Distribution of electrons and ions near a spherical dust particle

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Outline

Introduction

We scrutinize a kinetic collisional point-sink model of dust particle or spherical probe screnning, which described in papers:

- A. V. Filippov, A. G. Zagorodny, A. F. Pal', A. N. Starostin, A. I. Momot, Kinetic Description of the Screening of the Charge of Macroparticles in a Nonequilibrium Plasma, JETP Letters, v.86, pp.761–766 (2007).
- A.V. Filippov, A.G. Zagorodny, A.I. Momot, A.F. Pal, A.N. Starostin, Charge Screening in a Plasma with an External Ionization Source, J. Exp. Theor. Phys., v.104, pp.147–161 (2007).
- A.V. Filippov, A.G. Zagorodny, A.I. Momot, A.F. Pal, A.N. Starostin, Analysis of Macroparticle Charge Screening in a Nonequilibrium Plasma Based on the Kinetic Collisional Point Sink Model, J. Exp. Theor. Phys., v.125, pp.926–939 (2017).

Kinetic Collisional Point Sink Model

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{e_\sigma}{m_\sigma}\mathbf{E}\frac{\partial}{\partial \mathbf{v}} + \right.$$
$$\left. + \frac{1}{m_\sigma}\mathbf{F}_\sigma^{\text{ext}}\frac{\partial}{\partial \mathbf{v}}\right\} f_\sigma\left(X, t\right) = -\nu_\sigma\left[f_\sigma\left(X, t\right) - \right.$$
$$\left. - \Phi_\sigma\left(\mathbf{v}\right)\int d\mathbf{v} f_\sigma\left(X, t\right)\right] - S_\sigma(\mathbf{v}, t)\delta\left(\mathbf{r}\right)$$

$$\Delta\phi\left(\mathbf{r}, t\right) = -4\pi e q\left(t\right)\delta\left(\mathbf{r}\right) - 4\pi\sum_\sigma e_\sigma n_\sigma\int d\mathbf{v}\delta f_\sigma\left(X, t\right).$$

$$\sigma_\sigma\left(v\right) = \pi r_0^2 \times \begin{cases} 1 - \dfrac{2e_\sigma\phi_0}{m_\sigma v^2}, & v^2 > 2e_\sigma\phi_0/m_\sigma; \\ 0, & v^2 < 2e_\sigma\phi_0/m_\sigma; \end{cases}$$

For stationary corrections to unperturbed distribution functions

$$\delta f_{\sigma,\mathbf{k}}(\mathbf{v}) = \phi_{\mathbf{k}} \frac{e_\sigma}{m_\sigma} \frac{\partial f_{0\sigma}(\mathbf{v})}{\partial \mathbf{v}} \frac{\mathbf{k}}{\mathbf{k}\mathbf{v} - i\nu_\sigma} + \\ + \frac{iS_\sigma^{(0)}(\mathbf{v})}{\mathbf{k}\mathbf{v} - i\nu_\sigma} - i\mathbf{k}\phi_{\mathbf{k}} \frac{e_\sigma}{m_\sigma} \frac{\nu_\sigma \Phi_\sigma(\mathbf{v})}{\mathbf{k}\mathbf{v} - i\nu_\sigma} \\ \times \frac{1}{1 + I_{\Phi\sigma}(k)} \int \frac{1}{\mathbf{k}\mathbf{v}' - i\nu_\sigma} \frac{\partial f_{0\sigma}(\mathbf{v}')}{\partial \mathbf{v}'} d\mathbf{v}' \\ + \frac{\nu_\sigma \Phi_\sigma(\mathbf{v})}{\mathbf{k}\mathbf{v} - i\nu_\sigma} \frac{1}{1 + I_{\Phi\sigma}(k)} \int \frac{S_\sigma^{(0)}(\mathbf{v}')}{\mathbf{k}\mathbf{v}' - i\nu_\sigma} d\mathbf{v}',$$

After integration by Maxwell distribution functions

$$\delta n_{\sigma,\mathbf{k}} \equiv n_{0\sigma} \int \delta f_{\sigma,\mathbf{k}}(\mathbf{v})\, d\mathbf{v} = -\phi_{\mathbf{k}} \frac{e_\sigma n_{0\sigma}}{T_\sigma} + \frac{I_{S\sigma}(k)}{1 + I_{\Phi\sigma}(k)} n_{0\sigma}$$

where integrals $I_{\Phi\sigma}(k)$ and $I_{S\sigma}(k)$ are defined as

$$I_{\Phi\sigma}(k) = i\nu_\sigma \int \frac{\Phi_\sigma}{\mathbf{kv} - i\nu_\sigma} d\mathbf{v} \equiv -\frac{4\pi}{k} \int\limits_0^\infty \Phi_\sigma \nu_\sigma \arctan\left(\frac{kv}{\nu_\sigma}\right) v dv$$

$$I_{S\sigma}(k) = i \int \frac{S_\sigma^{(0)}(\mathbf{v})}{\mathbf{kv} - i\nu_\sigma} d\mathbf{v} = -\frac{4\pi}{k} \int\limits_0^\infty f_{0\sigma} \sigma_\sigma \arctan\left(\frac{kv}{\nu_\sigma}\right) v^2 dv$$

Fourier transform of potential

$$\phi_{\mathbf{k}} = \frac{4\pi e q_0}{k^2 \varepsilon(\mathbf{k}, 0)} - \frac{4\pi}{k^2 \varepsilon(\mathbf{k}, 0)} \times$$
$$\times \sum_{\sigma} e_{\sigma} n_{\sigma} \int d\mathbf{v} \int d\mathbf{v}' W_{\sigma\mathbf{k}}(\mathbf{v}, \mathbf{v}') S_{\sigma}^{(0)}(\mathbf{v}')$$

With Maxwell distribution functions

$$\phi_{\mathbf{k}} = \frac{4\pi e q_0}{k^2 + k_D^2} + \frac{4\pi}{k^2 + k_D^2} \sum_{\sigma} e_{\sigma} n_{0\sigma} \frac{I_{S\sigma}}{1 + I_{\Phi\sigma}}$$

After inverse Fourier transform

$$\phi(r) = \frac{eq_0 e^{-k_D r}}{r} + \frac{2}{\pi r}\sum_\sigma e_\sigma n_\sigma \int \frac{I_{S\sigma}(k)}{1 + I_{\Phi\sigma}(k)} \frac{\sin(kr)}{k^2 + k_D^2} k\mathrm{d}k,$$

$$\delta n_\sigma(r) = -n_{0\sigma}\frac{e_\sigma \phi(r)}{T_\sigma} + \frac{n_{0\sigma}}{2\pi^2 r}\int \frac{I_{S\sigma}(k)}{1 + I_{\Phi\sigma}(k)} \sin(kr)\, k\mathrm{d}k.$$

$$I_{\Phi\sigma}(k) = -(\pi)^{1/2}\frac{k_{\nu\sigma}}{k}\mathrm{Erfc}\left(\frac{k_{\nu\sigma}}{k}\right)\exp\left(\frac{k_{\nu\sigma}^2}{k^2}\right)$$

$$k_{\nu\sigma} = \nu_\sigma\sqrt{\frac{m_\sigma}{2T_\sigma}}$$

The OML approach

- Ya. L. Al'pert, A. V. Gurevich, and L. P. Pitaevskii, Space Physics with Artificial Satellites (Plenum Press, New York, 1965).

Without absorbtion for electrons

$$n_e\left(r\right) = n_{e0}\exp\left(-\varphi\right),$$

and with absorbtion

$$n_e\left(r\right) = \frac{n_{e0}}{2}\left\{1 + \mathrm{Erf}\left(\sqrt{\varphi_0-\varphi}\right) + \right.$$
$$\left. + \sqrt{1-\frac{a^2}{r^2}}\,\mathrm{Erfc}\left(\sqrt{\frac{\varphi_0-\varphi}{1-a^2/r^2}}\right)\exp\left[\left(\varphi_0-\varphi\right)\frac{a^2}{r^2-a^2}\right]\right\}\exp\left(-\varphi\right)$$

Here $\mathrm{Erf}(x)$ is the error function: $\mathrm{Erf}\left(x\right) = \frac{2}{\sqrt{\pi}}\int\limits_0^x e^{-t^2}dt$, $\mathrm{Erfc}(x)$ is the complementary error function: $\mathrm{Erfc}(x) = 1 - \mathrm{Erf}(x)$, $\varphi(r) = -e\phi(r)/T_e$, $\varphi_0 = -e\phi_0/T_e$.

Ion distribution

Case 1. $|e\phi(r)| \sim 1/r^{2-\delta}$, $\delta > 0$

Without absorbtion

$$n_i(r) = n_{i0}\left[\frac{2}{\sqrt{\pi}}\sqrt{z\beta\varphi} + e^{z\beta\varphi}\mathrm{Erfc}\left(\sqrt{z\beta\varphi}\right)\right],$$

and with absorbtion

$$n_i(r) = n_{i0}\left\{\sqrt{\frac{z\beta\varphi}{\pi}}\left[1 + \sqrt{1 - \frac{a^2}{r^2}\frac{\varphi_0}{\varphi}}\right] + \frac{1}{2}e^{z\beta\varphi}\mathrm{Erfc}\left(\sqrt{z\beta\varphi}\right) + \right.$$
$$\left. + \frac{\sqrt{1 - a^2/r^2}}{2}e^{z\beta\widetilde{\varphi}}\mathrm{Erfc}\left(\sqrt{z\beta\widetilde{\varphi}}\right)\right\},$$

$\widetilde{\varphi} = \left(\varphi - \varphi_0 a^2/r^2\right)/\left(1 - a^2/r^2\right)$, $\beta = T_e/T_i$.

Note that in the case 1 at all distances

$$\frac{r^2\varphi(r)}{a^2} = \varphi_0\left(\frac{r}{a}\right)^{\delta} \geq \varphi_0,$$

Case 2. $|e\phi(r)| \sim 1/r^{2-\delta}$ at small distances and $|e\phi(r)| \sim 1/r^{2+\epsilon}$ at large distances r ($\delta > 0$, $\epsilon > 0$)

For this case without absorption

$$n_i(r) = n_{i0}\left[e^{z\beta\varphi}\mathrm{Erfc}\left(\sqrt{z\beta\varphi}\right) + I(r,\infty)\right],$$

where $I(r,b)$ is defined by

$$I(r,b) = \frac{1}{2}\sqrt{\frac{z\beta}{\pi}}\int\limits_{\varrho_0(r)}^{b}\frac{\left(r_0^2/r^2-1\right)\left(3\frac{d\varphi}{dr_0}+r_0\frac{d^2\varphi}{dr_0^2}\right)}{\left[F(r,r_0)\right]^{1/2}}\times$$

$$\times\exp\left[z\beta\left(\varphi(r_0)+\frac{r_0}{2}\frac{d\varphi(r_0)}{dr_0}\right)\right]dr_0,$$

$$F(r,r_0) = \varphi(r)-\varphi(r_0)+\frac{r_0}{2}\frac{d\varphi}{dr_0}\left(\frac{r_0^2}{r^2}-1\right),$$

ϱ_0 is the maximal root of the following equation:

$$\varphi(\varrho_0)-\varphi(r)-\frac{\varrho_0}{2}\left.\frac{d\varphi}{dr}\right|_{r=\varrho_0}\left(\frac{\varrho_0^2}{r^2}-1\right)=0.$$

With ion absorption by probe in the case 2

$$n_i(r) = \frac{1}{2} n_{i0} \left\{ e^{z\beta\varphi} \mathrm{Erfc}\left(\sqrt{z\beta\varphi}\right) + \right.$$

$$+ \sqrt{1 - \frac{a^2}{r^2}} \mathrm{Erfc}\left(\sqrt{ -\frac{z\beta\varrho_0^3(a)}{2a^2} \left. \frac{d\varphi}{dr} \right|_{r=\varrho_0(a)} - \frac{z\beta\left[\varphi_0 - \varphi(r)\right]}{1 - a^2/r^2} } \right) \times$$

$$\left. \times \exp\left[-z\beta \frac{a^2\varphi_0 - r^2\varphi(r)}{r^2 - a^2} \right] + I\left(r, \varrho_0(a)\right) + I(r, \infty) \right\},$$

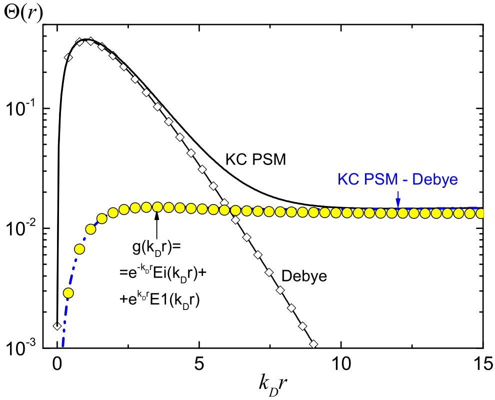
Modified OML

$$n_i(r) = n_{i0}\left\{\sqrt{\frac{z\beta\varphi}{\pi}}\left[1+\sqrt{1-\frac{a^2}{r^2}\frac{\varphi_0}{\varphi}}\right]+\frac{1}{2}e^{z\beta\varphi}\mathrm{Erfc}\left(\sqrt{z\beta\varphi}\right)+\right.$$
$$\left.+\frac{\sqrt{1-a^2/r^2}}{2}e^{z\beta\widetilde{\varphi}}\mathrm{Erfc}\left(\sqrt{z\beta\widetilde{\varphi}}\right)\theta\left(\widetilde{\varphi}\right)\right\},$$

$$\widetilde{\varphi} = \left(\varphi-\varphi_0 a^2/r^2\right)/\left(1-a^2/r^2\right)$$

- X.-Z. Tang and G. L. Delzanno, Orbital-motion-limited theory of dust charging and plasma response. Phys. Plasmas, v.21, 123708 (2014).
- T. Bystrenko and A. Zagorodny. Effects of bound states in the screening of dust particles in plasmas. Phys. Lett. A, v.299, 383 (2002).

Argon, $p = 0.1$ Pa, $a = 1\,\mu$m, $E/N = 1$ Td, $M = 2^{17}$

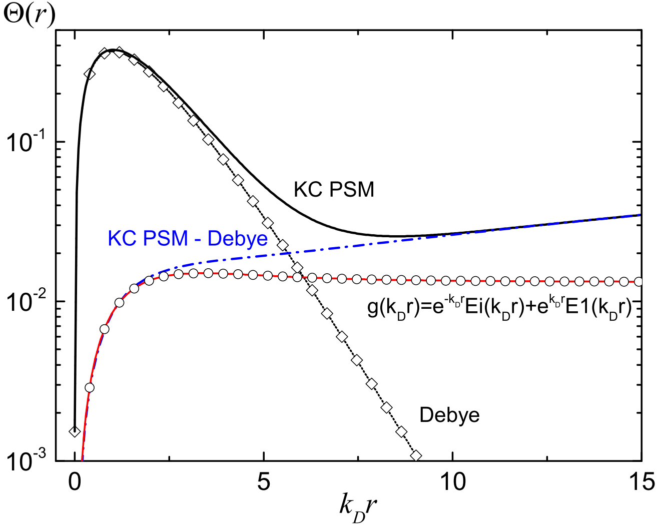


$$\Theta(r) = k_D r^2 \phi(r)/\phi_0,\ \phi_g(r) = e(\widetilde{Q}_e + \widetilde{Q}_i) g(k_D r)/r$$

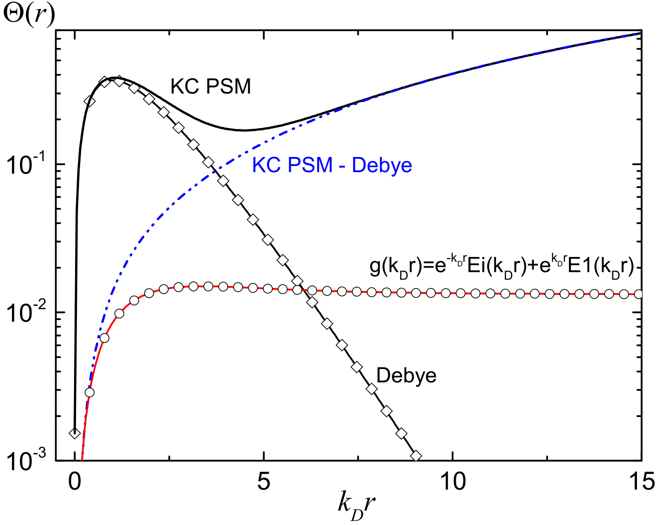
$$g(x) = \left[e^{-x}\mathrm{Ei}(x) + e^{x}\mathrm{E}_1(x)\right],\ \widetilde{Q}_\sigma = \frac{2\pi z_\sigma n_{\sigma 0}}{k_D} \int_0^\infty f_{\sigma 0}\sigma_\sigma v^2 \mathrm{d}v$$

At $x \to \infty$, $g(x) \to 2/x$

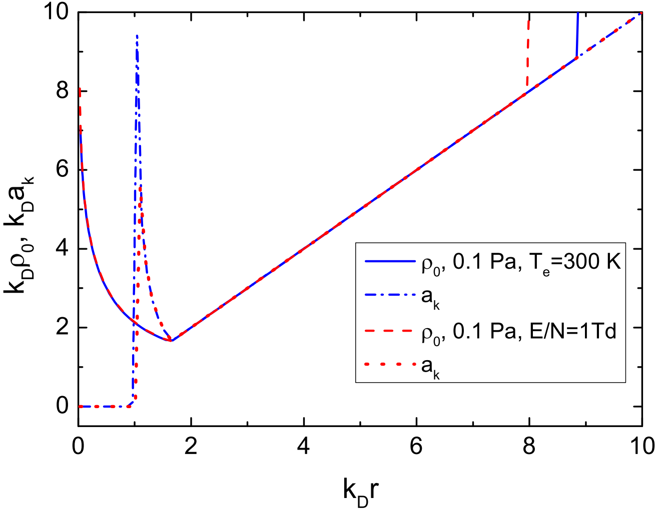
$p = 1$ Pa


Θ(r)
10-1
10-2
10-3
0
5
10
15
k_D r
KC PSM
KC PSM - Debye
g(k_D r)=e^{-k_D r}Ei(k_D r)+e^{k_D r}E1(k_D r)
Debye

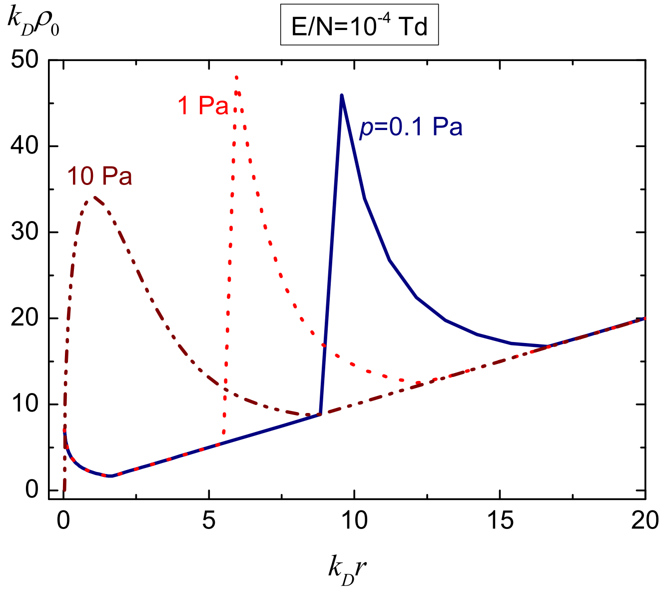
$p = 10$ Pa


Θ(r)
KC PSM
KC PSM - Debye
g(k_D r)=e^{-k_D r}Ei(k_D r)+e^{k_D r}E1(k_D r)
Debye
10^{-1}
10^{-2}
10^{-3}
0
5
10
15
k_D r

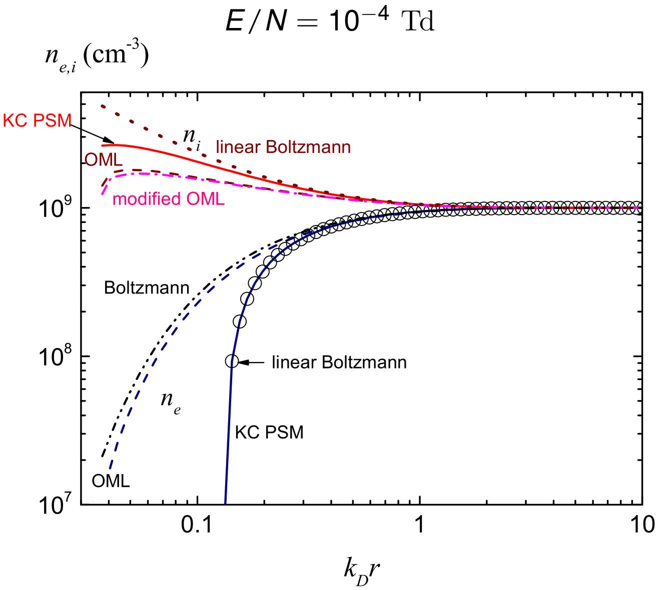
Ar, $a = 1\,\mu\mathrm{m}$, $E/N = 10^{-4}$ Td



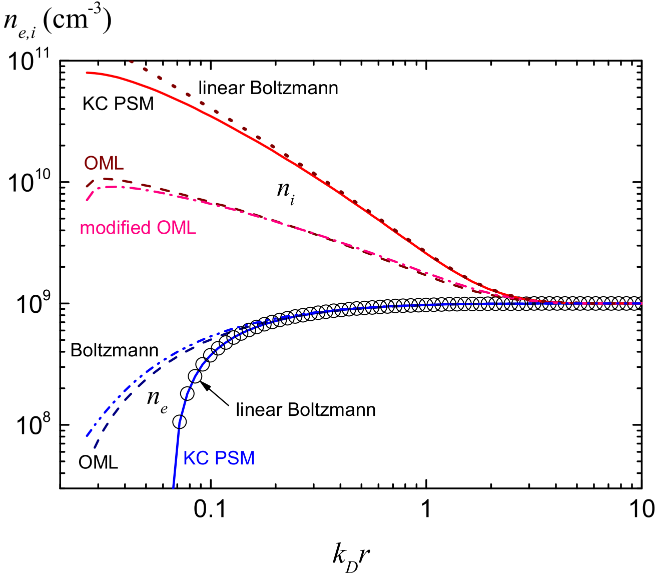
Ar, $a = 1\,\mu$m



Ar, $a = 1\,\mu\mathrm{m}$, $p = 0.1$ Pa

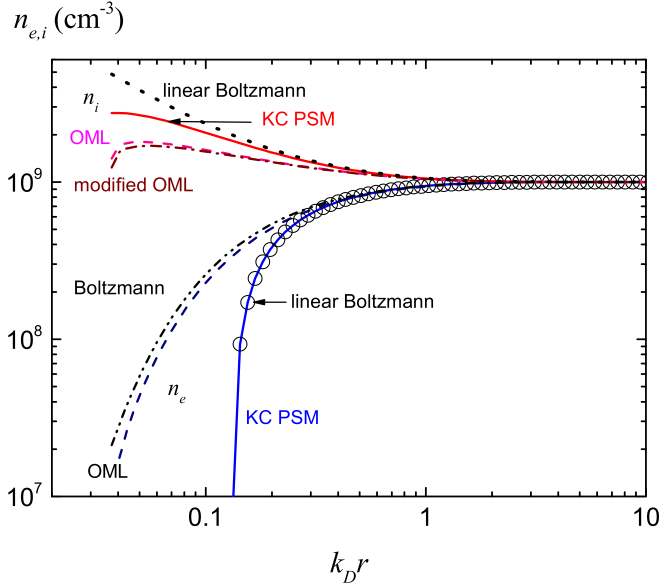


$E/N = 1$ Td

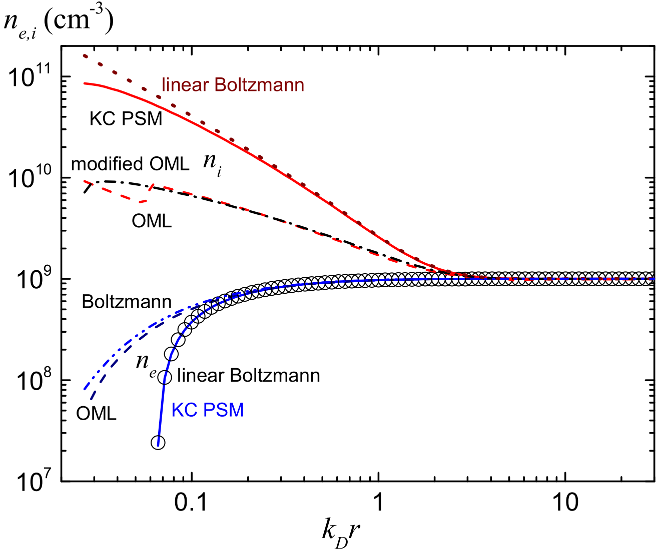


$n_{e,i}$ (cm^{-3})
10^{11}
10^{10}
10^{9}
10^{8}
linear Boltzmann
KC PSM
OML
n_i
modified OML
Boltzmann
n_e
linear Boltzmann
OML
KC PSM
0.1
1
10
$k_D r$

Ar, $a = 1\,\mu$m, $p = 1$ Pa
$E/N = 10^{-4}$ Td



$E/N = 1$ Td


$n_{e,i}$ (cm^{-3})
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
linear Boltzmann
KC PSM
modified OML
n_i
OML
Boltzmann
n_e
linear Boltzmann
OML
KC PSM
0.1
1
10
$k_D r$

Case 1.

$$S_e = \pi r_0^2 n_{e0} \sqrt{\frac{8T_e}{\pi m_e}} \exp\left(\frac{e\phi_0}{T_e}\right),\ S_i = \pi r_0^2 z n_{i0} \sqrt{\frac{8T_i}{\pi m_i}} \left(1 - \frac{ez\phi_0}{T_i}\right)$$

Cases 2 and 3.

$$S_i = \pi r_0^2 z n_{i0} \sqrt{\frac{8T_i}{\pi m_i}} \left\{ C_0 + \frac{\varrho_0^2(a)}{a^2} - \left[\frac{\varrho_0^2(a)}{a^2} - 1\right] \times \right.$$
$$\times \exp\left[z\beta\varphi\left(\left.\varrho_0\right|_{r=a}\right) + \frac{z\beta\varrho_0(a)}{2} \left.\frac{d\varphi}{dr}\right|_{r=\varrho_0(a)} \right] +$$
$$\left. + \frac{2}{a^2} \int\limits_{\varrho_0(a)}^{\infty} r_0 \left[1 - \exp\left(z\beta\varphi(r_0) + \frac{r_0 z\beta}{2}\frac{d\varphi(r_0)}{dr_0}\right)\right] dr_0 \right\}$$

In the case 2, $C_0 = 0$

The ion sink to a dust particle with $a = 1\,\mu$m at $p = 1$ Pa

Газ	$E/N = 10^{-4}$ Td		$E/N = 1$ Td	
	S_i (case 1)	S_i (case 2,3)	S_i (case 1)	S_i (case 2,3)
He	$1.60 \cdot 10^7$	$2.01 \cdot 10^7$	$1.34 \cdot 10^8$	$1.14 \cdot 10^8$
Ne	$8.30 \cdot 10^6$	$1.07 \cdot 10^7$	$3.86 \cdot 10^8$	$4.33 \cdot 10^8$
Ar	$6.25 \cdot 10^6$	$1.10 \cdot 10^7$	$2.06 \cdot 10^8$	$4.97 \cdot 10^8$
Kr	$4.58 \cdot 10^6$	$8.58 \cdot 10^6$	$1.41 \cdot 10^8$	$3.81 \cdot 10^8$
Xe	$3.79 \cdot 10^6$	$2.04 \cdot 10^6$	$1.00 \cdot 10^8$	$3.33 \cdot 10^8$

Выводы

- Приближение ограниченных орбит (ПОО) в инертных газах применимо только в пределе низких давлений, а с ростом давления кулоновская асимптотика потенциала, пропорциональная частоте столкновений электронов и ионов с нейтральными атомами (молекулами), делает неприменимыми формулы приближения ограниченных орбит для распределения ионов.
- Распределения электронов и ионов, полученные в рамках столкновительной кинетической модели точечных стоков, оказались близки к линеаризированным распределениям Больцмана. Это позволяет сделать вывод, что область применимости столкновительной кинетической модели точечных стоков близка к области применимости теории Дебая-Гюккеля.

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Более подробно см. Филиппов А.В. ЖЭТФ, т.159(1), с.176-188 (2021)