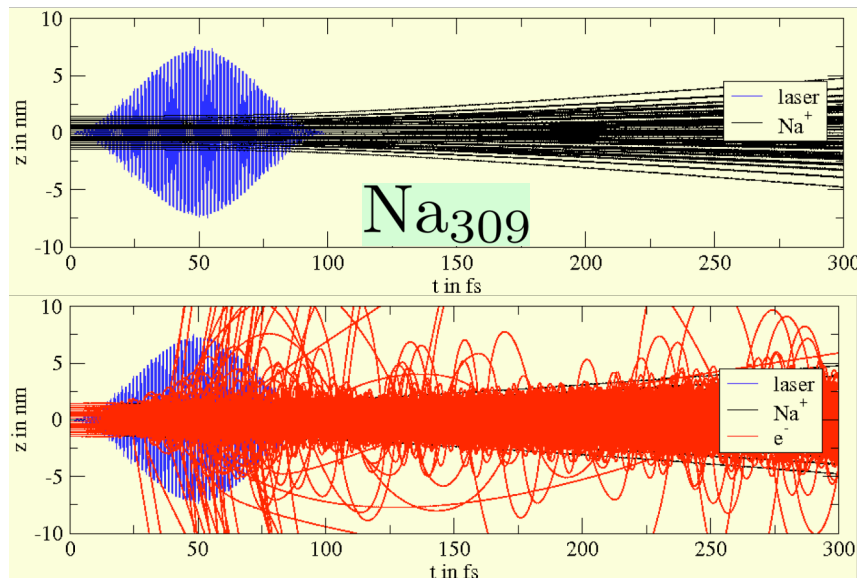


Finite size plasmas

Nanoplasma in laser excited clusters

sodium as exemplary system

- initial conditions: room temperature, solid density ($2.8 \cdot 10^{22} \text{ cm}^{-3}$)
- isocahedral structure (Na_{55} , Na_{147} , Na_{309}) and randomly distributed (Na_{309} , Na_{1000})
- intense short pulse laser ($\lambda=436 \text{ nm}$, $\Delta t = 50 \text{ fs}$, $I=10^{12} \text{ W cm}^{-2}$)



time evolution investigated via
MD simulations:
inner ionization, charging of cluster,
density, temperature

correlation functions -> optical properties

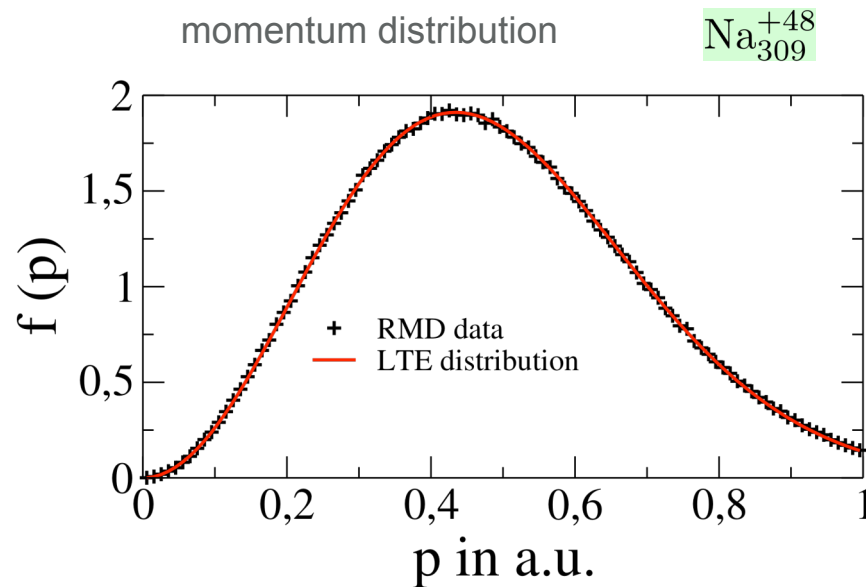
Raitza et al, JMPB 21 (2007) 2460; PRE 84, 214048 (2011)

Two-time distributions: RMD

Restricted MD simulation scheme

- observed local thermal equilibrium justifies restricted MD (RMD) ensemble average \rightarrow time average of trajectories for frozen ion configuration at fixed time

$T = 6.35$ eV (parameters for $t = 100$ fs)



- auto correlation functions
- bi-local correlation matrix

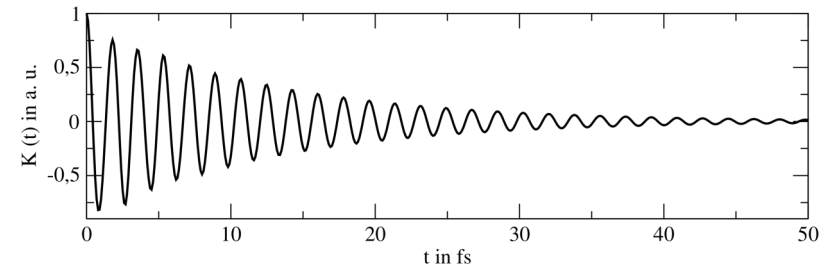


correlation effects

- collective excitations
- collisional damping

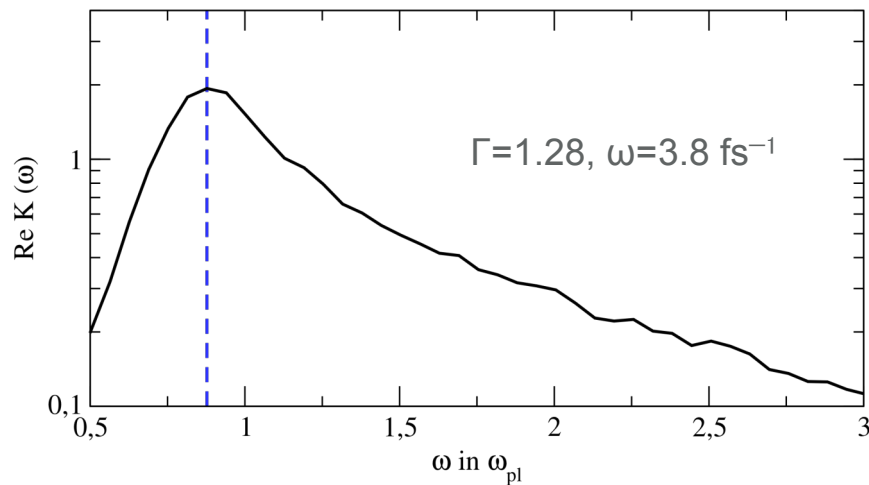
Frequency spectrum of correlation functions

$$K(t) = \frac{1}{\langle J^2 \rangle} \frac{1}{\delta} \int_0^\delta d\tau J^z(t + \tau) J^z(\tau)$$



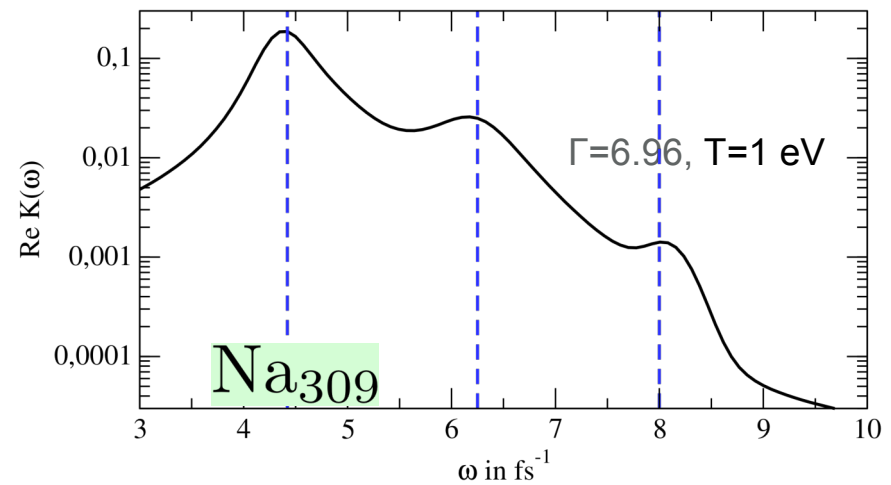
- Laplace transform $K(\omega)$ of total current density auto-correlation functions

bulk MD



H. Reinholz et al., PRE 69 (2004)

RMD cluster

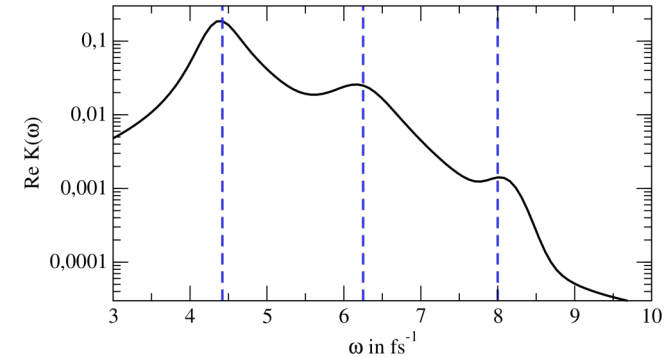


T. Raitza et al., JPA 42 (2009)

Plasma excitations

Resonances in finite systems

- multiple resonance structure obtained from RMD
- decomposition into spatially resolved correlations functions in order to find cross correlations

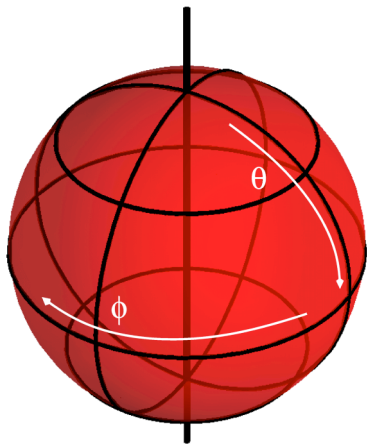


$$K(\vec{r}, \vec{r}', t) = \frac{1}{N_\tau} \sum_{a=1}^{N_\tau} J(\vec{r}, t + a \cdot \tau) \cdot J(\vec{r}', a \cdot \tau)$$

$$K(\omega) = \int \int d^3\vec{r} d^3\vec{r}' K(\vec{r}, \vec{r}', \omega)$$

$$= \int \int d^3\vec{r} d^3\vec{r}' \sum_{\nu} \Psi_{\nu}(\vec{r}, \omega) K_{\nu}(\omega) \Psi_{\nu}(\vec{r}', \omega)$$

- solving eigenproblem for resonance strength K_{μ} and spatial mode structure Ψ_{μ}

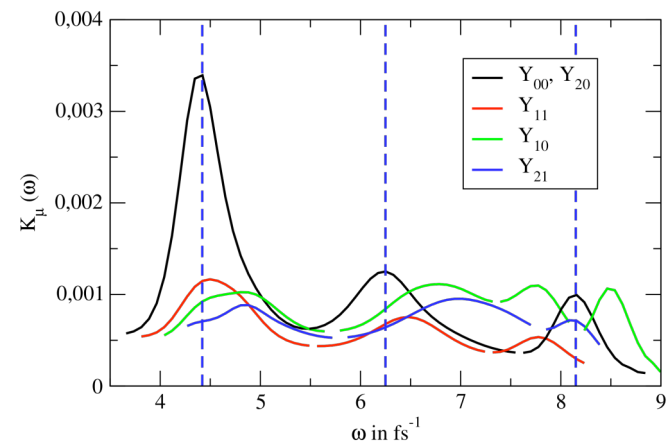
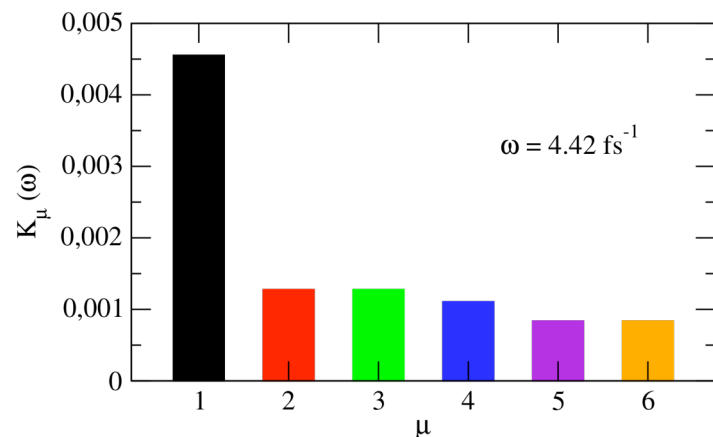


Plasma excitations

Eigenvalue spectrum

taking into account symmetries in bi-local correlation matrix

$$\int dr' K(\vec{r}, \vec{r}', \omega) \Psi_{\mu}(\vec{r}', \omega) = K_{\mu}(\omega) \Psi_{\mu}(\vec{r}, \omega)$$



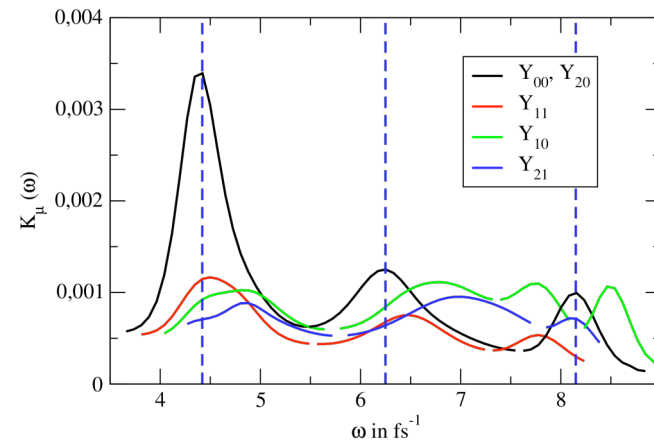
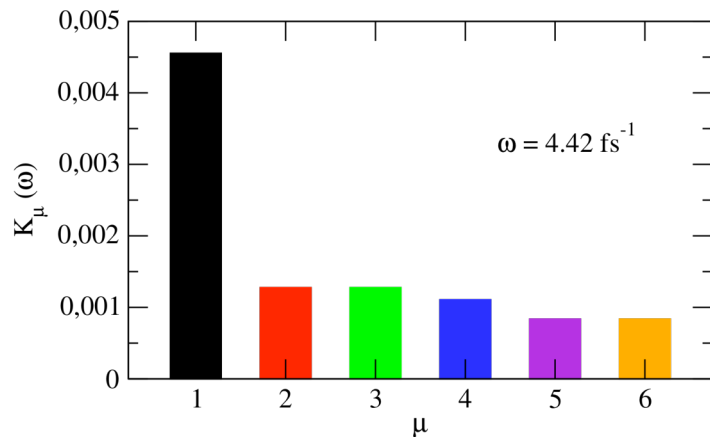
Na_{309} @ $T_e = 1.96 \text{ eV}$ and $n_e = 5.2 \cdot 10^{21} \text{ cm}^{-3}$

Plasma excitations

Eigenvalue spectrum

$$\int dr' K(\vec{r}, \vec{r}', \omega) \Psi_\mu(\vec{r}', \omega) = K_\mu(\omega) \Psi_\mu(\vec{r}, \omega)$$

$$= K_\mu(\omega) \sum_{n=1}^{N_r} \sum_{l=0}^{N_\theta} \sum_{m=-l}^l S_{nlm} j_l(k_{nl}r) Y_{lm}(\theta, \phi)$$

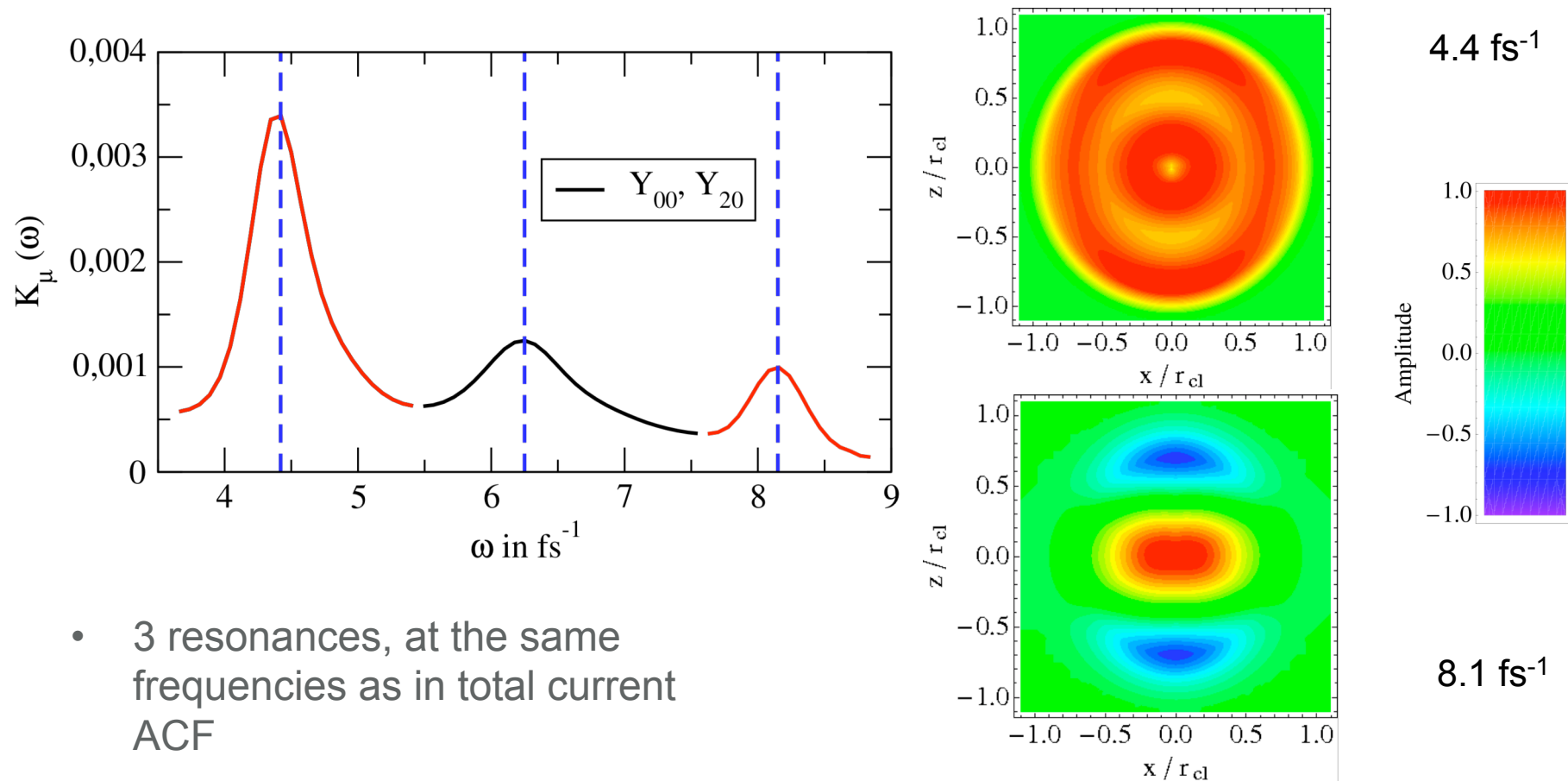


Na_{309} @ $T_e = 1.96 \text{ eV}$ and $n_e = 5.2 \cdot 10^{21} \text{ cm}^{-3}$

T. Raitza et al., PRE 84, 036406 (2011)

Plasma excitations

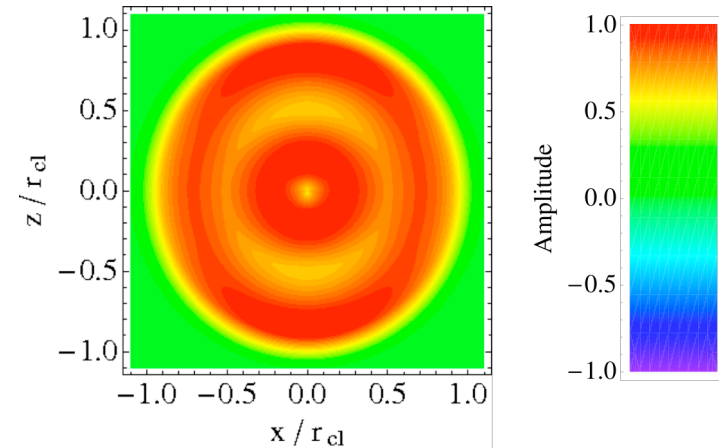
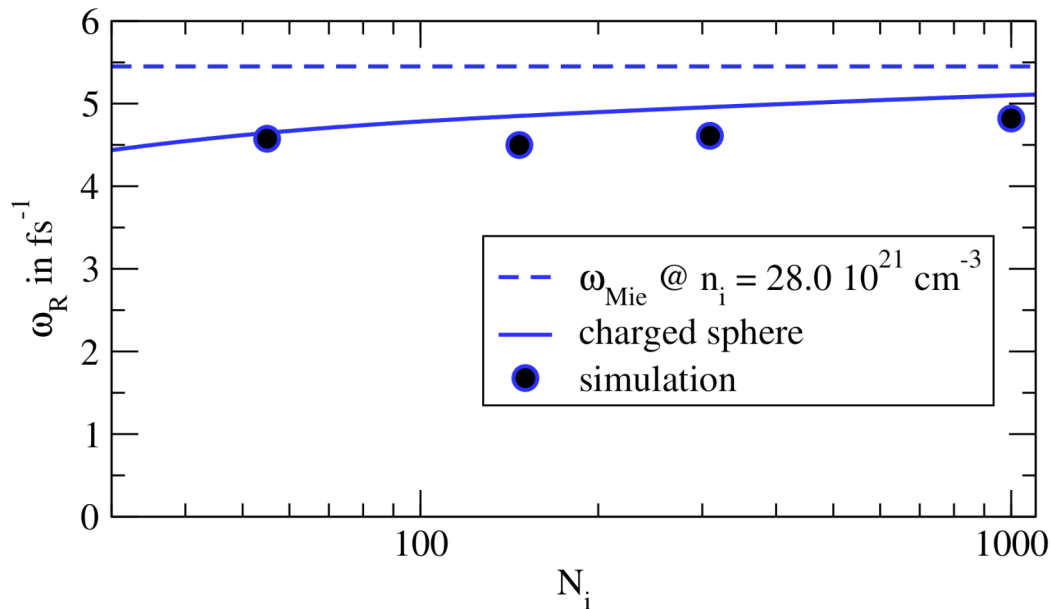
Fundamental dipole mode



- 3 resonances, at the same frequencies as in total current ACF

Plasma excitations

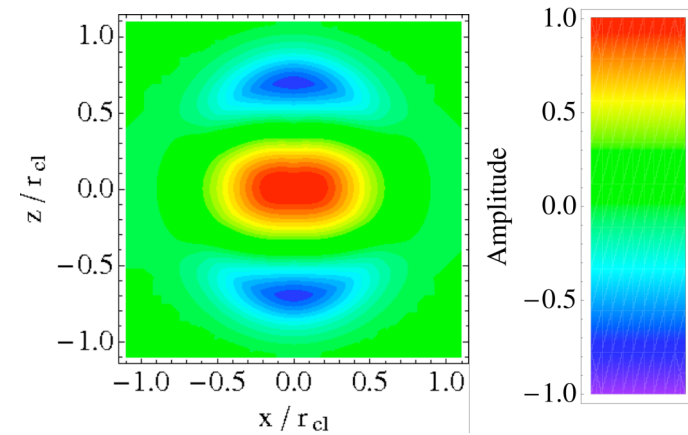
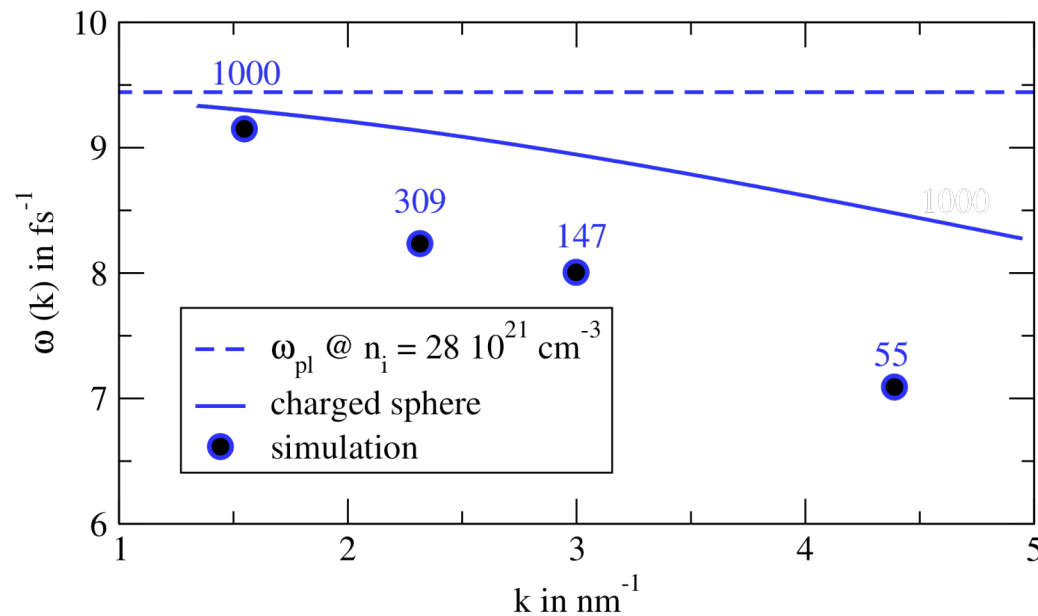
Fundamental dipole resonance



- rigid oscillation of electron density profile in external ion potential
- resonance of surface plasmon in analogy to Mie resonance

Plasma excitations

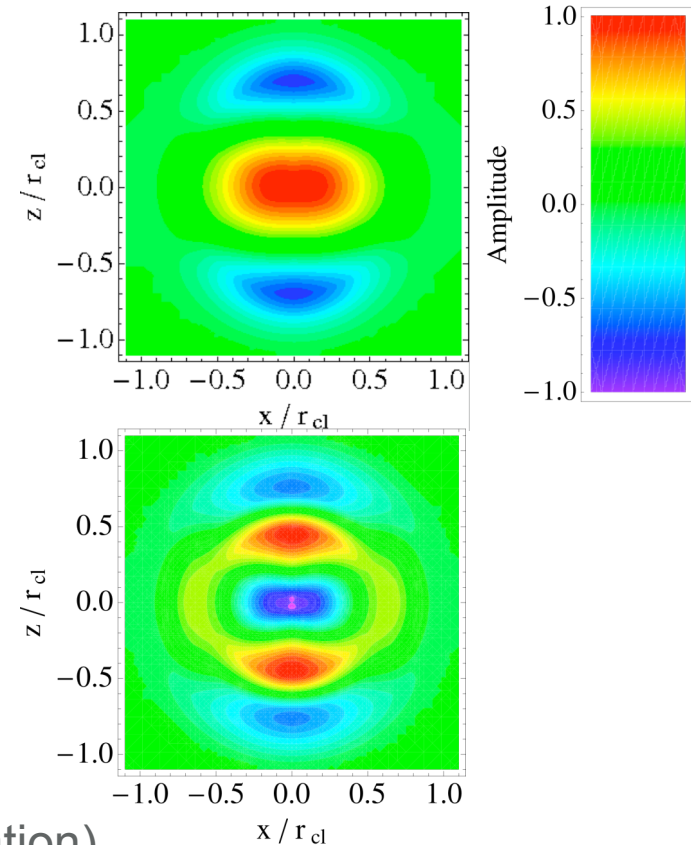
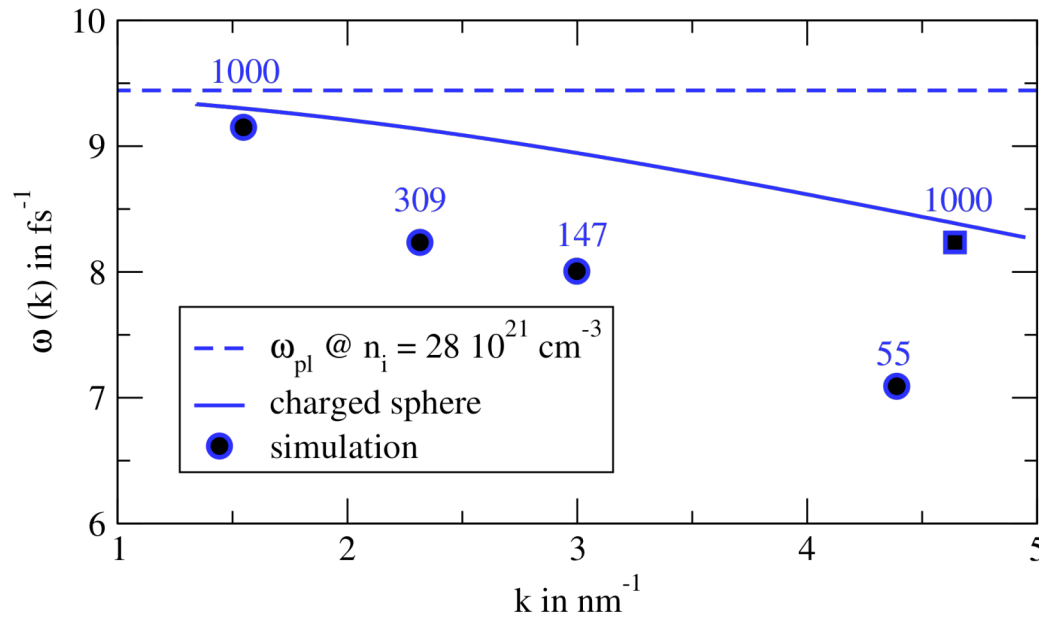
Dispersion of plane wave mode



- analogy to volume plasmon resonance in bulk
- plane wave confined within sphere (Euler equation)

Plasma excitations

Dispersion of plane wave mode



- analogy to volume plasmon resonance in bulk
- plane wave confined within sphere (Euler equation)
- for large clusters bulk limit obtained

Some Applications

Collective excitations in metal clusters

- photoabsorption by excited clusters (pump-probe experiments)
- scattering of light
- electron spectra

Collective excitations in other finite many-particle systems

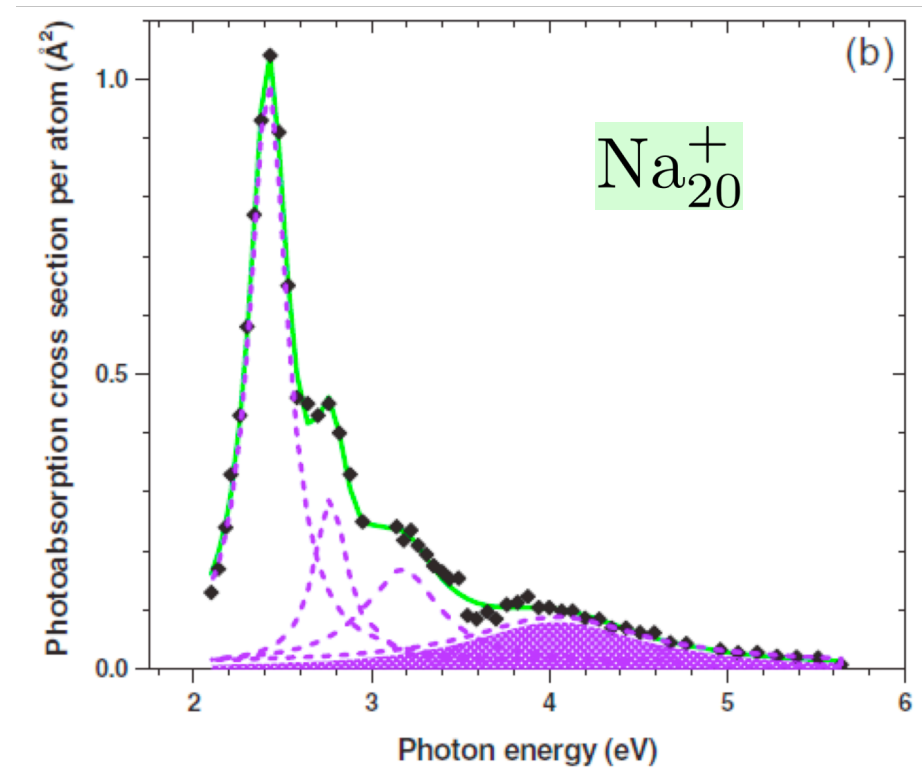
- atomic nuclei (giant dipole resonance, etc.)
- electrons in quantum dots
- dusty plasmas
- etc.

Experimental signatures

Na cluster

experiments on photodepletion
absorption cross section:

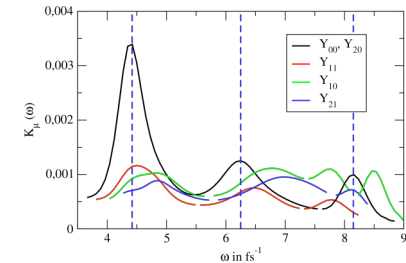
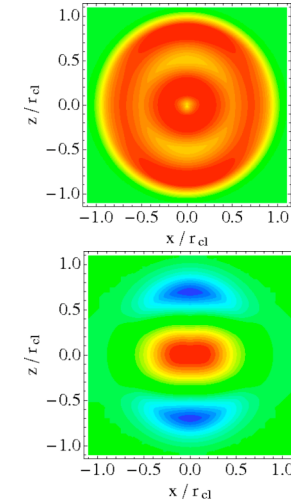
- multiple resonance structure for energies beyond Mie frequency
- fit by sum of Lorentzian profiles
- shaded area: volume plasmon (becomes dipole-active in finite systems)
- oscillator strength $f=0.71$



C. Xia, C. Yin, V.V. Kresin, PRL 102 (2009) 156802

Summary

- from bulk (wave vector k) to cluster (modes)
 - ✓ resonance modes beyond Mie/plasmon frequency
 - ✓ spatially resolved electron excitations
 - ✓ dependence on size, charge, density, temperature
 - ✓ change with cluster expansion (time evolution)
- damping behaviour of collective modes beyond Lorentzian fit and the relation to collision frequency
- discussion of higher modes, density vs. current (breathing modes, quadrupole modes, ..)
- some more technical issues
 - ✓ oscillator strength and link to experiments (electron- and photoabsorption spectroscopy)
 - ✓ going beyond non-degenerate systems (e.g. TD-LDA for electronic structure)
 - ✓ considering laser excitation processes for real systems



Thanks

T. Raitza, H. Reinholz, A. Wierling

Universität
Rostock



Traditio et Innovatio

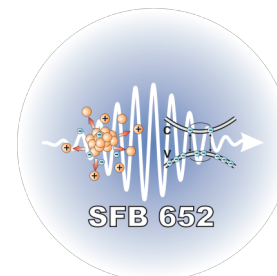
I. Morozov, (Moscow/Russland)



E. Suraud (Toulouse, Frankreich), G. Zwicknagel, P.G. Reinhard (Erlangen/Germany)

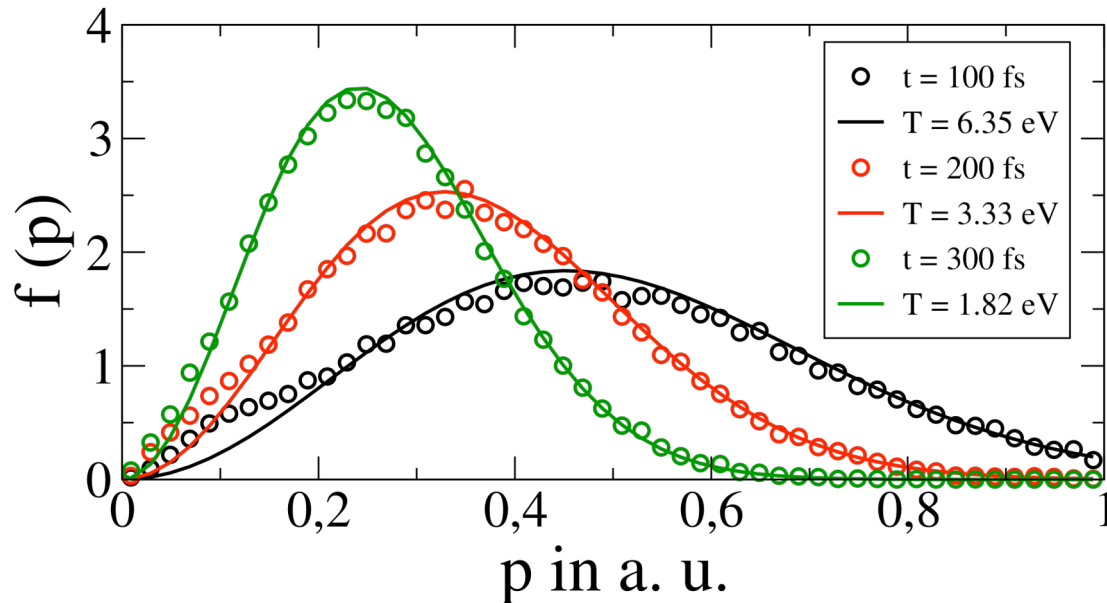
M. Winkel (Jülich/Germany), V. Fortov, V. Mintsev, V. Gryaznov, (Tschernogolovka/Russia)

Finances: SFB 652 (DFG)



Single-time distributions: LTE

Momentum distribution

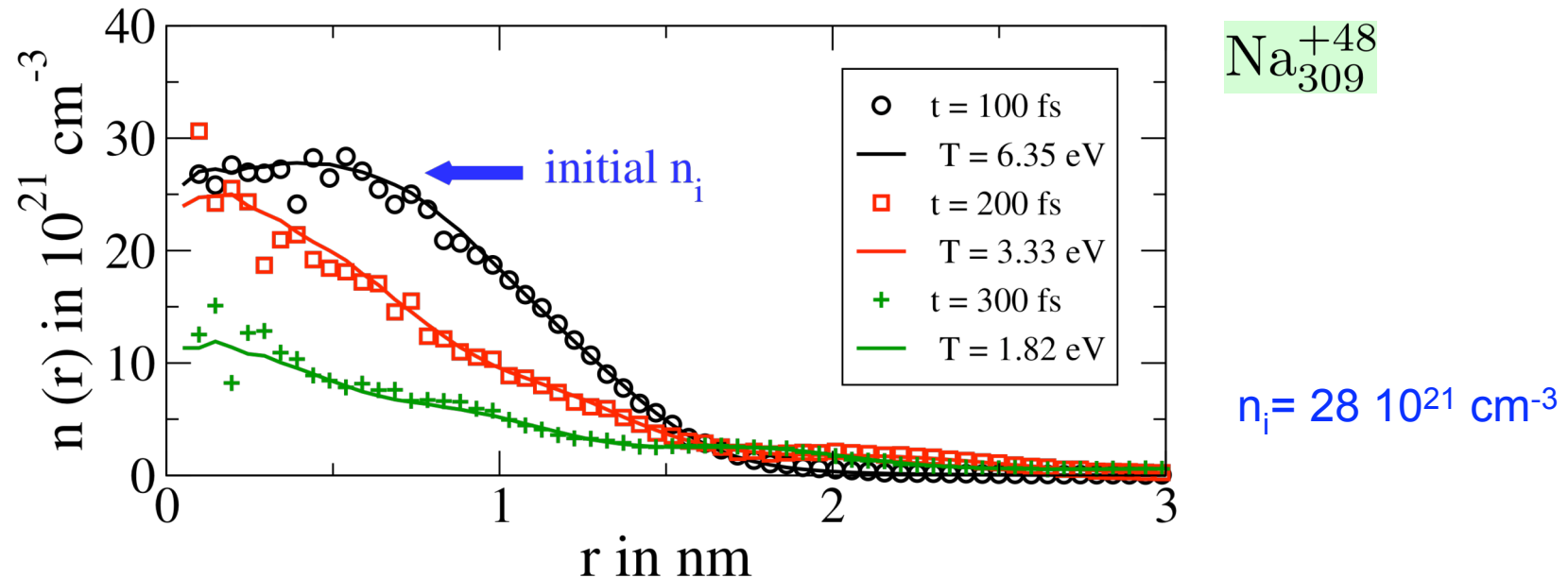


Na^{+48}_{309}

- isotropic distribution of inner free electrons observed
- agreement between temperatures due to mean kinetic energy $k_{\text{B}}T = \frac{3}{2} \langle E_{\text{kin}} \rangle$ and fit to Maxwell-Boltzmann distribution

Single-time distributions: LTE

Radial electron density profile

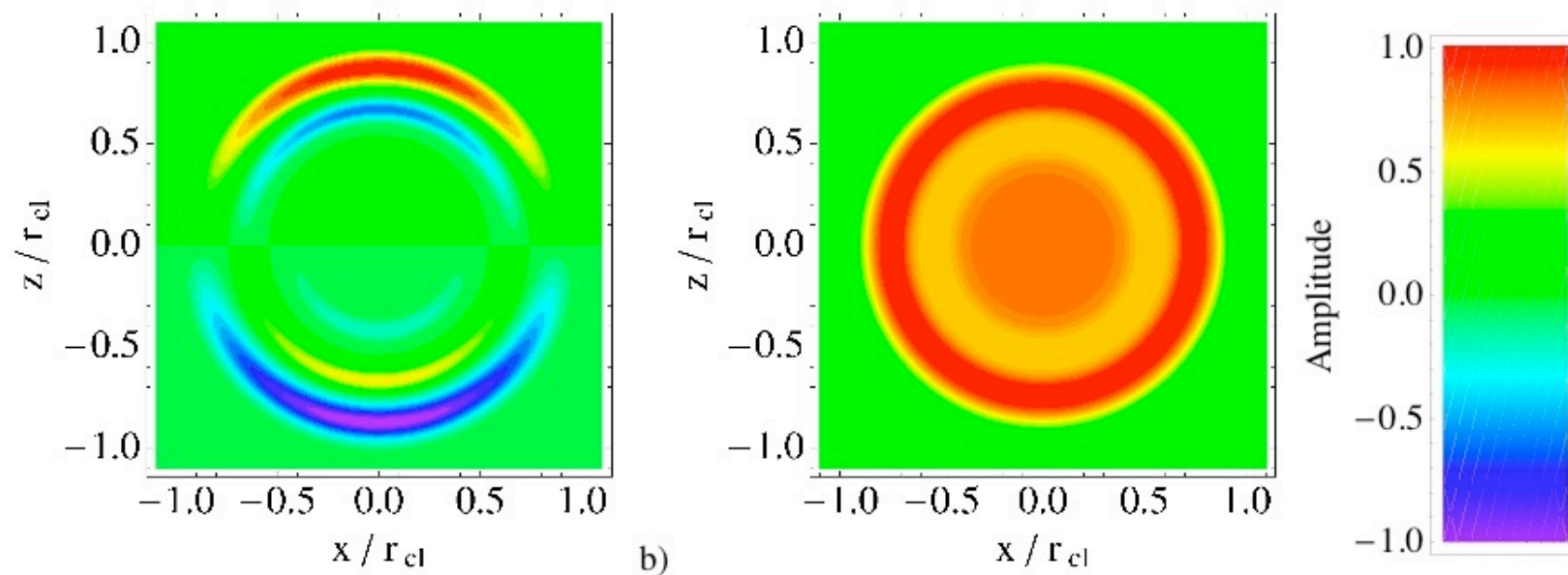


- simulation data can be interpreted as spatially distributed according to **mean field** $U(r)$ due to forces on electron at radial distance r from ions and electrons

$$n(r) = n_0 \exp \left\{ -\frac{U(r)}{k_B T} \right\}$$

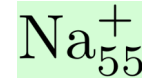
Bilocal density and current ACF

- Transformation from density ACF to curl-free current ACF



Experimental signatures

Na cluster



small to medium size at low temperatures

- vibrationally broadened electronic transitions
- onset of plasmon peak @ higher energies for $R \uparrow$

photoabsorption for sphere of radius R [2]

$$\alpha(\omega) \propto R^3 \text{Im} \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}$$

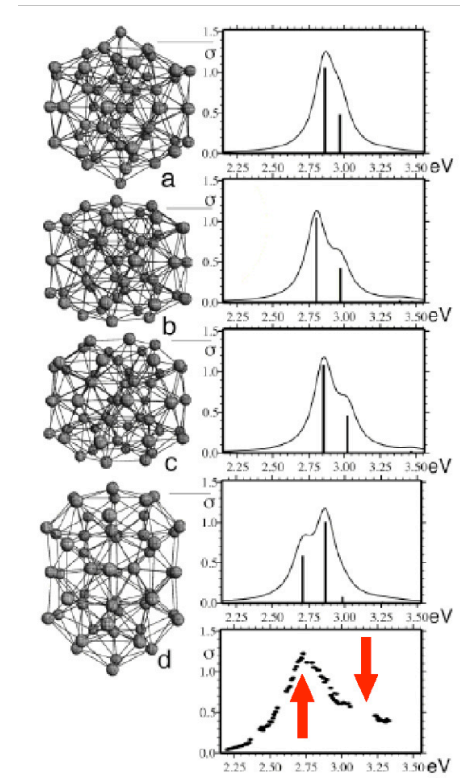
resonance frequency:

$$\omega_{\text{exp}} = 3.27 \text{eV} \quad [3]$$

homogeneous charge distribution

$$\omega_{\text{Mie}} = 3.41 \text{eV}$$

$$\omega_{\text{pl}} = \sqrt{3} \omega_{\text{Mie}} = 5.66 \text{eV}$$



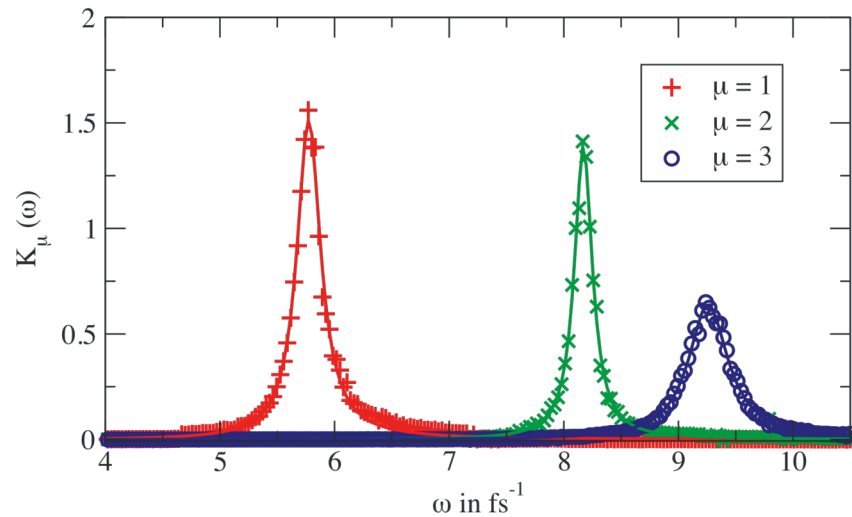
close to icosahedral core for E_{min} [1]

[1] Kümmel et al, PRB 62 (2000) 7602,

[2] Haberland, in: *Metal Clusters* (ed. Ekardt, 1999)

[3] Smith PR 183 (1969) 634

Broadening of collective excitations



example:
1-d chain with 55 ions

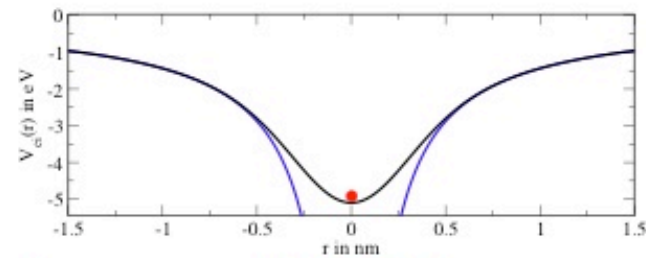
- eigenmode spectra can be considered separately
- Lorentz fit to each mode (resonance)

$$K_\mu(\omega) = K_0 \frac{\omega_\mu^2 \omega}{\nu_\mu \omega - i(\omega^2 - \omega_\mu^2)}$$

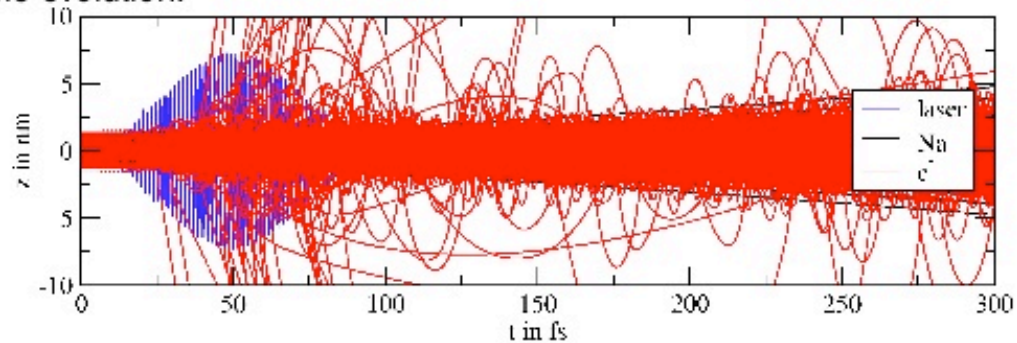
- clusters up to 1000 atoms
 - different ion structures
 - one electron per singly charged ion
 - initially electrons on top of ions

- error function potential:

$$V^{\text{erf}}(r) = -\frac{Ze^2}{r} \text{erf}\left(\frac{r}{\sigma}\right)$$

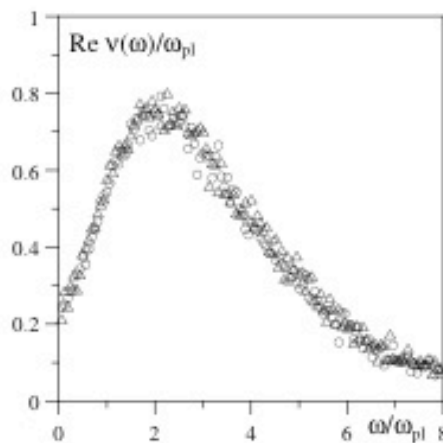


- time evolution:



- total current-density: $j_k(t) = \frac{1}{V} \frac{e}{m} \sum_i p_{i,k}(t)$
- current-density auto-correlation function (ACF):

$$K(k, t) = (j_k(t); j_k(0)) = \lim_{N_\tau \rightarrow \infty} \frac{1}{N_\tau} \sum_{i=0}^{N_\tau} j_k(t + \tau \cdot i) \cdot j_k(\tau \cdot i)$$



$$\Gamma = 1.27, T = 3 \text{ eV}$$

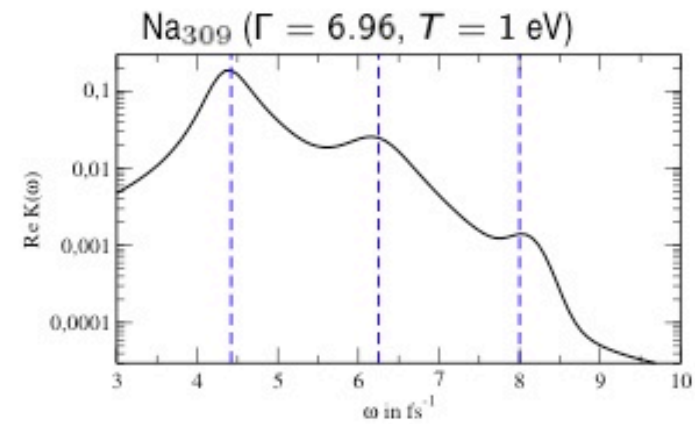
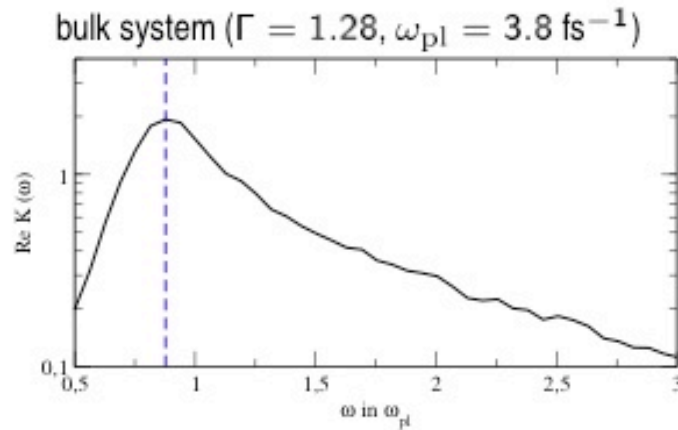
Reinholz *et al.*; *PRE* **62**, 066412 (2004)

- generalized Drude expression in long wavelength limit for bulk

$$K(\omega) = \frac{\omega}{\nu(\omega)\omega - i(\omega^2 - \omega_{pl}^2)}$$

- related to dynamical collision frequency $\nu(\omega)$ (see figure)

total current of:



H. Reinholz, *et al.*; *PRE* **69**, 066412 (2004)

T. Raitza, *et al.*; *JPA* **42**, 214048 (2009)

- bulk: single resonance described by generalized Drude model ($k = 0$):

$$K(\omega) = \frac{\omega}{\nu(\omega)\omega - i(\omega^2 - \omega_{pl}^2)}, \quad \omega_{pl}^2 = \frac{n_e e^2}{\epsilon_0 m_e}$$

- finite system: multiple resonances

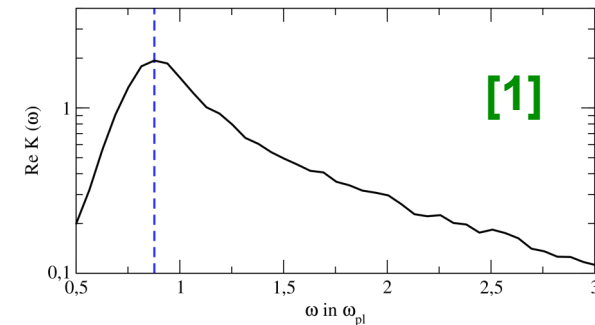
Plasma excitations

Plasmon resonance in bulk

- single resonance described by generalized Drude formula (linear response) [2]

$$K(\omega) = \frac{1}{\nu(\omega) - i \left(\omega - \frac{\omega_{pl}^2}{\omega} \right)},$$

- width of resonance peak due to collisional broadening described by dynamical collision frequency $\nu(\omega)$
- couples to longitudinal field, but not to transversal field (light)

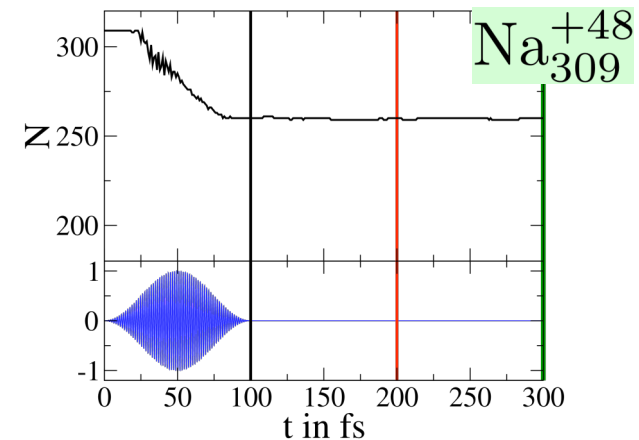
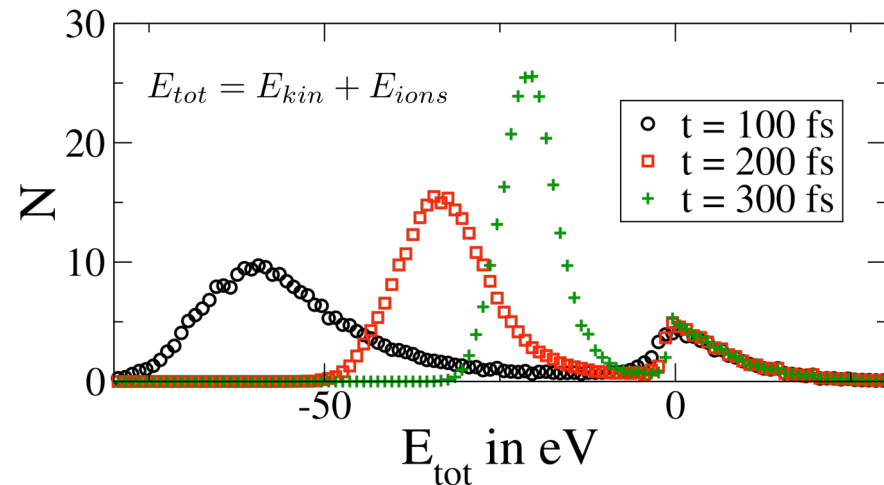
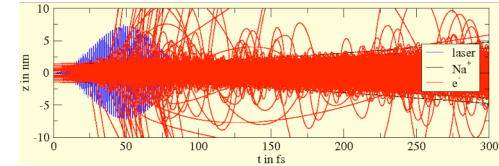


[1] Reinholz et al., PRE 69 (2004), [2] Reinholz et al, PRE 62 (2000)

Cluster parameters

Total energy of electrons

- cluster expansion process analyzed via ensemble average



- electrons with $E_{tot} < 0$ found in local thermal equilibrium distribution

$$f(r, p) = \frac{4\pi p^2}{Z} \exp \left\{ -\frac{1}{k_B T} \left(\frac{p^2}{2m} + U(r) \right) \right\}$$

Some Applications

- **reflectivity** *Morozov, Raitza, HR et al. 2005*
- **optical line shapes** (H, He⁺, Li²⁺) *Omar, Lorenzen, Wierling, HR et al. 2008, 2009*

VUV to X-ray necessary for diagnostics of warm dense matter

$$\omega > \omega_{\text{pl}} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$$

- **Thomson scattering** *Thiele, HR et al. PRE 78 (2008); Fäustlin, HR et al. 2009*
- **bremsstrahlung** *Zastrau, HR et al. PRE 78 (2008), Fortmann, HR et al. 2006, 2009*
- **X-ray emission lines** (K_α, K_β) *Sengebusch, HR et al. 2008, 2009*

Optical Properties

Dielectric function, homogeneous systems: wave vector k

$$\epsilon(\vec{k}, \omega) = 1 - \frac{1}{\epsilon_0 k^2} \left(\Pi_1(\vec{k}, \omega) + \Pi_2(\vec{k}, \omega) + \dots \right)$$

cluster expansion of polarization function:
contributions of free electrons Π_1 and bound states Π_2

- optical information: reflection, absorption

$$\lim_{k \rightarrow 0} \epsilon_t(\vec{k}, \omega) = \left(n(\omega) + \frac{ic}{2\omega} \alpha(\omega) \right)^2$$

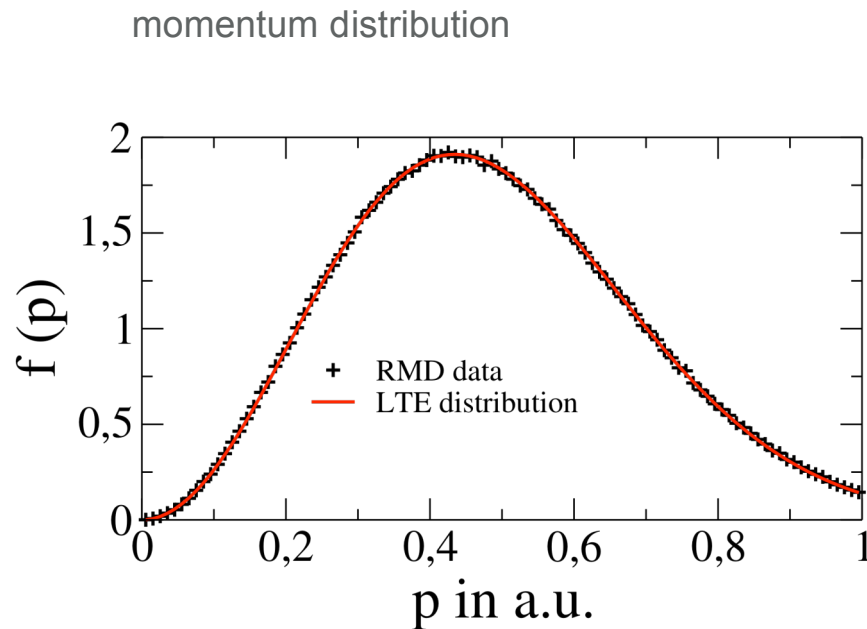
- dynamical structure factor (Thomson scattering)

$$S(\vec{k}, \omega) = \frac{1}{\pi V(k)} \frac{1}{e^{-\beta \hbar \omega} - 1} \text{Im} \epsilon_l^{-1}(\vec{k}, \omega)$$

Two-time distributions: RMD

Restricted MD simulation scheme

- observed local thermal equilibrium justifies restricted MD (RMD) ensemble average \rightarrow time average of trajectories for frozen ion configuration at fixed time



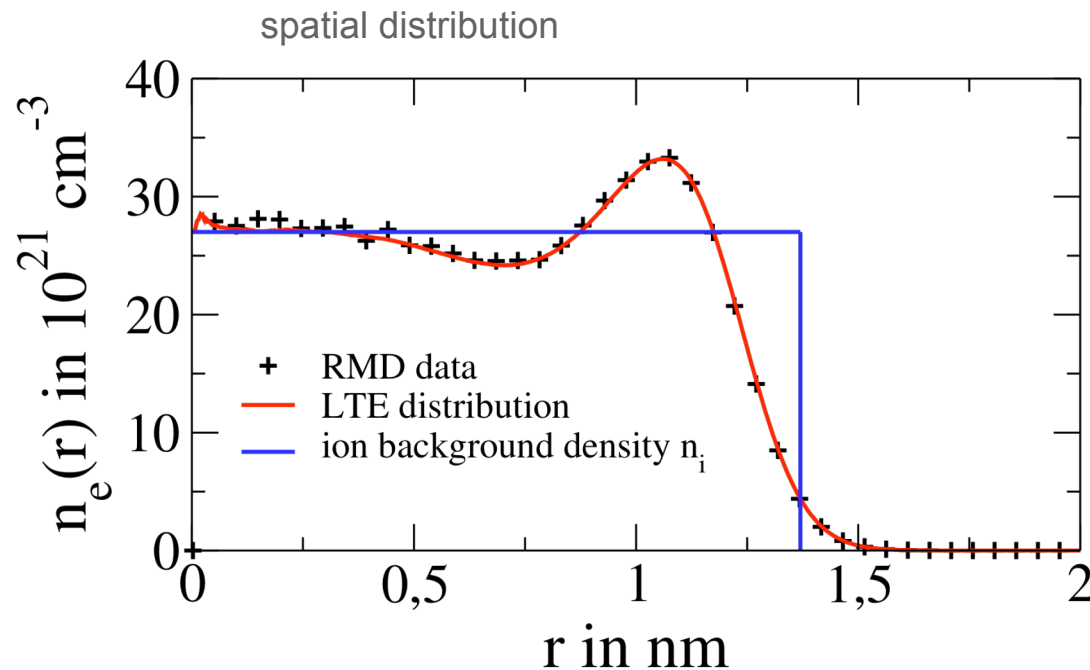
Na_{309}^{+48}

@ $T = 6.35$ eV (parameters for $t = 100$ fs)

Two-time distributions: RMD

Restricted MD simulation scheme

- observed local thermal equilibrium justifies restricted MD (RMD) ensemble average \rightarrow time average of trajectories for frozen ion configuration at fixed time

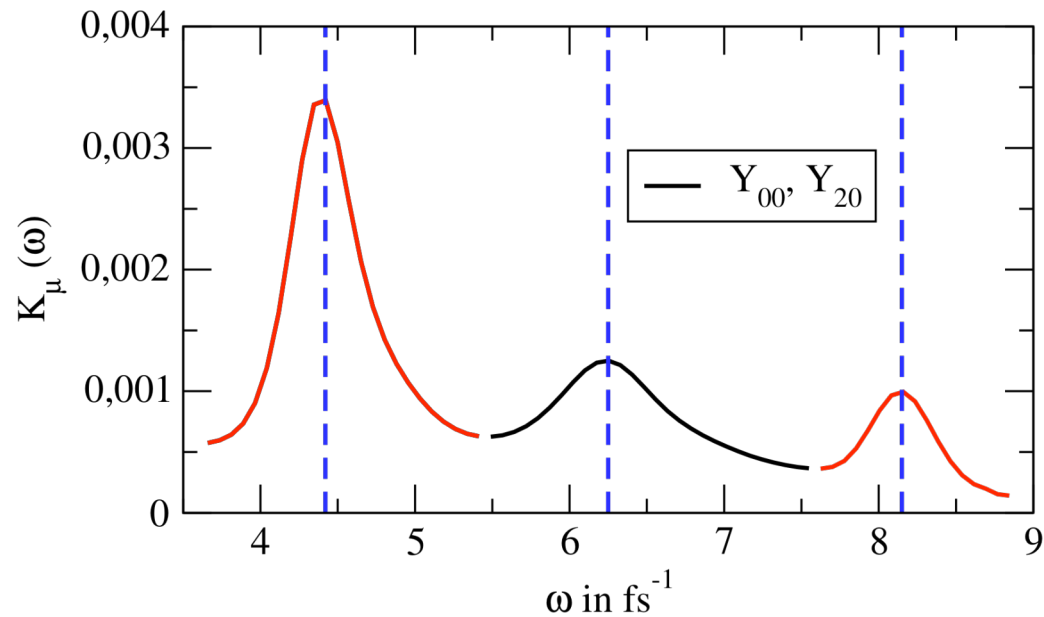


Na_{309}^{+48}

@ T= 6.35 eV (parameters for t=100 fs)

Plasma excitations

Fundamental dipole mode ($\mu=1$)



- 3 resonances, at the same frequencies as in total current ACF

