The electrostatic interaction of a charged spherical dielectric macroparticle with a point charge in a plasma in the presence of an external uniform electric field has been considered. The electrostatic force and the torque acting on the macroparticle have been determined, and the form of the interaction potential has been established for a nonuniform distribution of free charge on the macroparticle surface. A simple (for calculations) expression for the interaction potential that describes well the exact potential at all interparticle distances has been proposed. The angular velocity of the spinning of dust particles caused by a uniform distribution of free charge over their surface has been estimated.


1. Introduction

Dusty plasma is a convenient object for an investigation of properties of nonideal systems. The interaction potential of charged particles determines many properties of such systems. This paper is devoted to studying the electrostatic interaction of an arbitrarily charged dielectric spherical dust particle of radius $r_0$ and dielectric constant $\varepsilon_r$ with a point particle of charge $q_2$ immersed in plasma in presence of a uniform electric field $E_0$ with a point charge in plasma in presence of a uniform electric field.

2. The Field of a Charged Macroparticle and a Point Charge in Plasma

Let us introduce a coordinate system as shown in Fig.1. Due to linearity of the problem of the potential, the electrostatic potential can be written as:

$$\phi = \begin{cases} \phi_1, & r < a_1 \\ \phi_2 = \phi_1 + \phi_3, & r > a_1 \end{cases}$$

where $\phi_0 = -E_0 r$ is the constant electric field potential; $\phi_1$ is the Debye self-consistent potential of a point particle in plasma; $\phi_2$ is the potential inside the spherical particle (defined by the Laplace equation); $\phi_3$ is the self-consistent potential of the dust particle in plasma and defined by the linearized Poisson-Boltzmann equation.

After expanding $\phi_1$ and $\phi_3$ in series of spherical harmonics and using the boundary conditions on the macroparticle, we find the unknown coefficients and, therefore, the distribution of the potential.

The interaction potential of dust particles in isothermal plasma with the constant number of electrons and ions coincides with the free energy [1, 2]. The force acting on the point charge is defined by

$$F = -q_2 V_0 = -q_2 \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{r \sin \phi} \frac{\partial \phi}{\partial \phi} \right)$$

Being independent from the path of integration the interaction potential can be obtained by integrating the force component over $R$.

$$U(R, \theta, \phi) = U_{\text{DLVO}} + U_{\text{mK}}$$

$U_{\text{DLVO}} = \frac{1}{2} \frac{q_2 q_1}{R} \left( \frac{1}{1 + \frac{2 a_0}{R}} - 1 \right)$

$U_{\text{mK}} = \frac{1}{2} \frac{q_2 q_1}{R} \left( \frac{1}{1 + \frac{2 a_0}{R}} - 1 \right)$

where $a_0 = \frac{8 \pi}{3} \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \frac{\theta_0}{\varepsilon_r}$.

3. The Torque Acting on a Macroparticle

On the assumption that the medium is in mechanical and thermal equilibrium, the torques per unit area of dielectric [3]:

$$M = \frac{e}{4 \pi} \left[ r \times E_0 (nE_0 - |E_0|^2 r \times n) \right] = \frac{e}{4 \pi} \left[ E_0 q_2 - E_0 q_1 \right]$$

By making use of recurrence for the associated Legendre polynomials and their derivatives one can find projections of the moment on Cartesian axes.

Let us consider in detail an axisymmetric along the direction of the external electric field the free field charge distribution on the surface of the dust particle taking into account only the monopole and dipole terms, i.e. $\phi = \phi_0 + \phi_1 \cos \phi$. In this case, there is the $z$-component of force that tends to turn the point charge so that the direction of the line connecting the two particles coincided with that of the external electric field. In these electric fields.

$$\omega = \frac{2 \pi}{\tau}$$

where $\omega$ is the angular velocity of the macroparticle, $\tau$ is the relaxation time.

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Let us now discuss the spinning of dust particles, which was reported in [4–6]. In the absence of a magnetic field it can be expected that the distribution of surface charge on dust particles on the average in time is to be axisymmetric along the direction of the external electric field. For such distribution, it follows from (6) that the torque on an isolated dust particle in the region of levitation equals zero.

Further, suppose that the electron charge density on the surface of dust grain follows the angular dependence of the flow of electrons, whereas for ions follows the ion flow. Then,

$$\alpha(\theta, \phi) = \alpha_0 + \alpha_1 e^{-\alpha_2 \phi} \cos \phi$$

For such distribution of the surface charge only $M_z$ is nonzero. For weak screening, in this case one can find:

$$M_z = \frac{3 \pi e^2}{2 \varepsilon_r}$$

According to [7], the moment of a drag force of gas on a spinning particle is as follows:

$$M_{\text{F}} = \frac{2 \pi}{3 \varepsilon_r} \rho_{\text{gas}} v_{\text{gas}}$$

where $\rho_{\text{gas}}$ is the gas density, $v_{\text{gas}}$ is the thermal velocity of gas particles, $\alpha_0$ is the angular velocity of the dust grain.

Eventually, after equalizing (7) and (8) and using the condition of levitation of dust particles on gas:

$$\alpha_0 = \frac{3 \pi e^2}{2 \varepsilon_r}$$

where $\alpha_0$ is the density of the material of the dust particle, $\rho_{\text{gas}}$ is a parameter determining the degree of anisotropy of the electron charge distribution on the surface of dust particle.

Let us estimate the angular velocity of dust grains for the experimental conditions of [6], where the spinning of hollow spherical glass particles in neon at the pressure of $p = 0.15$ Torr was observed. Let us assume that $S_0 = 10^{-2}, T = 300 K, \rho_{\text{gas}} \sim 2 \text{g/cm}^3, \varepsilon_1 = 6$. From (9) one can finally find that the angular velocity around the $x$-axis is $\omega_{\text{x}} \sim 500 \text{ rad/s}$. This value of angular velocity and the direction of the rotation axis agree with those reported in [6], with such a large value being provided by only 1% of the electron charge anisotropy.

A more precise quantitative comparison requires a more rigorous examination of the surface charge distribution in the experiments.

References


Figure 1: The geometry of the problem

Figure 2: Potential energy of the macroparticle-point charge interaction versus $L$. For $q_2 = 10^{-7}, \varepsilon_r = 10, \mu_0 = 10^4 \mu_0, \varepsilon_r = 100, E_0 = 10^{2} \text{V/cm}$, $\varepsilon_1 = 0.10, E_0 = q_2/4\varepsilon_0 R_0$ at various values of the permitivity: $\varepsilon_r = 2, 2 - 4, 4 - 3\pi > 1, \varepsilon_r > 1$

Figure 3: Potential energy of the macroparticle-point charge interaction versus $L$. For $q_2 = 10^{-7}, \varepsilon_r = 10, \mu_0 = 10^4 \mu_0, \varepsilon_r = 100, E_0 = 10^{2} \text{V/cm}$, $\varepsilon_1 = 0.10$, $E_0 = q_2/4\varepsilon_0 R_0$ at various values of the permitivity: $\varepsilon_r = 2, 2 - 4, 4 - 3\pi > 1, \varepsilon_r > 1$. The solid, dash-dotted and dashed curves are for $b_0 = 0, \varepsilon_1 = 0, \varepsilon_1 = 0$, respectively.