The Stokes-Einstein (SE) relation between the self-diffusion and shear viscosity coefficients operates in sufficiently dense simple fluids not too far from the fluid-solid phase transition. By considering two simple model charged (Coulomb and Debye-Hückel or screened Coulomb) fluids as well as two simple model neutral fluids (Lennard-Jones and Coulomb) we demonstrate where exactly on the respective phase diagrams the SE relation holds. It appears that the reduced excess entropy $s_{\text{ex}}$ can be used as a suitable indicator of the validity of the SE relation. In all cases considered the onset of SE relation validity occurs at approximately $s_{\text{ex}} \lesssim -2$ [1]. In addition, we discuss the location of the crossover between the gas-like and liquid-like transport properties of simple fluids.