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Numerical Analysis of Anomalous Wave Processes at Quark-Hadron Phase Transition

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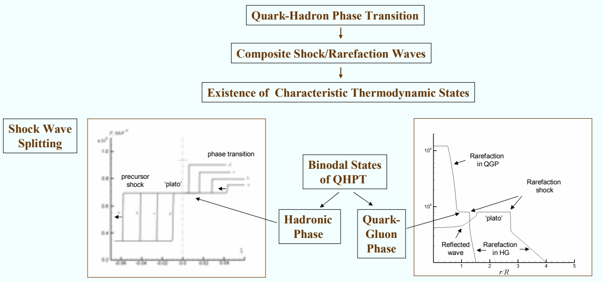
Abstract

The paper is devoted to the hydrodynamic description of the collective phenomena occurring at the collision of ultrarelativistic heavy ions and the expansion of quark-gluon plasma (QGP) cloud (so-called fireball). The equation of state of subhadronic matter describing the quark-hadronic phase transition has been calculated using the MIT-bag model variant (Cleymans J., 1986). It has been shown that the Taub adiabats passing through mixed phase have segments with an ambiguous representation of the shock discontinuity. The ambiguity is due to the implementation of the shock instability condition L<-1. Shocks belonging to such the segments split with formation of a composite compression wave (here two-wave structure). The thermodynamic condition $(\partial^2 p/\partial \tau^2)_S < 0$ can be performed in the transition region and the appearance of rarefaction shocks or composite rarefaction waves is expected. Such the behavior of shock and rarefaction waves is typical for the media with the phase transition of the first order. It has been found that for the EOS used the velocities of the precursor shock in hadronic phase and the shock wave of phase transition differ very slightly. Taking into account extremely short time of the interaction, the shock splitting may be masked by viscid and nonequilibrium effects. It makes very difficult to identify this phenomenon in collision events. It has been else found that the neutral stability condition is fulfilled only for shocks with the final state in the mixed phase. However, these shocks are unstable with respect to splitting and are not realized as unique wave. Thus, only the phase transition shocks being a part of the composite compression wave may be neutrally stable.

The collective phenomena in the expanding QGP cloud seems to be a marker of the reverse phase transition from the QGP to the hadronic state. The simulation of this process has been carried out in one- and two-dimensional formulations. In the latter case the central and peripheral collisions of nuclei were considered. The fireball expansion after the peripheral collision was modeled in the cylindrical and spherical coordinate systems. It has been found that in all cases the initial stage of the fireball expansion is characterized by the formation of the composite rarefaction wave including the plateau with practically zero gradients. In the one-dimensional case this plateau corresponds to the thermodynamic state on the binodal of the quark-hadron phase transition. Due to spatial expansion of the fireball in the two-dimensional formulations the plateau is the region of mixed states. In all cases the plateau exists during time ~1 R_0/c, where R_0 is the fireball radius, c is the velocity of light. A similar effect with the plateau formation and the same existance duration occurs after the reflection of the rarefaction wave from the centre. In this region the final transition to the hadronic phase is realized. Thus, the complete phase transition from the QGP to the hadronic phase lasts ~2 R_0/c. It is of interest that after the peripheral nuclei collision the fireball expands asymmetrically with the occurrence of so-called elliptic flows. According to existing ideas, it speaks about the collective nature of the behavior of particles in the quark-gluon plasma, manifesting itself as a continuous medium.

Introduction

Over the past three decades a large amount of experimental data has been accumulated (see [1]), showing that the hot QCD matter arising in the collision of ultrarelativistic heavy ions demonstrates a number of collective phenomena well and consistently described in terms of a nearly perfect (with low viscosity) relativistic hydrodynamics. In [2] a comparison of existing models for constructing the equation of state, which is required for the hydrodynamic description of such a substance, is given. The analysis contained in this paper clearly indicates the anomalous thermodynamic properties of QCD matter. Thus one can expect formation of the rarefaction shock waves in this substance. In addition, non-classical behavior both the shock waves of compression and the rarefaction shock waves is possible; this property is associated with the problem of their stability. The question of the stability of shock waves in subhadronic matter was considered in [3-10] . It has been suggested that the splitting of the shock wave, which leads to formation of the two-wave structure, can serve to identify the phase transition from hadronic matter to quark-gluon plasma [8]. In [10] the fundamental feasibility of neutral stability of shock waves was discussed. It should be noted that the necessary condition for a possible experimental detection of the composite compression wave is the big enough difference in velocity between the shock wave precursor in the hadronic matter and the shock wave of phase transition.



Equation of state and phase behavior

To describe the shock-wave processes in QCD matter, the equation of state, which describes the relationship between thermodynamic parameters adequately, is required. In the present paper the equation of state based on M.I.T.- bag model variant [11] is used. The model is based on the ideal gas description of both the hadronic and the quark gluon plasma phase.

The M.I.T.-bag model

HG : $P_h = P_\pi + P_N, \quad \varepsilon_h = \varepsilon_\pi + \varepsilon_N \qquad P_N(T,\mu) = \dfrac{2M^4}{3\pi^2}\int_0^1 \dfrac{u^4 du}{(1-u^2)^3}\left[f(u;T,\mu) + f(u;T,-\mu)\right]$ (1)

$\varepsilon_N(T,\mu) = \dfrac{2M^4}{\pi^2}\int_0^1 \dfrac{u^2 du}{(1-u^2)^3}\left[f(u;T,\mu) + f(u;T,-\mu)\right] \qquad n_h(T,\mu) = \dfrac{2M^3}{\pi^2}\int_0^1 \dfrac{u^2 du}{(1-u^2)^{5/2}}\left[f(u;T,\mu) - f(u;T,-\mu)\right]$

Mixed Phase

$P_h = P_p,$

$T_h = T_p,$

$\mu = 3\mu_q$

$P_\pi(T) = \dfrac{3m^2T^2}{2\pi^2}\sum_{k=1}^{\infty}\dfrac{K_2(km/T)}{k^2}, \quad \varepsilon_\pi(T) = 3P_\pi + \dfrac{3m^3T}{2\pi^2}\sum_{k=1}^{\infty}\dfrac{K_1(km/T)}{k}$ (2) $\quad f(u;T,\mu) = (1+\exp[(M/(1-u^2)^{1/2} - \mu)/T])^{-1}$ (3)

QGP : $P_p = \dfrac{37}{90}\pi^2T^4 + \mu_q^2T^2 + \dfrac{\mu_q^4}{2\pi^2} - B \qquad \varepsilon_p = 3P_p + 4B \qquad n_{Bp} = \dfrac{2}{3}\left(\mu_q T^2 + \dfrac{\mu_q^3}{\pi^2}\right)$ (4)

In the framework of this model the pressure, energy density and baryon number density of quark gluon plasma, which is considered as a gas of quarks and gluons with the number of flavors $N_f = 2$, number of colors $N_c = 3$ and the number of gluon color states $N_g = 8$. The pressure, energy density and baryon number density are given by (4), where μ_q is the baryonic chemical potential, which is supposed to be equal for u- and d- quarks. Hadronic phase is considered as an ideal relativistic gas of pions, nucleons and anti-nucleons. Contribution of the pion gas to the pressure, energy density as well as the pressure, energy density and baryon number density for gas of nucleons and antinucleons have been calculated in accordance with [11], see (1). The contribution of the pion gas is defined by the expressions (2), where K1 and K2 are the modified Bessel functions of the second kind, m is the pion mass. Pressure, energy density and baryon number density for the gas of nucleons and antinucleons can be expressed in terms of the integrals (3) (see [11]). The nucleon and pion mass were suggested to be M= 940 MeV and m=139.6 MeV, accordingly. Pressure and energy density of the hadronic phase contain contributions of pions and nucleons $P_h=P_\pi+P_N$ and $\varepsilon_h=\varepsilon_\pi+\varepsilon_N$. Taking into account that the nucleon consists of three quarks, the Gibbs conditions for the phase equilibrium gain the form (2). The dependence of the pressure on the temperature and baryon number density, P=P(n,T), which was calculated in accordance with (11), is shown in figure 1.

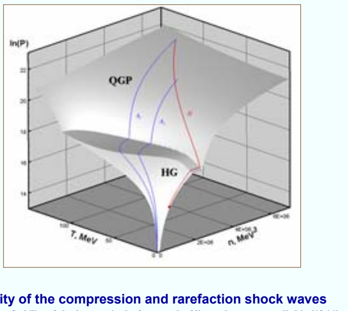


Figure 1 Dependence p(n,T) for the hadronic matter in the region of qurk-hadron phase transition calculated on the basis of M.I.T-bag model (Cleymans,Suhonen;1987) $-B^{1/4}$=153.8 MeV (H) is Hugoniot with phase transition A1, A2 are isentropes which meet the Hugoniot at states with temperatures 120 MeV and 75 MeV

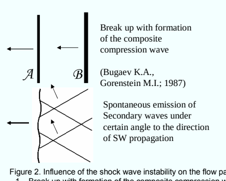


Figure 2. Influence of the shock wave instability on the flow pattern:
1 – Break up with formation of the composite compression wave (A-shock wave precursor, B- wave of the phase transition),
2- Spontaneous emission of waves under certain angle to the direction SW propagation

Stability of the compression and rarefaction shock waves

Stability of shock waves in the framework of linear theory was studied in [13,14] where the corresponding criteria have been obtained. The shock waves unstable according to the criteria of the linear theory are unstable with respect to break up with formation of the wave-split configuration of the outgoing waves. One can distinguish two different ways in which the shock wave instability can manifest itself in flow pattern of the ultrarelativistic nuclear collisions. The first one is related to the fact that in a certain range of parameters instead of a single shock wave the composite compression wave can be observed, which consists of several elements. This possibility is discussed in [7]. Another aspect is that under the condition of neutral stability (spontaneous sound emission), the secondary waves can take place, which propagate under certain angle to the direction of the shock wave. It should also be noted that in the presence of the first order phase transition during unloading of the shock-compressed material, the formation of combined rarefaction waves containing discontinuities can be observed.

Consider the condition of neutral stability condition as it follows from linear stability analysis [2]: $1 - v^2 - (v_0/v - 1)M\left(1 + M\left.\dfrac{\partial p}{\partial \varepsilon}\right|_n\right) < 0$ (5)

where v_0, v – is the pre-shock and post-shock 3-velocity in the shock wave rest frame, M=|v|/c_s is the post-shock Mach number, c_s – is the sound velocity defined as

$$\tilde{n}_s^2 = \left.\frac{\partial p}{\partial \varepsilon}\right|_{s/n} = \left.\frac{\partial p}{\partial \varepsilon}\right|_n + \frac{n}{p+\varepsilon}\left.\frac{\partial p}{\partial n}\right|_\varepsilon$$

The inequality (5) can be reformulated in the following form

$$\frac{n}{p}\left.\frac{\partial p}{\partial n}\right|_\varepsilon + \frac{\varepsilon}{p}\left.\frac{\partial p}{\partial \varepsilon}\right|_n - 1 + \frac{p_0}{p}\left(1 + \left.\frac{\partial p}{\partial \varepsilon}\right|_n\right) < 0 \qquad (6)$$

Here, all functions without subscript and derivatives correspond to the post-shock state. The application of this condition for the equation of state (1-4) has shown that the condition (5) is not satisfied in the region of the phase diagram, which corresponds to the quark-gluon plasma. Indeed, the relation between pressure and energy density in this case has the form (3) and the left-hand side of (6) is positive for any value of the initial pressure p_0. For the shock waves with the final state in the quark-gluon plasma, the shock waves considered as structureless discontinuity are stable (however, they can be unstable with respect to decay). Left side of (6) is also positive in the hadron phase , i.e. the shock waves with the hadronic post shock state are stable. Left side of (6) is negative in the two-phase region of the phase diagram that indicate possible neutral stability of shock waves with the final state in the two-phase region at a sufficiently high intensity of the shock wave. However, such shocks are unstable according to the criterion L<-1. They belong to the domain of ambiguous representation of shock wave discontinuity and break up.

The Taub shock adiabat [15] $n^2X^2 - n_0^2X_0^2 - (p - p_0)(X + X_0) = 0$

(where $X = (\varepsilon + p)/n^2$ is generalized specific volume) for considered equation of state is shown in figure 4 (right panel). The shock curve has the form characteristic for materials, which undergo a first-order phase transition [16]. It has a kink at the boundary of the two-phase region (points 1,3).

Conditions of shock wave instability (exponential growth of small perturbations) in linear theory are as follows [13]:

$$L \equiv m^2\left(\frac{\partial X}{\partial p}\right)_H < -1, \qquad (7) \qquad\qquad m^2\left(\frac{\partial X}{\partial p}\right)_H > \frac{1 + 2M + v_0 v}{1 - v_0 v}, \qquad (8)$$

Here $m = nv/(1-v^2)^{1/2}$ is the flux of the baryon number density through the SW front.

From geometrical consideration in (p, X)-plane it follows that provided (7) is fulfilled, the shock wave discontinuity can be represented by a system of wave elements (including shocks, isentropic waves and a contact discontinuity). Under this condition the initial value problem for relativistic hydrodynamic equations is ill-posed, because the infinitesimal smoothing of the shock front switches one solution to another. Another instability criterion (8) can not be fulfilled for considered equation of state (the example of the Hugoniot-Taub curve is shown in figure 5 right panel). Numerical solutions of the initial value problem for relativistic hydrodynamic equations with the shock wave as initial data have shown break up of the shock wave in the region of its ambiguous representation. Figure 5 (left panel) shows the solution of the problem with initial data that correspond to the shock- wave splitting in the region of ambiguous representation of the shock-wave discontinuity (the section of the Taub adiabat concluded between points 1 and 3). The post-shock state is characterized by temperatures 28.5 MeV (a), 29 MeV (b), 29.5 MeV (c), 30 MeV (d). The ordinate in the plot is the pressure, the abscissa is the similarity variable ξ = x/(ct). Difference between the speed of the first and second shock wave in the decay configuration is much smaller than the relativistic velocity of these waves relative to the matter. This means that the decay of the shock wave can be easily hidden by non-equilibrium and viscosity effects and it has not significant influence on the impact pattern. The isentrope crossing the region of the quark-hadron phase transition is shown in figure 6. The adiabat has two inflection points at the two-phase boundaries. Since the adiabat is not convex in (p,X)-plane, the composite rarefaction wave is formed instead of isentropic rarefaction wave in the problem of discontinuity break up. The composite wave includes isentropic rarefaction wave in the quark-gluon phase 0-1, the rarefaction shock (wave of phase transition) 1-3 and isentropic rarefaction 3-4. Point 3 is the point at which straight line drawn from point 1 is tangential to the adiabat.

The hydrodynamic description of the collective phenomena occurring at the collision of ultrarelativistic heavy ions and the expansion of quark-gluon plasma (QGP) cloud (so-called fireball) is considered. These high energy processes happen at extremely low background density of the baryon charge that imposes special demands on the numerical model used. The relativistic hydrodynamic codes permitting to simulate flows with regions with essentially lower density have been elaborated on the base of the HLLC method (Mignone, 2005). The equation of state of subhadronic matter describing the quark-hadronic phase transition has been calculated using the MIT-bag model variant (Cleymans J., 1986). It has been shown that the Taub adiabats passing through mixed phase have segments with an ambiguous representation of the shock discontinuity. The ambiguity is due to the implementation of the shock instability condition L<-1. Shocks belonging to such the segments split with formation of a composite compression wave (here two-wave structure). The thermodynamic condition $(\partial^2 p/\partial \tau^2)_S < 0$ can be performed in the transition region and the appearance of rarefaction shocks or composite rarefaction waves is expected. Such the behavior of shock and rarefaction waves is typical for the media with the phase transition of the first order. The predictions of the theoretical analysis are checked in calculations. Besides, the fireball expansion is simulated in one- and two-dimensional formulations. In the latter case the central and peripheral collisions of nuclei are considered. The fireball expansion after the peripheral collision is modeled in the cylindrical and spherical coordinate systems.

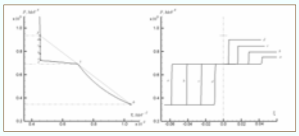


Figure 3 Dependence p(n,T) for the hadronic matter in the region of qurk-hadron phase transition calculated on the basis of M.I.T-bag model (Cleymans et al.;1986) - $B^{1/4}$=153.8 MeV

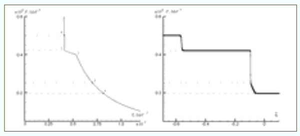
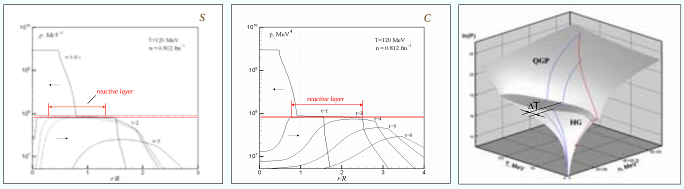


Figure 4. Left panel: isentrope in the region of quark-hadron transition; right panel: composite rarefaction wave which includes rarefaction shock (the segment 1-3), numerical calculation.

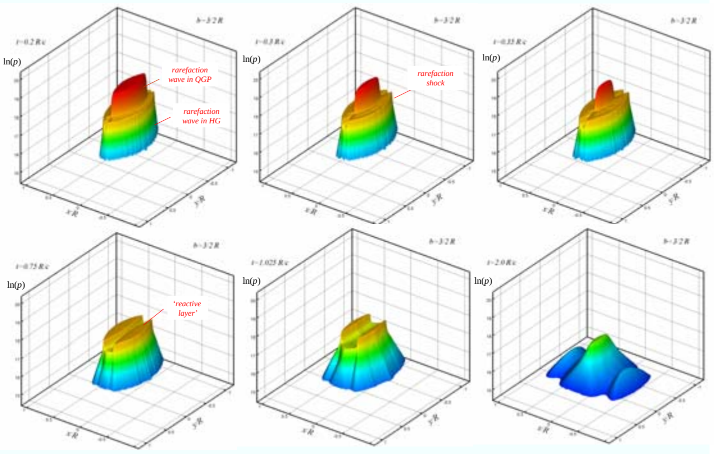
The problem is formulated under the following assumptions: local thermodynamic equilibrium, low initial Mach numbers (collective velocity is neglected at t=0), uniform initial distributions. Equations of relativistic hydrodynamics have been solved numerically. The equations of relativistic fluid dynamics include the energy-momentum conservation equation $\nabla_\beta T^{\alpha\beta} = 0,$ where ∇_β is covariant derivative in space-time; energy momentum tensor is given by $T^{\alpha\beta} = (\varepsilon + p)u^\alpha u^\beta - g^{\alpha\beta}p,$ ε and p are the energy density and pressure defined in the rest coordinate system; $g^{\alpha\beta} = diag(1,-1,-1,-1),$ is the metric tensor (for special theory of relativity) ;
The system is complemented by the conservation law for the baryon charge $\nabla_\beta(nu^\beta) = 0,$ and u is the 4-velocity vector.

Cases of cylindrical and spherical symmetry



The calculations have shown formation of 'plato'-like region that is better seen in pressure distribution. This region (we call it 'reactive layer') corresponds to mixed phase. The temperature distribution in this region is relatively small because of high specific heat of the mixed phase.

Case of asymmetry caused by peripheral character of collision



In the case of asymmetry caused by peripheral character of collision the effect of low gradients of the thermodynamic parameters has been also observed. The pressure distributions, which correspond to the succesive points in time are presented in the figure.

Conclusion

The equation of state of subhadronic matter describing the quark-hadronic phase transition has been built using the variant of the MIT-bag model [11]. The theoretical analysis of the EOS constructed has been carried out to check the fulfillment of the criteria of the instability and neutral stability of relativistic plane shock waves [13, 14]. It has been shown that the Taub adiabats passing through mixed phase have the segments, in which shock wave discontinuity has an ambiguous representation. The ambiguity is due to the implementation of the shock instability condition L<-1. In such segments the splitting of the initial shock with formation of composite compression wave takes place. The appearance of composite rarefaction waves, which include rarefaction shocks is expected. This behavior is quite common for the media with the phase transitions of the first order and was predicted for QHPT and analyzed in a number of works (see [6-8]). In the present work it is shown that for the considered equation of state the speed of the precursor shock in hadronic phase and the wave of phase transition have small relative velocities. This means that such wave splitting may be masked by viscosity and non-equilibrium effects and hardly has any observable consequences in collision events. It is found that for the MIT-bag equation of state the neutral stability condition is fulfilled only for the shock wave discontinuities with the final state in the mixed phase. These shocks are unstable with respect to splitting. Thus only the waves of phase transition being a part of the composite wave may violate Kontorovich stability condition [13]. The method of solution relativistic hydrodynamic equations closed by the subhadronic matter equation of state is developed to predict the interaction of such waves with non-uniformities of baryonic number density field.

The results of calculations have completely confirmed the predictions of the theoretical analysis. Besides, it has been found that for the EOS used the velocities of the precursor shock in hadronic phase and the shock wave of phase transition differ very slightly. Taking into account extremely short time of the interaction, the shock splitting may be masked by viscid and nonequilibrium effects. It makes very difficult to identify this phenomenon in collision events. It has been else found that the neutral stability condition is fulfilled only for shocks with the final state in the mixed phase. However, these shocks are unstable with respect to splitting and are not realized as unique wave. Thus, only the phase transition shocks being a part of the composite compression wave may be neutrally stable.

On the other hand, the collective phenomena in the expanding QGP cloud seems to be a marker of the reverse phase transition from the QGP to the hadronic state. The simulation of this process has been carried out in one- and two-dimensional formulations. In the latter case the central and peripheral collisions of nuclei were considered. The fireball expansion after the peripheral collision was modeled in the cylindrical and spherical coordinate systems. It has been found that in all cases the initial stage of the fireball expansion is characterized by the formation of the composite rarefaction wave including the plateau with practically zero gradients. In the one-dimensional case this plateau corresponds to the thermodynamic state on the binodal of the quark-hadron phase transition. Due to spatial expansion of the fireball in the two-dimensional formulations the plateau is the region of mixed states. In all cases the plateau exists during time ~1 R0/c, where R0 is the fireball radius, c is the velocity of light. A similar effect with the plateau formation and the same existance duration occurs after the reflection of the rarefaction wave from the centre. In this region the final transition to the hadronic phase is realized. Thus, the complete phase transition from the QGP to the hadronic phase lasts ~2 R0/c.

It is of interest that after the peripheral nuclei collision the fireball expands asymmetrically with the occurrence of so-called elliptic flows. According to existing ideas, it speaks about the collective nature of the behavior of particles in the quark-gluon plasma, manifesting itself as a continuous medium.

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